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PHYSICS

CLASS-XI

PART-I

STRICTLY ACCORDING TO THE NEW SYLLABUS

INDIA'S FIRST SMART BOOK

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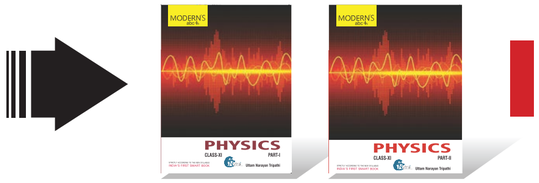


IMPORTANT FORMULAE

CHAPTER 2

Units and Measurement

- Angle $= \dfrac{\text{arc}}{\text{radius}}$

  or $\theta = \dfrac{a}{D}$,

  Here a is the length of arc and D is the radius.
- $1\ \text{AU} = 1.496 \times 10^{11}\ \text{m}$
- $1\ \text{Light year} = 9.46 \times 10^{15}\ \text{m}$

Errors in measurement

Let $x_1, x_2, x_3 \ldots\ldots x_n$ be the measured values of a given physical quantity x then

- $x_{\text{mean}} = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$

$$x_{\text{mean}} = \frac{\sum_{i=1}^{i=n} x_i}{n}$$

- Mean absolute error,

$$\Delta x_{\text{mean}} = \frac{|\Delta x_1| + |\Delta x_2| + \ldots + |\Delta x_n|}{n} = \frac{\sum_{i=1}^{i=n} |\Delta x_i|}{n}$$

where $|\Delta x_1| = |x_1 - x_{\text{mean}}|$, $|\Delta x_2| = |x_2 - x_{\text{mean}}|$ and so on.

- Fractional error/Relative error $= \dfrac{\Delta x_{\text{mean}}}{x_{\text{mean}}}$
- Percentage error $= \dfrac{\Delta x_{\text{mean}}}{x_{\text{mean}}} \times 100$

Combination of Errors

- If $Z = A + B$ or $Z = A - B$

  then $\Delta Z = \pm\Delta A \pm \Delta B$
- If $Z = AB$

$$\frac{\Delta Z}{Z} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

- If $Z = \dfrac{A}{B}$

$$\frac{\Delta Z}{Z} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

- For $Z = A^n$

$$\frac{\Delta Z}{Z} = \pm n\frac{\Delta A}{A}$$

- If $X = A^m B^n / C^r$, then

$$\frac{\Delta X}{X} = \left[\pm m\left(\frac{\Delta A}{A}\right) + n\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)\right]$$

Dimensional Analysis

Principle of homogenity of dimensions is obeyed if Dimensions of M, L, T on L.H.S. = Dimensions of M, L, T on R.H.S.

Conversion of unit of one system to another

For conversion of one system of units into another we use

$$n_2 = \frac{n_1 u_1}{u_2}$$

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

Here u_1 and u_2 represent two different system of units.

n_1 is the number for system of unit u_1.

n_2 is the number for system of unit u_2.

CHAPTER 3

Motion in a Straight Line

Average speed = $\dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

Average velocity = $\dfrac{\text{Total displacement}}{\text{Total time}}$

Instantaneous Velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{d\vec{x}}{dt}$$

Instantaneous Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = v\frac{dv}{dx}$$

Equations for Uniformly Accelerated Motion

(when a = constant)

$$v = u + at$$

$$x = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2ax$$

Displacement in n^{th} second

$$S^{n\text{th}} = u + \frac{a}{2}(2n - 1)$$

Motion Under Gravity

When an object is thrown vertically upwards with initial speed u.

- Maximum height attained $H = \dfrac{u^2}{2g}$
- Time taken to reach the topmost point $t = \dfrac{u}{g}$
- Time taken to return back to the initial point $t = \dfrac{u}{g}$
- Total time of flight $T = \dfrac{2u}{g}$
- Time taken by an object to reach the ground when released from a rest from height h $t = \sqrt{\dfrac{2h}{g}}$
- An object thrown upwards with initial velocity u, from the top of a building of height H.

$$t = \frac{u}{g} \pm \sqrt{\frac{u^2}{g^2} + \frac{2H}{g}}$$

(time taken by the object to reach ground).

Relative Velocity

Velocity of body B w.r.t A.

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

Velocity of body A w.r.t B.

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

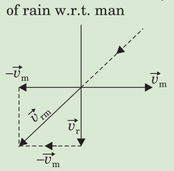
$\vec{v}_{AB} = -\vec{v}_{BA}$

- **Relative velocity of rain and man**

$\vec{v}_r$ = velocity of rain w.r.t. ground

$\vec{v}_m$ = velocity of man w.r.t. ground

$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$ = velocity of rain w.r.t. man



CHAPTER 4

Motion in a Plane

The resultant of two vectors inclined at an angle θ is

$$C = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Rectangular Components of a Vector

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$A_x = A\cos\theta, A_y = A\sin\theta$

- Unit vector of $\vec{A}$, $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

Dot product of two vectors: (scalar product)

- $\vec{A}\cdot\vec{B} = AB\cos\theta$
- If $\vec{A} \perp \vec{B}$ then $\vec{A}\cdot\vec{B} = 0$.
- $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{z}\cdot\hat{z} = 1$
- $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

Vector product of two vectors

- $\vec{A}\times\vec{B} = (AB\sin\theta)\,\hat{n}$

  $\hat{n}$ represents the direction of vector product.
- If $\vec{A} \parallel \vec{B}$ then $\vec{A}\times\vec{B} = 0$.
- $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$
- $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$
- Area of a parallelogram $= |\vec{A}\times\vec{B}|$

Motion in a plane Projectile Motion

- Maximum Height $H = \dfrac{u^2\sin^2\theta}{2g}$
- Time of Flight $T = \dfrac{2u\sin\theta}{g}$
- Horizontal Range $R = \dfrac{u^2\sin 2\theta}{g}$
- $R = 4H\cot\theta$

Equation of Trajectory in Projectile

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

- Equation of trajectory of Horizontal projectile $y = \left(\dfrac{g}{2u^2}\right)x^2$
- Time of flight for projectile $T = \sqrt{\dfrac{2h}{g}}$
- Horizontal range (R) $R = u\sqrt{\dfrac{2h}{g}}$
- At any instant, $v_x = u$, $v_y = gt$

  Instantaneous velocity (v) $v = \sqrt{u^2 + g^2t^2}$

$$\beta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{gt}{u}\right)$$

Circular motion Instantaneous angular velocity and angular acceleration

$$\omega = \frac{d\theta}{dt}, v = \frac{d\omega}{dt}, \alpha = \frac{d\omega}{dt}$$

For non-uniform circular motion

- Tangential acceleration $a_T = r\alpha$
- Centripetal acceleration (or radial acceleration) (a_c)

$$a_c = r\omega^2 = \frac{v^2}{r}$$

- Net acceleration

$$\vec{a} = \vec{a}_r + \vec{a}_C$$

$$a_{\text{net}} = \sqrt{a_T^2 + a_C^2}$$

- Relation between linear speed and angular speed: $v = r\omega$

CHAPTER 5

Laws of Motion

Linear momentum, $\vec{p} = m\vec{v}$, $\vec{F}_{\text{ext}} = \dfrac{\Delta\vec{p}}{\Delta t}$

$$\vec{F}_{\text{ext}} = m\vec{a}$$

Units of Force

$1\ \text{kgf} = 9.8\ \text{kg m s}^{-2} = 9.8\ \text{N}$.

- **Impulse of force,**

$$\vec{I} = \vec{F}\,\Delta t = \vec{p}_2 - \vec{p}_1$$

Conservation of linear momentum

- For an isolated system if $\vec{F}_{\text{ext.}} = 0$ then

$$\Rightarrow \vec{p}_1 = \vec{p}_2$$

- Thrust force of a Rocket,

$$F = -u\left(\frac{dm}{dt}\right)$$

$-\dfrac{dm}{dt}$ = Rate at which fuel is burnt.

Contact Force: (Normal Reaction and Friction)

- **Kinetic friction is,**

  $f_k = \mu_k N$

  μ_k = coefficient of kinetic friction
- **Angle of repose or angle of sliding**

  $\theta_0 = \tan^{-1}(\mu)$
- $\mu = \tan\theta$

  Here, θ is the angle of friction.

Centrifugal Force

$$f = mR\omega^2 = mR(2\pi\nu)^2 = 4\pi^2 mR\nu^2$$

(also known as Pseudo force)

Motion in a Lift

Apparent reading (R) of weighing machine in a lift

- (a) When the lift moves upwards with acceleration a: $R = m(g + a)$
- (b) When the lift moves downwards with acceleration a: $R = m(g - a)$
- (c) When the lift falls freely: $R = 0$
- (d) When the lift is at rest or $a = 0$: $R = mg$

Motion along a rough inclined plane

When a body moves down, its acceleration is

$a = g(\sin\theta - \mu\cos\theta)$

When a body moves up, its acceleration,

$a = g(\sin\theta + \mu\cos\theta)$

Motion of a vehicle on a level road:

For a vehicle taking circular turn of radius R on a level road, Maximum permissible velocity is

- (a) $v = \sqrt{\mu gR}$, when the speed is uniform
- (b) $v \le [(\mu^2 g^2 - a^2)R^2]^{1/4}$, when the speed is non-uniform

Banking of roads:

For a vehicle taking circular turn of radius R on a banked road,

- (a) Maximum permissible velocity of vehicle, in the absence of friction is $v_0 = \sqrt{Rg\tan\theta}$
- (b) In the presence of friction, maximum safe velocity is

$$v_{\max} = v_0\sqrt{\frac{1 + \mu\cot\theta}{1 - \mu\tan\theta}}$$

Bending of a cyclist:

In order to take a circular turn of radius R safely, a cyclist must bend himself at an angle θ such that

$$\tan\theta = \frac{v^2}{rg}$$

CHAPTER 6

Work, Energy and Power

- Work done $\Delta W = \vec{F}.\Delta\vec{x} = F\Delta x\cos\theta$.
- Work done by a variable force $\vec{F}$ for displacing an object from $x = x_1$ to $x = x_2$ is: $W = \int_{x_1}^{x_2} \vec{F}.d\vec{x}$

Work done by gravitational force

$\Delta W_{mg}^{A-B} = -mgh$

Work done by Spring Force

$W_s = -\dfrac{1}{2}Kx^2$

- If a conservative force F_c then $F_c = -\dfrac{\Delta U}{\Delta x}$

Motion in a Vertical Circle

- Tension at any position at an angular displacement θ $T = \dfrac{mv^2}{r} + mg\cos\theta$
- Minimum velocity at highest point for looping the loop $v = \sqrt{gr}$
- Minimum velocity at lowest point for looping the loop $v = \sqrt{5gr}$
- For $\theta = 90^\circ$ or when the string is horizontal then $v = \sqrt{3gr}$

Power

- Instantaneous power, $P = \dfrac{dW}{dt} = \vec{F}.\vec{v}$
- 1 hp = 746 watt

Various forms of energy

- K.E. $= \dfrac{1}{2}m\vec{v}.\vec{v} = \dfrac{1}{2}mv^2$
- Change in potential energy (ΔU) is negative of the work done by the conservative forces, that is $\Delta U = -W$.
- Gravitational potential energy of an object of mass m, at a height of h above the reference level, $\Delta U = mgh$
- Potential energy of a spring when it is compressed or stretched by a length x, $\Delta U = \dfrac{1}{2}kx^2$
- Work-energy theorem: $W = K_2 - K_1$
- Loss of energy in Head-on Collision between Inelastic Bodies

$$\Delta K_{\text{loss}} = \frac{m_1 m_2 (u_1 - u_2)^2}{2(m_1 + m_2)}$$

Collisions

- Two objects having masses m_1 and m_2 moving with velocities u_1 and u_2, respectively, suffer head-on collision then their final velocities are v_1 and v_2, respectively:

- **Perfectly elastic collision**

$$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \frac{2m_2}{(m_1 + m_2)}u_2$$

- **Purely inelastic collision:**

$$v_1 = v_2 = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

- **Coefficient of restitution**

$$-e = \frac{v_2 - v_1}{u_2 - u_1}$$

$e = 1$ for elastic bodies.

$e = 0$ for inelastic bodies.

$0 < e < 1$ for real bodies.

CHAPTER 7

Systems of Particles and Rotational Motion

Centre of mass

- For two particle system, the position vector of centre of mass, $\vec{r} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$
- For continuous mass $R_{cm} = \dfrac{\int r\,dm}{\int dm}$
- The coordinates of centre of mass are $X_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$

If centre of mass is at origin then,

$X_{cm} = Y_{cm} = Z_{cm} = 0$

$\Rightarrow m_1x_1 + m_2x_2 = 0$

$\Rightarrow \boxed{x_2 = -\dfrac{m_1x_1}{m_2}}$

- **Velocity of centre of mass of two particle system,**

$$\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

Rotation of rigid body

- Angular velocity, $\omega_{av} = \dfrac{\Delta\theta}{\Delta t}$
- Angular acceleration, $\alpha = \dfrac{d\omega}{dt}$
- The equations of kinematics for rigid body rotation:

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$

$\omega^2 = {\omega_0}^2 + 2\alpha\theta$

- **Torque**

$\vec{\tau} = \vec{r}\times\vec{F} = rF\sin\theta\hat{n}$

$\tau = I\alpha = F\cdot R$

Moment of Inertia

$$I = \sum_{i=1}^{i=n}(m_i r_i^2) = \int r^2\,dm$$

- Angular momentum, $L = I\omega = mvr$
- Angular Impulse $= I(\omega - \omega_0)$

Law of conservation of Angular momentum

$\vec{\tau} = \dfrac{d\vec{L}}{dt}$,

If $\tau = 0$, $\vec{L}$ = constant

$\therefore \boxed{I_1\omega_1 = I_2\omega_2 = \text{constant}}$

As $\omega = \dfrac{2\pi}{T}$

$\therefore \boxed{\dfrac{I_1}{I_2} = \dfrac{T_1}{T_2}}$

Rolling Motion

- Rotational kinetic energy $= \dfrac{1}{2}I\omega^2$
- Translational kinetic energy $= \dfrac{1}{2}mv^2$
- Total kinetic energy in rolling $= \dfrac{1}{2}I\omega^2 + \dfrac{1}{2}mv^2$
- **Acceleration of a cylinder rolling without slipping of an inclined plane,**

$$a = \frac{mg\sin\theta}{m + I/R^2}$$

- Moment of Inertia of a solid cylinder about its axis of symmetry $I = \dfrac{1}{2}mR^2$

$\therefore$ we get $a = \dfrac{2}{3}g\sin\theta$

Rolling friction

$F = \dfrac{Ia}{r^2} \Rightarrow F = \dfrac{1}{3}mg\sin\theta$

- Velocity of a body rolling down an inclined plane without slipping

$$v = \sqrt{\frac{2gh}{1 + k^2/R^2}}$$

k = radius of gyratition

CHAPTER 8

Gravitation

- **Newton's law of Gravitation**

$$F = \frac{Gm_1m_2}{r^2}$$

- **Gravitational field intensity,**

$$\vec{E} = \frac{\vec{F}}{m}$$

- **Gravitational potential,**

$$V = \frac{-GM}{r}$$

(r is the distance of the given point from the centre of earth)

- **Acceleration Due to Gravity**

$$\boxed{g = \frac{GM}{R^2} = G\frac{4}{3}\pi R\rho}$$

Variation of Acceleration due to Gravity

- **Effect of height**

$$g' \approx g\left(1 - \frac{2h}{R}\right) \quad R >>> h$$

- **Effect of depth**

$$\boxed{g' = g\left(1 - \frac{d}{R}\right)} \quad d <<< R$$

When $d = R$, then the acceleration due to gravity becomes zero

Effect of Rotation of Earth

$\boxed{g' = g - R\omega^2\cos\lambda}$, where R = Radius of earth.

- At Equator: $\lambda = 0^\circ$ $\therefore$ $\boxed{g' = g - R\omega^2}$
- At Pole: $\lambda = 90^\circ$ $\therefore$ $\boxed{g' = g}$

Gravitational Potential Energy of system

$$\boxed{U = -\frac{Gm_1m_2}{r}}$$

Total Energy of Planet-Satellite System

T.E. = K.E. + P.E.

$$\text{T.E.} = -\frac{GMm}{2r} = -\text{B.E.}$$

- Gravitational field intensity (E) due to a point mass M at a distance r is: $\vec{E} = \dfrac{GM}{r^2}\hat{r}$
- **Escape velocity,** $v_e = \sqrt{\dfrac{2GM}{R}} = \sqrt{2gR}$
- **Orbital velocity,** $v_0 = \sqrt{\dfrac{GM}{R + h}} = \sqrt{gR}$
- Period of revolution of a satellite $= \dfrac{2\pi r}{v_0}$

$$= 2\pi\sqrt{\frac{(R + h)^3}{GM}}$$

- **Time period of geostationary satellite,** $T = 24$ hour, $h = 36000$ km
- **Kepler's 2nd law:** $\dfrac{dA}{dt} = \dfrac{vr}{2} = \text{constant}$
- **Kepler's 3rd law:** $T^2 = ka^3$ $\because$ a is length of semi-major axis.

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PHYSICS CLASS-XI PART-I

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PART-I

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Preface & Acknowledgement

This book, the revised edition of Modern's abc + of Physics for class XI, has been a journey of dedication and an intellectual endeavour. In this book, it has been my sincere effort to kindle a sense of exploration with the concepts in students and nudge them towards self-study.

I have used my experience in teaching to design the book such that it gives a classroom feeling to students: as if the concepts are being explained by their teacher. I have explained the concepts in clear, simple and precise language. This book is a must-have for class XI students irrespective of their aspirations: Annual exams, competitive exams like JEE Main, JEE Advanced and NEET or any other exams for engineering entrance, or medical entrance.

Key Features of the Book:

- Spectrum of contents comprises well-connected topics with proper flow of precise information.
- This book has been arranged in a very systematic and interesting manner. Along with detailed explanation of concepts, the theory part also contains components such as Remember, Key Note, Watch Out, Learning & More, Competition Edge and Captivate Your Curiosity. These components help students develop a deeper understanding of the subject and enhance their learning.
- Plenty of solved examples and practice problems have been provided topic wise to help students check their command over concepts.
- A summary section has been provided at the end of the theory section. It has been arranged systematically into three categories: Key Terms and Laws, Facts and Relations. These components help students revise all the basic concepts and the formulae simultaneously.
- Tough and Tricky problems and questions on Higher Order Thinking Skills (HOTS) have been provided in every chapter. These thought-provoking problems are application-based questions. Practising these questions after finishing the chapter would make one ready for competitive exams.
- Every chapter has detailed solutions of NCERT and NCERT Exemplar.
- Revision exercises along with hints and solutions at the end of each chapter help check retention of the concepts learnt in the chapter. Very Short Answer questions, Fill in the Blanks, Short Answer questions, Long Answer questions and Numerical Problems.
- Objective questions of various types along with previous years' questions of AIPMT/NEET, AIEEE/JEE Main (online and offline), IIT JEE/JEE Advanced of previous years have been incorporated in Competition file. Students can enhance their understanding and application of concepts as they go through the detailed solutions. I have also provided tips and tricks to help students perform and score better in entrance exams.

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I would like to express my sincere gratitude to my mother. She is my first teacher. She helped me grow as a son, student and teacher, and now I have authored this book. I have been teaching since 1998; I have been learning a lot, too, while teaching, which has only augmented my Bachelor's Degree in Technology in Mechanical Engineering from IIT, Kanpur. I have tried to incorporate my understanding of the challenges students face with Physics into this book. I express my deep regards for the management of MBD Group, for always encouraging me and taking keen interest in this project. My special thanks to Mr Manik Juneja Director - Content & Production, who had faith in me and also the patience to make this project successful. Special thanks to Mr Debasish Subudhi and Mr B.S. Rawat for their consistent support. I would also like to thank Mr Ankit Pasricha and my team who supported immensely in the production of this book. I'd like to express deep gratitude to Mr Vishal Tanwar and Mr Jabi John who introduced me to MBD Group.

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We welcome suggestions and comments from teachers and students; their valuable suggestions will be analysed and incorporated in the next edition of the book.

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Syllabus

PHYSICS

CLASS XI (THEORY)

Time: 3 hrs. **Max Marks: 70**

		No. of Periods	Marks
Unit–I	**PHYSICAL WORLD AND MEASUREMENT**		
1.	Physical World	10	
2.	Units and Measurements		
Unit–II	**KINEMATICS**		23
3.	Motion in a Straight Line	24	
4.	Motion in a Plane		
Unit–III	**LAWS OF MOTION**		
5.	Laws of Motion	14	
Unit–IV	**WORK, ENERGY AND POWER**		
6.	Work, Energy and Power	12	
Unit–V	**MOTION OF SYSTEM OF PARTICLES AND RIGID BODY**		17
7.	System of Particles and Rotational Motion	18	
Unit–VI	**GRAVITATION**		
8.	Gravitation	12	
Unit–VII	**PROPERTIES OF BULK MATTER**		
9.	Mechanical Properties of Solids	24	
10.	Mechanical Properties of Fluids		
11.	Thermal Properties of Matter		
Unit–VIII	**THERMODYNAMICS**		20
12.	Thermodynamics	12	
Unit–IX	**BEHAVIOUR OF PERFECT GASES AND KINETIC THEORY OF GASES**		
13.	Kinetic Theory	08	
Unit–X	**OSCILLATIONS AND WAVES**		
14.	Oscillations	26	10
15.	Waves		
Total		**160**	**70**

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Unit–I	**PHYSICAL WORLD AND MEASUREMENT**	**10 Periods**
1.	**Physical World:** Physics-scope and excitement; nature of physical laws; Physics, technology and society.	
2.	**Units and Measurements:** Need for measurement: Units of measurement; systems of units; SI units, fundamental and derived units. Length, mass and time measurements; accuracy and precision of measuring instruments; errors in measurement; significant figures. Dimensions of physical quantities, dimensional analysis and its applications.	
Unit–II	**KINEMATICS**	**24 Periods**
3.	**Motion in a Straight Line:** Frame of reference, Motion in a straight line: Position-time graph, speed and velocity. Elementary concepts of differentiation and integration for describing motion, uniform and non-uniform motion, average speed and instantaneous velocity, uniformly accelerated motion, velocity - time and position-time graphs. Relations for uniformly accelerated motion (graphical treatment).	
4.	**Motion in a Plane:** Scalar and vector quantities; position and displacement vectors, general vectors and their notations; equality of vectors, multiplication of vectors by a real number; addition and subtraction of vectors, relative velocity, Unit vector; resolution of a vector in a plane, rectangular components, Scalar and Vector product of vectors. Motion in a plane, cases of uniform velocity and uniform acceleration-projectile motion, uniform circular motion.	
Unit–III	**LAWS OF MOTION**	**14 Periods**
5.	**Laws of Motion:** Intuitive concept of force, Inertia, Newton's first law of motion; momentum and Newton's second law of motion; impulse; Newton's third law of motion. Law of conservation of linear momentum and its applications. Equilibrium of concurrent forces, Static and kinetic friction, laws of friction, rolling friction, lubrication. Dynamics of uniform circular motion: Centripetal force, examples of circular motion (vehicle on a level circular road, vehicle on a banked road).	
Unit–IV	**WORK, ENERGY AND POWER**	**12 Periods**
6.	**Work, Engery and Power:** Work done by a constant force and a variable force; kinetic energy, work-energy theorem, power. Notion of potential energy, potential energy of a spring, conservative forces: conservation of mechanical energy (kinetic and potential energies); non-conservative forces: motion in a vertical circle; elastic and inelastic collisions in one and two dimensions.	
Unit–V	**MOTION OF SYSTEM OF PARTICLES AND RIGID BODY**	**18 Periods**
7.	**System of Particles and Rotational Motion:** Centre of mass of a two-particle system, momentum conservation and centre of mass motion. Centre of mass of a rigid body; centre of mass of a uniform rod. Moment of a force, torque, angular momentum, law of conservation of angular momentum and its applications. Equilibrium of rigid bodies, rigid body rotation and equations of rotational motion, comparison of linear and rotational motions. Moment of inertia, radius of gyration, values of moments of inertia for simple geometrical objects (no derivation). Statement of parallel and perpendicular axes theorems and their applications.	

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Unit–VI	**GRAVITATION**	**12 Periods**
8.	**Gravitation:** Kepler's laws of planetary motion, universal law of gravitation. Acceleration due to gravity and its variation with altitude and depth. Gravitational potential energy and gravitational potential, escape velocity, orbital velocity of a satellite, Geo-stationary satellites.	
Unit–VII	**PROPERTIES OF BULK MATTER**	**24 Periods**
9.	**Mechanical Properties of Solids:** Elastic behaviour, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear modulus of rigidity, Poisson's ratio; elastic energy.	
10.	**Mechanical Properties of Fluids:** Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes), effect of gravity on fluid pressure. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its applications. Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise.	
11.	**Thermal Properties of Matter:** Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity; Cp, Cv - calorimetry; change of state - latent heat capacity. Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wein's displacement Law, Stefan's law, Greenhouse effect.	
Unit–VIII	**THERMODYNAMICS**	**12 Periods**
12.	**Thermodynamics:** Thermal equilibrium and definition of temperature (zeroth law of thermodynamics), heat, work and internal energy. First law of thermodynamics, isothermal and adiabatic processes. Second law of thermodynamics: reversible and irreversible processes, Heat engine and refrigerator.	
Unit–IX	**BEHAVIOUR OF PERFECT GASES AND KINETIC THEORY OF GASES**	**08 Periods**
13.	**Kinetic Theory:** Equation of state of a perfect gas, work done in compressing a gas. Kinetic theory of gases - assumptions, concept of pressure. Kinetic interpretation of temperature; rms speed of gas molecules; degrees of freedom, law of equi-partition of energy (statement only) and application to specific heat capacities of gases; concept of mean free path, Avogadro's number.	
Unit–X	**OSCILLATIONS AND WAVES**	**26 Periods**
14.	**Oscillations:** Periodic motion - time period, frequency, displacement as a function of time, periodic functions. Simple harmonic motion (S.H.M) and its equation; phase; oscillations of a loaded spring-restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum derivation of expression for its time period. Free, forced and damped oscillations (qualitative ideas only), resonance.	
15.	**Waves:** Wave motion– Transverse and longitudinal waves, speed of travelling wave, displacement relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, Beats, Doppler effect.	

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Some Physical Constants

S. No.	Quantity	Symbol	Value
1.	Acceleration due to gravity on surface of Earth	g	$9.8\ m\ s^{-2}$
2.	Average distance of earth from Sun	d_{e-s}	$1.5 \times 10^{11}\ m$
3.	Average distance of Moon from Earth	d_{m-e}	$3.8 \times 10^{8}\ m$
4.	Avogadro's number	N	$6.02 \times 10^{23}\ mol^{-1}$
5.	Absolute zero of temperature	0 K	– 273.15 °C
6.	Atmospheric pressure on surface of Earth	P	$1.013 \times 10^{5}\ N\ m^{-2}$
7.	Atomic mass unit	amu or u	$1.66 \times 10^{-27}\ kg = 931\ MeV/C^2$
8.	Boltzmann's constant	k_B	$1.38 \times 10^{-23}\ J\ K^{-1}$
9.	Bohr's radius of hydrogen atom	a_0	$5.3 \times 10^{-11}\ m = 0.53\ Å$
10.	Bohr magneton	μ_B	$9.27 \times 10^{-24}\ A\ m^2$
11.	Charge on electron	$-e$	$-1.6 \times 10^{-19}\ C$
12.	Charge on proton	$+e$	$+1.6 \times 10^{-19}\ C$
13.	Coulomb's constant	$1/4\pi\varepsilon_0$	$9.0 \times 10^{9}\ Nm^2\ C^{-2}$
14.	Dielectric constant of vacuum	K	1
15.	Dielectric constant of air	K_{air}	1.005
16.	Equatorial radius of Earth	R_e	$6.378 \times 10^{6}\ m$
17.	Mass of Sun	M_s	$2.0 \times 10^{30}\ kg$
18.	Molar volume	V	$22.4 \times 10^{-3}\ m^3\ mol^{-1}$
19.	Mass of Earth	M_e	$5.98 \times 10^{24}\ kg$
20.	Mass of Moon	M_m	$7.347 \times 10^{22}\ kg$
21.	Nuclear magneton	μ_N	$5.05 \times 10^{-27}\ A\ m^2$
22.	Permittivity of free space	ε_0	$8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$
23.	Polar radius of Earth	R_p	$6.357 \times 10^{6}\ m$
24.	Permeability of free space	μ_0	$4\pi \times 10^{-7}\ T\ m\ A^{-1}$
25.	Planck's constant	h	$6.62 \times 10^{-34}\ Js$
26.	Rest mass of electron	m_e	$9.11 \times 10^{-31}\ kg$
27.	Rest mass of proton	m_p	$1.672 \times 10^{-27}\ kg$
28.	Rest mass of neutron	m_n	$1.674 \times 10^{-27}\ kg$
29.	Radius of Sun	R_s	$6.95 \times 10^{8}\ m$
30.	Rydberg's constant	R_H	$1.097 \times 10^{7}\ m^{-1}$
31.	Specific charge (Charge to mass ratio) of electron	e/m_e	$1.7588 \times 10^{11}\ C\ kg^{-1}$
32.	Solar constant	S	$1.366 \times 10^{3}\ W\ m^{-2}$
33.	Speed of light in vacuum	c	$3.0 \times 10^{8}\ m\ s^{-1}$
34.	Stefan's constant	σ	$5.67 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$
35.	Universal gas constant	R	$8.31\ J\ mol^{-1}\ K^{-1}$
36.	Universal gravitational constant	G	$6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$
37.	Wien's constant	b	$2.898 \times 10^{-3}\ m\ K$

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The Greek Alphabet

Letter	Upper	Lower	Letter	Upper	Lower
Alpha	A	α	Omega	Ω	ω
Beta	B	β	Psi	Ψ	ψ
Chi	X	χ	Phi	Φ	ϕ
Delta	Δ	δ	Pi	Π	π
Epsilon	E	ε	Rho	P	ρ
Eta	H	η	Sigma	Σ	σ
Gamma	Γ	γ	Tau	T	τ
Kappa	K	κ	Theta	Θ	θ
Lambda	Λ	λ	Upsilon	Y	υ
Mu	M	μ	Xi	Ξ	ξ
Nu	N	ν	Zeta	Z	ζ

Multiples and Prefixes For Metric Units

Multiple	Prefix	Abbreviation	Pronunciation
10^{15}	peta	P	pet'a
10^{12}	tera	T	ter'a
10^{9}	giga	G	ji'ga
10^{6}	mega	M	meg'a
10^{3}	kilo	k	kil'o
10^{2}	hecto	h	hek'to
10	deka	da	dek'a
10^{-1}	deci	d	des'i
10^{-2}	centi	c	sen'ti
10^{-3}	milli	m	mil'li
10^{-6}	micro	μ	mi'kro
10^{-9}	nano	n	nan'oh
10^{-12}	pico	p	pe'ko
10^{-15}	femto	f	fem'toe

SI Base Units

S. No.	Physical quantity	Unit	Symbol	Dimensional Symbol
1.	Length	metre	m	L
2.	Mass	kilogram	kg	M
3.	Time	second	s	T
4.	Temperature	kelvin	K	K
5.	Electric current	ampere	A	A
6.	Charge	coulomb	C	C
7.	Luminous intensity	candela	cd	
8.	Quantity of matter	mole	mol	

Some Non-SI Units

S. No.	Physical quantity	Unit / Symbol	Value
1.	Force	kilogram force (kgf)	1 kgf = 9.8 N
2.	Energy	calorie	1 cal = 4.186 J
3.	Energy	kilowatt hour (kWh)	1 kWh= 3.6×10^{6} J
4.	Energy	electron volt (eV)	1 eV = 1.6×10^{-19} J
5.	Length	par sec (pc)	1 pc = 3.08×10^{16} m
6.	Volume	litre (l)	1 litre = 10^{-3} m^3
7.	Length	light year (ly)	1 ly = 9.46×10^{15} m
8.	Length	Astronomical unit (AU)	1AU = 1.496×10^{11} m
9.	Length	angstrom (Å)	1Å = 10^{-10} m
10.	Length	fermi (fm)	1fm = 10^{-15} m
11.	Pressure	bar	1 bar = 10^{5} Pa
12.	Power	horse power (hp)	1hp = 746 W

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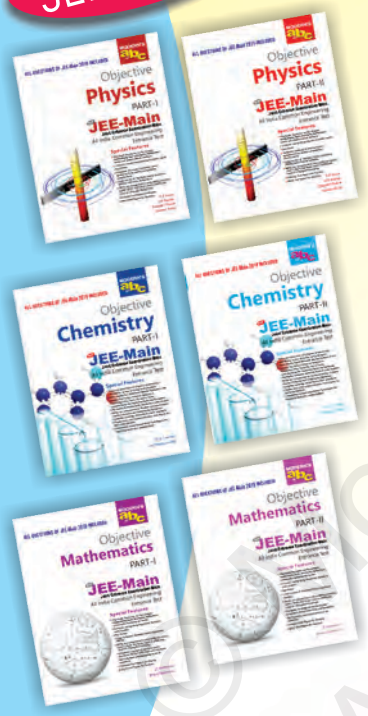
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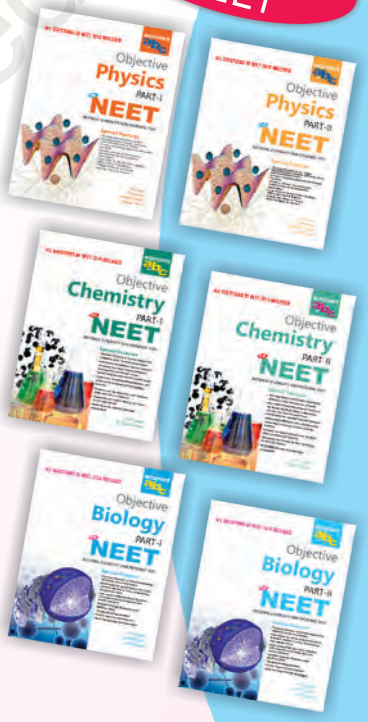
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CHAPTER

0 MATHEMATICAL TOOLS

1. INTRODUCTION

In order to do analysis of the physical situations we need the laws of physics and to apply the laws of physics the knowledge of basic mathematics is necessary. In this chapter we shall study about the basic mathematics that we will use in the analysis while applying the laws of physics.

2. ALGEBRAIC IDENTITIES

1. $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$

2. $(a - b)(a + b) = a^2 - b^2$

3. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

4. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

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5. $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 - ab + b^2)$

6. $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + ab + b^2)$

7. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

3. QUADRATIC EQUATIONS AND EXPRESSIONS

3.1. Polynomial

An expression of the form:

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots + a_1 x + a_0$,

where x is a variable, n a positive integer and a is constants.

(*i*) Degree of Polynomial: The highest power of x in $f(x)$ is called the degree of the polynomial $f(x)$.

(*ii*) Polynomial of degree 1, 2 and 3 are called linear, quadratic and cubic polynomial respectively.

(*iii*) Forms of Polynomials

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- Quadratic Equation 2

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Geometrical Progression 2
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Degree	Name of Polynomial	Form of Polynomial
0	Constant polynomial	$f(x) = a$, being a constant
1	Linear polynomial	$f(x) = ax + b$, $a \neq 0$
2	Quadratic polynomial	$f(x) = ax^2 + bx + e$, $a \neq 0$
3	Cubic polynomial	$f(x) = ax^3 + bx^2 + ex + d$, $a \neq 0$
4	Biquadratic polynomial	$f(x) = ax^4 + bx^3 + ex^2 + dx + e$, $a \neq 0$

3.2. Linear Equation

The first degree equation of the form [$bx + c = 0$ ($b \neq 0$)] is called linear equation.

- $ax + by + c = 0$ is also straight line and its graph on axis is straight line.
- This equation has two variables x and y. We must have at least two equations in order to find their values.

3.3. Quadratic Equation

The second degree (maximum power of x is called degree) equation of the form, $ax^2 + bx + c = 0$ ($a \neq 0$), is called quadratic equation. Here 'a' and 'b' are called the coefficients of x^2 and x respectively, 'c' is a constant term.

- **Roots of the equation.** The value of the variable 'x' that satisfies the equation is called root of the equation. A quadratic equation has two roots.
- **Solution of this quadratic** is $x = \alpha, \beta$

  $D = b^2 - 4ac$

  If $D > 0$, both roots are real and unequal that is

  $$x = \alpha = \frac{-b + \sqrt{D}}{2a}, \beta = \frac{-b - \sqrt{D}}{2a}$$

  If $D = 0$, both roots are real and equal that is

  $$x = \alpha = \frac{-b}{2a}, \beta = \frac{-b}{2a}$$

  If $D < 0$, both roots are imaginary and unequal that is

  $$x = \alpha = \frac{-b + i\sqrt{D}}{2a}, \beta = \frac{-b - i\sqrt{D}}{2a}$$

- Quadratic equation in terms of roots

  $x^2 + (\alpha + \beta)x + \alpha\beta = 0$

Sum of the roots = $\alpha + \beta = -b / a$
Product of the roots = $\alpha\beta = c / a$

SOLVED EXAMPLE

Example 1.

Solve the equation $2x^2 + x - 3 = 0$.

Solution: Here $a = 2, b = 1, c = -3$

$D = b^2 - 4ac \Rightarrow (1)^2 - 4(2)(-3) = 25 > 0$, so it has real and unequal root that is

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm \sqrt{25}}{4}$$

$$= \frac{-5 \pm 5}{4}$$

Hence $x = \left(\frac{-5}{2}, 0\right) \in (\alpha, \beta)$

4. ARITHMETICAL PROGRESSION

A group of numbers in which the difference between two consecutive numbers is constant is known as arithmetic series. The constant difference is called the common difference 'd'.

General arithmetic progression is $a, (a + d), (a + 2d), (a + 3d) \ldots a + (n - 1)\, d$.

(*i*) **General term. $a_n = a + (n - 1)d$,**

where a_n is the n^{th} term, a is the first term, d the common difference and l is the last term

(*ii*) **Sum of n terms.** $S_n = \frac{n}{2}[2a + (n - 1)d]$.

(*iii*) **Arithmetic Means.** A.M. between a and $b = \frac{a + b}{2}$

The sum of n arithmetic means between a and b is n times $\left(\frac{a + b}{2}\right)$

5. GEOMETRICAL PROGRESSION

A group of numbers in which every number is obtained by multiplying the previous number by the same constant quantity is called geometric series or progression (G.P). The constant quantity is called common ratio 'r'.

General geometric progression is:

$a, ar, ar^2, ar^2 \ldots ar^{n-1}$

(*i*) **General n^{th} term.** $a_{n^{th}} = ar^{n-1}$, where a is the first term and r the common ratio.

(*ii*) **Sum of n^{th} terms.** $S_{n^{th}} = \frac{a(1 - r^n)}{1 - r}$ when $r < 1$

and $S_{n^{th}} = \frac{a(r^n - 1)}{r - 1}$ when $r > 1$.

(*iii*) **Geometric Means.** G.M. between a and $b = \sqrt{ab}$.

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The product of n geometric means between a and $b = (\sqrt{ab})^n$.

(*iv*) For five terms, take $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.

(*v*) If G.P. is of infinite number of terms, then

$$S_n = \frac{a}{1-r} \text{ if } r < 1$$

5.1. Series

If the number of the sequence is connected together by sign of addition (+), we get a series.

Standard Results

(*i*) $\sum_{k=1}^{n} k = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

(*ii*) $\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

(*iii*) $\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$

6. BINOMIAL THEOREM AND EXPANSION SERIES

6.1. Binomial Theorem

Theorem. If n is a non-negative integral exponent, then:

$$(x + y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 \times ... + {}^nC_r x^{n-r} y^r + ... + {}^nC_n y^n.$$

(*i*) General Term. $a_{r+1} = {}^nC_r x^{n-r} y^r$.

(*ii*) $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + ... + C_n x^n$

(*iii*) The Binomial's theorem states that if $|x| << 1$ then,

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + ...$$

where n is any number, positive, negative, integer, or a fraction.

Here $n! = n(n-1)(n-2) ..... 3 \times 2 \times 1$.

It is read as n – factorial.

(*iv*) When $|x|$ is very very small as compared to 1. The higher powers of x tend to zero or are negligible. Hence the expansion may be written as:

$$(1 + x)^n = 1 + nx$$
$$(1 + x)^{-n} = 1 - nx$$

SOLVED EXAMPLES

Example 1.

Write the first three terms of the expansion $(1 + q^2)^{10}$.

Solution: Here $n = 10$ and $x = q^2$. Therefore:

$$(1 + q^2)^{10} = 1 + 10\, q^2 + \frac{10(10-1)}{2!} q^4 + \frac{10(10-1)(10-2)}{3!} q^6 + ...$$
$$= 1 + 10\, q^2 + 45\, q^4 + 120\, q^6 + ...$$

Example 2.

The acceleration due to gravity at a height h is given by $g_h = g\left(\frac{R}{R+h}\right)^2$, *where g is the acceleration due to gravity on the surface of earth. For h << R, find the value of g using the Binomial theorem.*

Solution: $g_h = g\frac{R^2}{(R+h)^2}$

$$= g\left(1 + \frac{h}{R}\right)^{-2}$$

Since $h/R << 1$, therefore using Binomial expansion,

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

PRACTICE PROBLEMS 1

1. Write the first four terms of the expansion $(1 + q)^{-2}$.

2. Evaluate $(1003)^{1/3}$ upto five places of decimal.

3. Find the value of $g' = g\left(1 - \frac{2}{291}\right)^{1/2}$

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7. EXPONENTIAL EXPANSION

The exponential function is defined as

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots + \frac{x^n}{n!}$$

(*i*) If $x << 1$ then $e^x = 1 + x$

(*ii*) $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} \ldots$

(*iii*) If $x << 1$ then $e^{-x} = 1 - x$

Also this gives the value of '*e*' by putting $x = 1$

$$e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots = 2.718$$

8. LOGARITHMS AND SERIES EXPANSION

Logarithm: Every positive real number n can be expressed in exponential form as

$$n = a^x \qquad \ldots(1)$$

Hence logarithm of a number to some base is the exponent by which the base must be raised in order to get the number. a is raised what power to get n.

KEY NOTE

Logarithm of zero does not exist and logarithm of (negative) base is not defined in the system of real numbers, where 'a' is also a positive real different than unity and is called the base and 'x' is called the exponent.

We can write the relation (1) in logarithmic form as

$$a^x = n$$
$$\log_a n = x$$

where $n > 0$, $a \neq 1$

Important points:

(*a*) Unity has been excluded from the base of the logarithm as in this case

$\log_1 n$: will not be possible and if $n = 1$

$\log_1 1$: will have infinitely many solutions and will not be unique which is necessary in the functional notation.

(*b*) $a^{\log_a n} = n$ is an identify for all $n > 0$ and $a > 0$ and $a \neq 1$ [**Example:** $3^{\log_3 5} = 5$]

(*c*) The number n is called the antilog of 'x' to the base 'a'.

Hence if $\log_3 243$ is 5 $\Rightarrow$ $\text{antilog}_3 5$ is equal to $3^5 = 243$

(*d*) Using the basic definition of log we have 3 important deductions:

(*i*) $\log_n n = 1$, i.e. logarithm of a number to the same base is 1.

(*ii*) $\log_{\frac{1}{n}} n = -1$, i.e. logarithm of a number to its reciprocal is –1.

(*iii*) $\log_a 1 = 0$, i.e. logarithm of unity to any base is zero.

(basic constraints on number and base must be observed.)

(*e*) $a \log_n n = a$ is an identify for all $n > 0$ and $a > 0$; $a \neq 1$ [**Example:** $5 \log_2 2 = 5$]

(*f*) Whenever the number and base are on the same side of unity then logarithm of that number to the base is (positive), however if the number and base are located on different sides of unity then logarithm of that number to the base is (negative).

Example: (*i*) $\log_{10} 100 = 2$

(*ii*) $\log_{1/10} 100 = -2$

(*g*) For a non-negative number 'a' and $n \geq 2$, $n \in N$ $\sqrt[n]{a} = a^{1/n}$

8.1. Other Logarithm Definitions

(*i*) Definition of **Natural Logarithm (ln*x*):**

In this class of logarithms, the base is e, where

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots = 2.7182818$$

Logarithm with a base of e are called **natural logarithms**. It is customary to write $\log_e x$ as $\ln(x)$.

(*ii*) Definition of **Common Logarithm (log *x*):**

Logarithm with a base of 10 are called common logarithm. It is customary to write $\log_{10} x$ as $\log x$. They are very useful in numerical calculations.

The natural logarithm may be converted to common logarithm using the following relation:

$$\log_e a = 2.3026 \log_{10} a$$
$$\ln N = 2.3026 \log_{10} N$$

(*iii*) Discussion

(*a*) $\log_a a = 1$ because $a^1 = a$

(*b*) $\log_a 1 = 0$ because $a^0 = 1$

(*c*) Logarithm of a negative number is meaningless, because it does not exist.

8.2. Standard Formulae of Logarithm

(*i*) $\log_a (m \times n \times p \times q) = \log_a m + \log_a n + \log_a p + \log_a q$

(*ii*) $\log_a (m/n) = \log_a m - \log_a n$.

(*iii*) $\log_a m^n = n \log_a m$.

(*iv*) $\log_a m = \log_b m \times \log_a b$.

(*v*) $\log_a b = \frac{\log_e b}{\log_e a}$.

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8.3. Logarithmic Expansion

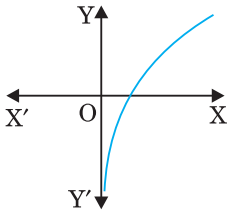
$$\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 \ldots (|x| < 1)$$

If $x << 1$, then $\log(1+x) \approx x - \frac{x^2}{2}$

8.3.1. Logarithmic Function

A function 'f' defined by $f(x) = \log x$, $x > 0$ is called a logarithmic function.

The graph of the function is as shown in the adjoining figure.



Change of Base: $\log_b a = \frac{\log_c a}{\log_c b} = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$

8.4. Characteristic and Mantissa of Common Logarithms

The logarithm of a number consists of two parts as follows:

(*i*) **Characteristic.** It is the integral part of the logarithm.

(*ii*) **Mantissa.** It is the decimal part of the logarithm.

Example: $\log_a n = 3.2459$. Then characteristic is 3 and Mantissa is 0.2459.

8.4.1. Important Points:

(*i*) The integral part or **characteristic** of the logarithm may be **positive, zero or negative.**

(*ii*) The decimal part or the **mantissa** of the logarithm is **always positive**.

(*iii*) In case the logarithm of a number is negative, then, we rearrange the characteristic and mantissa in such a way as to make the decimal part positive. **Example:** suppose $\log_{10} n = -1.2318$. Then we can write it as:

$$\log_{10} n = -1.2318$$
$$= (-1) + (-0.2318)$$
$$= (-1-1) + (-0.2318+1) = -2 + (0.7682)$$

Hence, characteristic is – 2 and mantissa is 0.7682.

(*iv*) If the characteristic is negative, we denote it with a bar. That is characteristic – 2 is denoted as pronounced as 2 bar. **Example:** the logarithm as expressed above may be written as:

$$\log_{10} n = \bar{2}.7682.$$

8.4.2. Finding the Logarithm of the Given Number

To obtain the logarithm of a given number n, we find the characteristic by inspection and obtain the mantissa from the logarithm table. Then $\log_{10} n$ is given by the characteristic as the integral part and mantissa as the decimal part.

Example: $\log_{10}$ 2387400 = 6.3779. Here characteristic is 6 and mantissa is 3779.

Example: $\log(5438) = \log(5.438 \times 10^3)$

$$= \log(5.438) + 3 \quad \ldots(1)$$

Find the number at the intersection of the row beginning with the first two digits(54) and the column headed by the third digit(3) is 7348 in log table end of the book.

Summation of mean difference column headed by fourth digit(8) is 6.

Total value is 7348 + 6 = 7354 but it can be written in way is 0.7354.

So, we can write in such way of equation (1).

log (5438) = log(5.438) + 3 = 0.7354 + 3 = **3.7354**.

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PRACTICE PROBLEMS 2

Find the logarithm of the following number:

(*i*) 5438 (*ii*) 543.8 (*iii*) 54.38
(*iv*) 5.438 (*v*) 0.5438 (*vi*) 0.05438
(*vii*) 0.005438.

Answers to Practice Problems

(*i*) 3.7354 (*ii*) 2.7354 (*iii*) 1.7354
(*iv*) 0.7354 (*v*) $\bar{1}$.7354 (*vi*) $\bar{2}$.7354
(*vii*) $\bar{3}$.7354

Note: $\bar{1}.7354 = -1 + 0.7354 = -0.2646$
$\bar{2}.7354 = -2 + 0.7354 = -1.2646$

8.4.3. Antilogarithm

The number a whose logarithm is x is called antilogarithm of x. That is if

$$\log a = x$$

Then $a = \text{antilog } x$

Since $\log 2387400 = 6.3779$.

Hence 2387400 = antilog 6.3779.

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8.4.4. Finding Antilogarithm

The antilogarithm is obtained from the table as follows. Suppose, we wish to find antilog 6.3779. We proceed as described below:

Step 1: Leave the characteristic part and take the mantissa (M). Here mantissa is M = 3779.

Step 2: Locate the first two digits (in this case .37) of the mantissa in the first left-hand column of the antilog table.

Step 3: Find the number at the intersection of the row beginning with the first two digits and the column headed by the third digit (viz. 7) of M.

Let it be q'. Here q' for M = .3779 is equal to 2382.

Step 4: Also find the number at the intersection of the row beginning with the first two digits (here 37) and the mean difference column headed by the fourth digit (9) of the mantissa.

Let it be r'. For M = .377(9), we find $r' = 5$

Step 5: Add q' and r'. Thus for $M = 3779$, we find $q' + r' = 2382 + 5 = 2387$.

Step 6: So this can be written as 2.387.

Step 7: If the characteristic ($c = 6$) is positive, it can be written as 2.387×10^6 antilog 6.3779 $= 2.387 \times 10^6$.

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SOLVED EXAMPLES

Example 1.

Find the value of antilog $\bar{3}.8914$.

Solution: Refer to table of antilog, at intersection of row (0.89) and column (1) = 7780

Mean difference column headed by fourth digit (4) of mantissa = 7

Thus, value can be added

$$x = 7787$$

Antilog $(\bar{3}.8914) = 7.787 \times 10^{-3}$

Example 2.

Find the value of $x = 22.89 \times \dfrac{0.0273}{58.3 \times 0.0021}$.

Solution: Given $x = 22.89 \times \dfrac{0.0273}{58.3 \times 0.0021}$

$$\log x = \log 22.89 + \log 0.0273 - \log 58.3 - \log 0.0021$$

$$= \log (2.289 \times 10) + \log (2.73 \times 10^{-2}) - \log (5.83 \times 10^{1}) - \log 2.1 \times 10^{-3})$$

$$= 1.3596 + \bar{2}.4362 - 1.7657 - \bar{3}.3222$$

$$= 1.3596 + [(-2) + 0.4362] - 1.7657 - [(-3) + 0.3222]$$

$$= 0.7079$$

Note that in $\bar{2}.4362$, the characteristic is negative but the mantissa is positive.

$\therefore$ $\log x = 0.7079$

Hence x = antilog [0.7079]

= antilog [0.7079] $= 5.104 \times 10^0$

Example 3.

Find the value of $y = (0.05246)^{1/8}$.

Solution: Given $y = (0.05246)^{1/8}$

Taking logarithm, we find

$$\log y = \frac{1}{8} \log (0.05246) = \frac{1}{8} \log (5.246 \times 10^{-2})$$

$$= \frac{1}{8} [0.7198 + (-2)]$$

$$= \frac{1}{8} [-1.2802] = -0.1600$$

$$= (-1) + [1 - 0.01600]$$

or $\log y = \bar{1}.8400$

$\therefore$ y = antilog $\bar{1}.8400 = 6.918 \times 10^{-1}$

PRACTICE PROBLEMS 3

1. Find the value of $\dfrac{1}{200}\left[6500 \times \dfrac{981}{0.024}\right]^{1/2}$
2. Evaluate: $(0.0043)^{1/7}$
3. Evaluate: $\dfrac{17.6 \times 0.51}{0.02 \times 3}$
4. Given that $v = \sqrt{\dfrac{2GM}{R}}$. Find $\log v$.
5. Write the following in common logarithms

   $W = RT \log_e V_2 / V_1$
6. Expand the following $N = N_0 e^{-\lambda t}$.

Answers to Practice Problems

1. 81.51 **2.** 0.4591 **3.** 149.6

4. $\log v = \dfrac{1}{2}$ [log 2 + log G + log M – log R]

5. $W = 2.3026 \left[RT \log_{10} \dfrac{V_2}{V_1}\right]$

6. $\log N = \log N_0 - lt \log e$.

9. TRIGONOMETRY

9.1. Angle

Angle is the measure of the revolution of a line about a point. The sense of rotation is marked by the arrow. The angle is said to be positive if the line revolves in the anticlockwise direction and it is said

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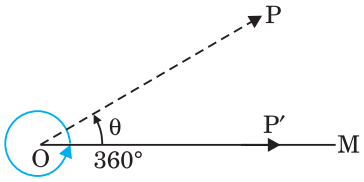
to be negative if the line revolves in the clockwise direction.

$\angle MOP = \theta$, while $\Delta POM = 360° - \theta$

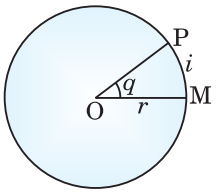
Systems of measuring the angle



Circular System. In the circular system, the angle is measured as the ratio of the length of the arc and the radius of the circle described by the terminus of the revolving line.

$$\theta = \frac{\text{arc MP}}{\text{OP}} = \frac{l}{r}$$

The unit of the angle in circular measure is called radian (symbol: rad).



The radian is the angle subtended by an arc of length equal to its radius.

2π rad = 360°.

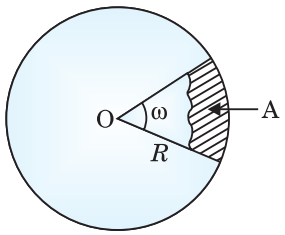
KEY NOTE

It is customary to express angle in radian if there is numeric digits.

Solid Angle: The unit of solid angle is **Steradian**.

One **steradian** is defined as the solid angle subtended at the centre of a sphere by an area (on the surface) equal to the square of radius of the sphere.

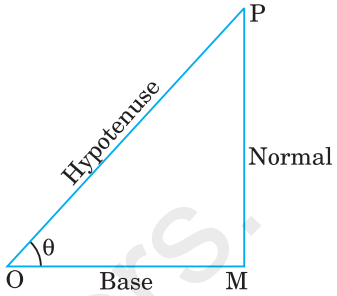
If A be the area on the surface of a sphere of radius R which subtends a solid angle 'ω' at the centre.



$$\omega = \frac{A}{R^2} \text{ Sr}$$

Note that the complete solid angle in space is 4π steradian $\left[\because \omega = \frac{4\pi R^2}{R^2} = 4\pi\right]$.

9.2. Trigonometric Ratios



$$\text{sine } \theta = \frac{p}{h} = \frac{\text{MP}}{\text{OP}} \qquad \text{cotangent } \theta = \frac{b}{p} = \frac{\text{OM}}{\text{MP}}$$

$$\text{cosine } \theta = \frac{b}{h} = \frac{\text{OM}}{\text{OP}} \qquad \text{secant } \theta = \frac{h}{b} = \frac{\text{OP}}{\text{OM}}$$

$$\text{tangent } \theta = \frac{p}{b} = \frac{\text{MP}}{\text{OM}} \qquad \text{cosecant } \theta = \frac{h}{p} = \frac{\text{OP}}{\text{MP}}$$

The sine θ is abbreviated as **sin** θ, cosine θ as **cos** θ, tangent θ as **tan** θ, cotangent θ as **cot** θ, secant θ as **sec** θ and cosecant θ as **cosec** θ.

It is evident that:

$$\tan\theta = \frac{\sin\theta}{\cos\theta};\ \cot\theta = \frac{\cos\theta}{\sin\theta};\ \sec\theta = \frac{1}{\cos\theta};$$

$$\text{cosec } \theta = \frac{1}{\sin\theta}$$

Values of trigonometry ratios for certain angles are given in the table below:

Angle	0°	30°	45°	60°	90°	120°	135°	150°	180
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	−1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$\frac{1}{-\sqrt{3}}$	−1	$-\frac{1}{\sqrt{3}}$	0

9.3. Trigonometric Identities

(*i*) $\sin^2\theta + \cos^2\theta = 1$.

(*ii*) $\sec^2\theta - \tan^2\theta = 1$.

(*iii*) $\text{cosec}^2\theta - \cot^2\theta = 1$.

(*iv*) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$.

(*v*) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$.

(*vi*) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$.

From the above relations, putting $B = A$, we find:

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(*vii*) $\sin 2A = 2 \sin A \cos A$.

(*viii*) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$.

(*ix*) $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

Also it can be proved that.

(*x*) $2 \sin A \cos B = \sin (A + B) + \sin (A - B)$.

(*xi*) $2 \cos A \cos B = \cos (A + B) + \cos (A - B)$.

(*xii*) $2 \sin A \sin B = \cos (A + B) - \cos (A - B)$.

(*xiii*) $\sin C + \sin D = 2 \sin \left(\dfrac{C + D}{2}\right) \cos \left(\dfrac{C - D}{2}\right)$.

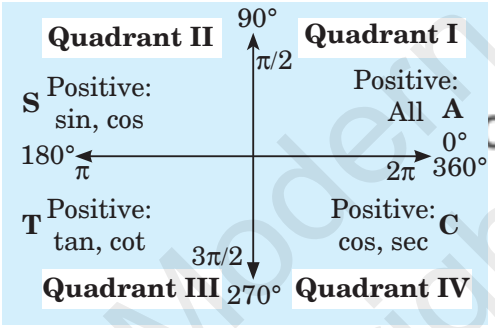
(*xiv*) $\sin C - \sin D = 2 \cos \left(\dfrac{C + D}{2}\right) \sin \left(\dfrac{C - D}{2}\right)$.

(*xv*) $\cos C + \cos D = 2 \cos \left(\dfrac{C + D}{2}\right) \cos \left(\dfrac{C - D}{2}\right)$.

(*xvi*) $\cos C - \cos D = 2 \sin \left(\dfrac{C + D}{2}\right) \sin \left(\dfrac{D - C}{2}\right)$.

9.4. Signs and Trigonometry Formulas

The Cartesian coordinate axes divide the plane into four parts, which are called quadrants.



For $0° \le \theta \le 90°$, we have the first quadrant.

For $90° \le \theta \le 180°$, we have the second quadrant.

For $180° \le \theta \le 270°$, we have the third quadrant.

For $270° \le \theta \le 360°$, we have the fourth quadrant.

Trigonometry Formulas

(*i*) $\sin (-\theta) = -\sin \theta$

(*ii*) $\cos (-\theta) = \cos \theta$

(*iii*) $\tan (-\theta) = -\tan \theta$

(*iv*) $\sin \left(\dfrac{\pi}{2} - \theta\right) = \cos \theta$

(*v*) $\cos \left(\dfrac{\pi}{2} - \theta\right) = \sin \theta$

(*vi*) $\tan \left(\dfrac{\pi}{2} - \theta\right) = \cot \theta$

(*vii*) $\sin \left(\dfrac{\pi}{2} + \theta\right) = \cos \theta$

(*viii*) $\cos \left(\dfrac{\pi}{2} + \theta\right) = -\sin \theta$

(*ix*) $\tan \left(\dfrac{\pi}{2} + \theta\right) = -\cot \theta$

(*x*) $\sin (\pi - \theta) = \sin \theta$

(*xi*) $\cos (\pi - \theta) = -\cos \theta$

(*xii*) $\tan (\pi - \theta) = -\tan \theta$

(*xiii*) $\sin (\pi + \theta) = -\sin \theta$

(*xiv*) $\cos (\pi + \theta) = -\cos \theta$

(*xv*) $\tan (\pi + \theta) = \tan \theta$

(*xvi*) $\sin (\pi - \theta) = -\sin \theta$

(*xvii*) $\cos (\pi - \theta) = \cos \theta$

(*xviii*) $\tan (\pi - \theta) = -\tan \theta$

(*xix*) $\sin (\pi + \theta) = \sin \theta$

(*xx*) $\cos (\pi + \theta) = \cos \theta$

(*xxi*) $\tan (\pi + \theta) = \tan \theta$.

SOLVED EXAMPLE

Example 1.

Find the vlue of sin 120°.

Solution: $= \sin (90° + 30°) = \cos 30° = \dfrac{\sqrt{3}}{2}$

[Because $\sin (90 + \theta) = \cos \theta$]

PRACTICE PROBLEMS 4

1. Find the values of the following allied angles:

(*i*) sin (–30°) (*ii*) cos 120° (*iii*) tan 210°

(*iv*) sin 300° (*v*) sin 330° (*vi*) sin 225°

(*vii*) cos 135°

Answers to Practice Problems

(*i*) $-\dfrac{1}{2}$ (*ii*) $-\dfrac{1}{2}$ (*iii*) $\dfrac{1}{\sqrt{3}}$

(*iv*) $-\dfrac{\sqrt{3}}{2}$ (*v*) $-\dfrac{1}{2}$ (*vi*) $-\dfrac{1}{\sqrt{2}}$

(*vii*) $-\dfrac{1}{\sqrt{2}}$

9.5. Inverse-Trigonometric Functions

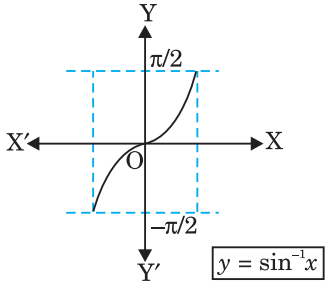
(*i*) Definition:

The inverse-sine function is defined as:

$$y = f^{-1}(x) = \sin^{-1} x$$

iff $x = \sin y$ and $y \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

The graph of $\sin^{-1} x$ is as shown.



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Function	Principal Value Branch	
	Domain	Range
(i) $y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
(ii) $y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
(iii) $y = \tan^{-1} x$	$-\infty < x < +\infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

9.6. Trigonometric Series Expansions

(i) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots.$

If x is small, i.e. $x \approx 10°$, then $\sin x \approx x$

(ii) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots..$

If x is small, i.e. $x \approx 10°$, then $\cos x \approx 1$

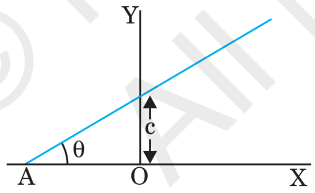
(iii) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots.$

If x is small, i.e. $x \approx 10°$ then $\tan x \approx x$.

10. COORDINATE GEOMETRY

(a) **Equation of straight line:** The equation is stated as $y = mx + c$ where $m = \tan \theta$ = slope of line; c = intercept on Y-axis.

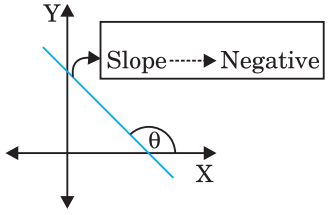
Further if $c = 0$ then $y = mx$ is a line passing through the origin and if $\theta = 45°$, i.e. line is equally inclined to axes, then $y = x$ as $m = \tan 45°$



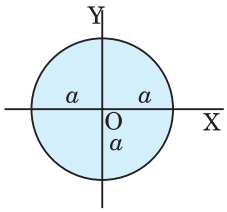
Slope-intercept form

$$y = \left(\frac{-b}{a}\right)x + \left(\frac{-c}{a}\right)$$

Slope of line (inclination with respect to X-axis) is $m = \left(\frac{-b}{a}\right)$

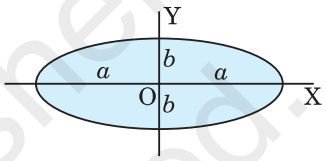


(b) **Equation of a circle:** It is stated as $x^2 + y^2 = a^2$



where 'a' is the radius and centre is the origin of circle.

(c) **Equation of ellipse:** It is stated as: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



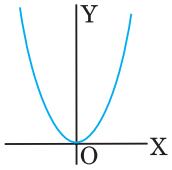
where a = length of semi-major axis

b = length of semi-minor axis and centre is the origin

(d) **Equation of Parabola:**

(i) Symmetrical to X-axis; then equation is $y^2 = kx$ where k is constant.

(ii) Symmetrical to Y-axis; then $x^2 = ky$ represents the equation of rectangular hyperbola:



10.1. Geometry

(i) Area of a triangle = $\frac{1}{2}$ (base × height)

(ii) Area of a circle = $\pi \times (\text{radius})^2$

(iii) Circumference of a circle = $2\pi \times$ radius = $\pi \times$ diameter

(iv) Area of a sphere (surface) = $4\pi \times (\text{radius})^2$

(v) Volume of a sphere = $4/3\pi \times (\text{radius})^3$

(vii) Curved surface area of a closed cylinder = 2π (radius) × length + 2π (radius)^2.

(viii) Volume of a cylinder = $\pi \times (\text{radius})^2 \times$ length.

(ix) Surface area of a cube = $6 \times (\text{length})^2$

(x) Volume of a cube = $(\text{length})^3$

(xi) Volume of a parallelopiped = length × breadth × height

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(*xii*) Volume of a cone = $\frac{1}{3}\pi$ (radius)^2 × height

11. DIFFERENTIATION

11.1. Function of a Variable

A quantity, the value of which may change, is called **variable.** It may be denoted by x, y, z, t, etc.

If corresponding to one value of the variable x, there is one and only one value of the variable y, then y is said to be a **function of** x. Mathematically, we represent this relation as:

$$y = f(x)$$

It is read as "y is a function of x".

Here $f(x)$ is an expression in x. That is, we may have:

$$y = f(x) = \frac{x^2 + x - 2}{x - 1}$$

When $x = 2, y = 4$ and when $x = 3$, then $y = 5$. Thus, for every value of x, y has definite value. Here $f(x)$ is a composite expression. It is not the product of f and x.

The small change in the value of the variable is called increment of variable. If variable is x, we represent the increment in x by Δx or δx (read as delta x). Similarly increment in y is represented as Δy or δy.

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KEY NOTE

Here Δx, δx, Δy, δy are all composite expressions and are not to be taken simple multiplication of Δ or δ with x or y.

The ratio of increments of two variables is called **quotient of increments.** For example, the quotient of increment of x and y is:

$$\frac{\Delta y}{\Delta x} \text{ or } \frac{\delta y}{\delta x}$$

11.2. Limit of a Variable

If a variable x approaches a value 'k' in such a manner as given in expression is less than any positive number, however small, then k is said to be limit of x. We represent it symbolically as $x \to k$. It is read as "x approaches k" or "x tends to k".

The value of the function $f(x)$ as the variable approaches certain value, say k, is called limit of the function. It is mathematically expressed as:

$$\underset{x \to k}{\text{Limit}}\ f(x) \text{ or } \underset{x \to k}{\text{Lt}}\ f(x)$$

11.2.1. Limits

(*i*) $\underset{x \to 0}{\text{Lt}} (\sin x) = 0$ (*ii*) $\underset{x \to 0}{\text{Lt}} \cos x = 1$

(*iii*) $\underset{x \to 0}{\text{Lt}} \left(\frac{\sin x}{x}\right) = 1$ (*iv*) $\underset{x \to 0}{\text{Lt}} \left(\frac{\tan x}{x}\right) = 1$

(*v*) $\underset{x \to 0}{\text{Lt}} \left(\frac{a^x - 1}{x}\right) = \log_e a, \quad a > 0$

11.3. Derivative or Differentiation

If $y = f(x)$, then limit of the quotient of the increments of 'y' and 'x' as the increment of 'x' approaches zero is called the **differential co-efficient** of y w.r.t. x.

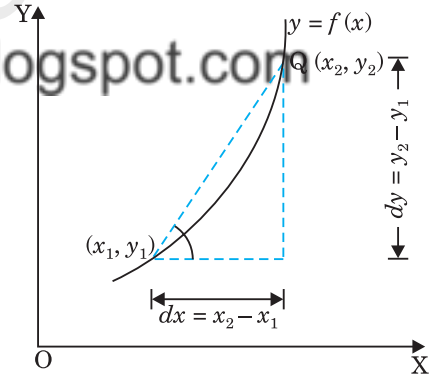
It is written as $\frac{dy}{dx}$ and also known as derivative of y w.r.t. x.

Differential co-efficient of y with respect to x

$$f(x) = \underset{\delta x \to 0}{\text{Lt}} \left(\frac{\delta y}{\delta x}\right) = \frac{dy}{dx}$$

The process of obtaining differential co-efficient of a function is called differentiation. It means we can study about the change in y when we can change x at very small level where our eyes are not able to predict the change.

Physical meaning of differentiation:



Let $y = f(x)$. If x and y represent the co-ordinates of a point in the Cartesian plane, then by obtaining values of y for different values of x, we can plot a graph for $y = f(x)$. The graph is shown in Let P(x_1, y_1) and Q (x_2, y_2) be two points on the graph.

Slope of the line joining P to Q is:

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\delta y}{\delta x}$$

If Q approaches P, the line PQ will tend to be tangent at P. In this process, $x_2 - x_1 = dx$ and $y_2 - y_1 = dy$, will also become very small and will finally tend to zero.

Hence in such a case we may write:

$$\tan\theta = \underset{\delta x \to 0}{\text{Lt}} \left(\frac{\delta y}{\delta x}\right) = \frac{dy}{dx}$$

That is, differential co-efficient gives the slope of the graph $y = f(x)$.

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It also represents the rate of change of y w.r.t. x at various points of the graph.

11.3.1. Formulas of Differentiation

1. The differential co-efficient of a constant is zero. If 'c' is a constant, then = 0
2. If $y = u \pm v \pm \ldots$ where u and v etc. and y are the functions of x, then
$$\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx} \pm \ldots..$$
3. If $y = x^n$, where n is a real number, then
$$\frac{dy}{dx} = \frac{d}{dx}\left(x^n\right) = nx^{n-1}$$
4. If $y = cu$, where c is a constant and u is function of x, then
$$\frac{dy}{dx} = c\frac{du}{dx}$$
5. If $y = uv$, where u and v are both functions of x, then:
$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$
6. If $y = f(u)$ and $u = f(x)$, then:
$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$
It is also called the chain rule.
7. If $y = (ax + b)^n$, then:
$$\frac{dy}{dx} = n(ax + b)^{n-1} \times \frac{d}{dx}(ax + b)$$
8. If $y = \frac{u}{v}$ where $u = f(x)$ and $v = f(x)$, then:
$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$
9. If $y = u^n$, where u is function of x, then :
$$\frac{dy}{dx} = \frac{d}{dx}\left(u^n\right) = nu^{n-1}\frac{du}{dx}$$

SOLVED EXAMPLES

❑ Example 1.

Given that $y = x^2$, *find* $\frac{dy}{dx}$.

Solution: $\frac{du}{dx} = \frac{d}{dx}\left(x^2\right) = 2x^{2-1} = 2x.$

❑ Example 2.

Given that $y = x^5 + x^4 + 7$. *Find* $\frac{dy}{dx}$.

Solution: $y = x^5 + x^4 + 7$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^5\right) + \frac{d}{dx}\left(x^4\right) + \frac{d}{dx}(7)$$
$$= 5x^4 + 4x^3 + 0 = 5x^4 + 4x^3.$$

❑ Example 3.

Given that $y = x^2 + 4x^{-\frac{1}{2}} - 3x^{-2}$, *find* $\frac{dy}{dx}$.

Solution: $y = x^2 + 4x^{-\frac{1}{2}} - 3x^{-2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(4x^{-\frac{1}{2}}\right) - \frac{d}{dx}\left(3x^{-2}\right)$$

or $$\frac{dy}{dx} = 2x + 4 \times \left(-\frac{1}{2}x^{-\frac{3}{2}}\right) - 3\left(-2x^{-3}\right)$$
$$= 2x - 2x^{-\frac{3}{2}} + 6x^{-3}$$

❑ Example 4.

Given that $y = (3x^2 + 7)(6x + 3)$. *Find* $\frac{dy}{dx}$.

Note: This problem can be solved by using Theorem 5.
Here $u = 3x^2 + 7$ and $v = 6x + 3$.

Solution: $y = (3x^2 + 7)(6x + 3)$

$$\frac{dy}{dx} = (3x^2 + 7)\frac{d}{dx}(6x + 3) + (6x + 3)\frac{d}{dx}(3x^2 + 7)$$
$$= (3x^2 + 7)(6) + (6x + 3)(6x)$$
$$= (18x^2 + 42) + (36x^2 + 18x)$$
$$= 54x^2 + 18x + 42.$$

❑ Example 5.

Obtain the derivative of $\sqrt{1 + x^3}$.

Solution:

Let $y = \sqrt{1 + x^3} = \left(1 + x^3\right)^{\frac{1}{2}}$

Put $1 + x^3 = u$

Then $y = u^{1/2}$

Now $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

But $\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2}\left(1 + x^3\right)^{-\frac{1}{2}}$

and $\frac{du}{dx} = 0 + 3x^2 = 3x^2$

or $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$$= \frac{1}{2}\left(1 + x^3\right)^{-1/2} 3x^2 = \frac{3}{2}x^2\left(1 + x^3\right)^{-1/2}$$

Alternative

$\therefore$ $y = (1 + x^3)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}\left(1 + x^3\right)^{-1/2} \times \frac{d}{dx}\left(1 + x^3\right)$$
$$= \frac{1}{2}\left(1 + x^3\right)^{-1/2} \times \left(0 + 3x^2\right).$$
$$= \frac{3}{2}x^2\left(1 + x^3\right)^{-\frac{1}{2}}$$

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❑ **Example 6.**

Given that $y = \dfrac{x^2 + 1}{x - 2}$, find $\dfrac{dy}{dx}$

Solution: This example can be solved using theorem 8. Here

$$y = \frac{u}{v} \text{ and } u = x^2 + 1,\ v = x - 2$$

$$\therefore \quad \frac{du}{dx} = 2x \text{ and } \frac{dv}{dx} = 1$$

Now,
$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$= \frac{(x-2)\,2x - \left(x^2+1\right) \times 1}{(x-2)^2}$$

$$= \frac{x^2 - 4x - 1}{(x-2)^2}$$

PRACTICE PROBLEMS 5

Obtain $\dfrac{dy}{dx}$ *for the following:*

1. $y = x^{-3}$

2. $y = x^{7/2}$

3. $y = 5x^4 + 6x^{3/2} + 9x$

4. $ax^2 + bx + c$

5. $3x^5 - 3x - \dfrac{1}{x}$

6. $(9x^3 - 8x + 7)\,(3x^5 + 5)$

7. (*i*) $(3 - 4x^2)^2$ (*ii*) $\sqrt{\left(3 + x^2\right)}$

8. (*i*) $\dfrac{1+x}{x}$ (*ii*) $\dfrac{1}{(1+x)^2}$

8. $\dfrac{x^2 + 2}{x^3 + 5}$

10. $\dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}$

11. Given that $x^2 + y^2 = 9$

show that $\dfrac{dy}{dx} \cdot \dfrac{dx}{dy} = 1.$

12. Given that $s = t^2 + 2t + 3$. Find $\dfrac{ds}{dt}$.

13. Given that $y = 6t$ and $x = 9t^2$. Find $\dfrac{dy}{dx}$.

$$\left[\textbf{Hint. } \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}\right]$$

14. Given that $s = ut + \dfrac{1}{2}\,at^2$ where u and a are constants.

Obtain the value of $\dfrac{ds}{dt}$.

15. The area '*A*' of a blot of ink is growing such that after '*t*' second.

$$A = 3t^2 + \frac{t}{5} + 7$$

Calculate the rate of increase of area after five seconds.
[**Hint:** Obtain dA/dt and substitute $t = 5$]

16. The area of a circle is given by $A = \pi r^2$, where r is the radius. Calculate the rate of increase of area with radius. [**Hint:** Calculate dA/dr]

Note *that* dA/dr means increase in area per unit increase in radius.

Answers to Practice Problems

1. $-3x^4$

2. $\dfrac{7}{2}\,x^{\frac{5}{2}}$

3. $20x^3 + 9x^{1/2} + 9$

4. $2ax + b$

5. $15x^4 - 3 + \dfrac{1}{x^2}$

6. $216x^7 - 144x^5 + 105x^4 + 135x^2 - 40$

7. (*i*) $-16x\,(3 - 4x^2)$ (*ii*) $x(3 + x^2)^{-1/2}$

8. (*i*) $-x^{-2}$ (*ii*) $-\dfrac{2}{(1+x)^3}$

9. $\dfrac{1}{2}\,x^{-\frac{1}{2}}(1+x)^{-\frac{3}{2}}$

10. $\dfrac{-x^{-\frac{1}{2}}}{(1+\sqrt{x})^2}$

12. $2\,(t + 1)$

13. $x^{-\frac{1}{2}}$

14. $u + at$

15. 30.2 per second

16. $2\pi r$.

11.4. Differential Coefficients of Trigonometric Functions

(*i*) $\dfrac{d}{dx}(\sin x) = \cos x$

(*ii*) $\dfrac{d}{dx}(\cos x) = -\sin x$

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(iii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(iv) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(v) $\frac{d}{dx}(\sec x) = \tan x \sec x$

(vi) $\frac{d}{dx}(\operatorname{cosec} x) = -\cot x \operatorname{cosec} x.$

Note: If $y = \sin u$, where $u = f(x)$

Then $\frac{dy}{dx} = \cos u \frac{du}{dx}$

Similarly if $y = \cos u$, where $u = f(x)$

Then $\frac{dy}{dx} = -\sin u \frac{du}{dx}$

SOLVED EXAMPLES

Example 1.

Obtain the differential coefficient of sin 6x.

Solution: $\frac{d}{dx}(\sin 6x)$

$= [\cos 6x] \frac{d}{dx}(6x) = [\cos 6x]\, 6 = \mathbf{6 \cos 6x}$

Example 2.

Differentiate $x^2 \cos x$.

Solution: Using the theorem

$$\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$\frac{d}{dx}(x^2 \cos x) = (\cos x) \times \frac{d}{dx}(x^2) + x^2 \frac{d}{dx}(\cos x)$$

$$= (\cos x) \times 2x + x^2 \times -\sin x$$

$$= \mathbf{2x \cos x - x^2 \sin x}.$$

Example 3.

Differentiate $\tan^4 x$.

Solution: $\frac{d}{dx}(\tan^4 x) = 4 \tan^3 x \frac{d}{dx}(\tan x)$

$$= 4 \tan^3 x \times \sec^2 x$$
$$= \mathbf{4 \tan^3 x \sec^2 x}.$$

Example 4.

Given that $y = \frac{\sin x}{1 + \cos x}$. *Find* $\frac{dy}{dx}$.

Solution: Using the theorem

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x}{1 + \cos x}\right)$$

$$= \frac{(1+\cos x)\frac{d}{dx}(\sin x) - (\sin x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$= \frac{(1 + \cos x)(\cos x) - \sin x (0 - \sin x)}{(1 + \cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$$

$$= \frac{\cos x + 1}{(1 + \cos x)^2}$$

$$= \frac{\mathbf{1}}{\mathbf{1 + \cos x}}.$$

Example 5.

Given $x = a \cos \theta, y = b \sin \theta$, *find* $\frac{dy}{dx}$.

Solution:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \quad \ldots(i)$$

Now $\frac{dy}{d\theta} = b \cos \theta$

And $\frac{dx}{d\theta} = -a \sin \theta.$

Substituting in (i), we get

$$\frac{dy}{dx} = \frac{b \cos \theta}{-a \sin \theta} = -\frac{\boldsymbol{b}}{\boldsymbol{a}} \mathbf{cot}\ \theta.$$

PRACTICE PROBLEMS 6

Differentiate the following:

1. (i) $\tan 6x$ (ii) $\sec(ax + b)$ (iii) $\cot \sqrt{x}$

2. (i) $\cos^2 x$ (ii) $\sin^2(ax + b)$ (iii) $\sin(ax + b)^2$

3. (i) $x^2 \cos x$ (ii) $(1 + x) \sin x$ (iii) $\cos^2 x \sin x^2$

4. (i) $\sin(x^2 + 1)$ (ii) $\sin x^3$ (iii) $\sin(3x^2 + 7)$

5. (i) $\frac{\theta}{\tan \theta}$ (ii) $\frac{\cos \theta}{\theta + \sin \theta}$ (iii) $\frac{\tan \theta}{\tan \theta + \sec \theta}$

6. Given that $x = a \tan \theta; y = b \sec \theta$. Find $\frac{dy}{dx}$.

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Answers to Practice Problems

1. (*i*) $6 \sec^2 6x$

(*ii*) $a \sec (ax + b) \tan (ax + b)$

(*iii*) $-\frac{1}{2} x^{-\frac{1}{2}} \operatorname{cosec}^2 \sqrt{x}$

2. (*i*) $-2 \cos x \sin x$

(*ii*) $2a \sin (ax + b) \cos (ax + b)$

(*iii*) $2a (ax + b) \cos (ax + b)^2$

3. (*i*) $2x \cos x - x^2 \sin x$

(*ii*) $\sin x + \cos x + x \cos x$

(*iii*) $-2 \sin x \cos x \sin x^2 + 2x \cos^2 x \cos x^2$

4. (*i*) $2x \cos (x^2 + 1)$ (*ii*) $3x^2 \cos x^3$

(*iii*) $6x \cos (3x^2 + 7)$

5. (*i*) $\cot \theta - \theta \operatorname{cosec}^2 \theta$

(*ii*) $\frac{-(\theta \sin \theta + \cos \theta + 1)}{(\theta + \sin \theta)^2}$

(*iii*) $\frac{\sec \theta}{(\tan \theta + \sec \theta)^2}$

6. $\frac{b}{a} \sin \theta$.

11.4.1. Differential Coefficients of Logarithmic and Exponential Functions

(*i*) $\frac{d}{dx} \log_a (x) = \frac{1}{x} \log_a (e)$

(*ii*) $\frac{d}{dx} \log_e (x) = \frac{1}{x} \log_e (e) = \frac{1}{x}$

Because $\log_e (e) = 1$

(*iii*) $\frac{d}{dx} (a^x) = a^x \log_e (a)$

(*iv*) $\frac{d}{dx} (e^x) = e^x \log_x (e) = e^x$

(*v*) When $u = f(x)$

$$\frac{d}{dx} (\log u) = \frac{1}{u} \frac{du}{dx}$$

(*vi*) When $u = f(x)$

$$\frac{d}{dx} (e^u) = e^u \frac{du}{dx}$$

KEY NOTE

When base of the logarithmic function is not mentioned, it should be treated as *e*.

SOLVED EXAMPLES

❑ **Example 1.**

Given that $y = \log (ax + b)$. *Find* $\frac{dy}{dx}$.

Solution: Using formula (*v*)

$$\frac{dy}{dx} = \frac{d}{dx} [\log (ax + b)]$$

$$= \frac{1}{ax + b} \frac{d}{dx} (ax + b)$$

$$= \frac{a}{ax + b}.$$

❑ **Example 2.**

Given that $y = e^{\tan x}$, *find* $\frac{dy}{dx}$.

Solution: Using formula (*vi*)

$$\frac{dy}{dx} = \frac{d}{dx} \left[e^{\tan x}\right] = e^{\tan x} \frac{d}{dx} (\tan x)$$

$$= e^{\tan x} \sec^2 x$$

PRACTICE PROBLEMS 7

Obtain the differential coefficient of the following:

1. $\log \frac{1+x}{1-x}$ **2.** $\log (3x + 4)^2$

3. $\log (\cos x)$ **4.** $\log (\tan x + \sec x)$

5. $\frac{\log x}{1 + \log x}$

6. (*i*) e^{nx} (*ii*) a^{nx}

Answers to Practice Problems

1. $\frac{2}{1 - x^2}$ **2.** $\frac{6}{3x + 4}$

3. $-\tan x$ **4.** $\sec x$

5. $\frac{1}{x (1 + \log x)^2}$

6. (*i*) ne^{nx} (*ii*) $na^{nx} \log a$.

12. APPLICATION OF DERIVATIVES

12.1. Tangents and Normal

(*i*) If $y = f(x)$, then the **slope of the tangent** at P($x = c$) is

$$m = \tan \theta = \left.\frac{dy}{dx}\right]_{x = c}$$

(*ii*) **Equation of the tangent** at P(x', y') to the curve $y = f(x)$ is

$$y - y' = \left(\frac{dy}{dx}\right)_P (x - x').$$

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12.2. Maxima and Minima

(*i*) **Method to find absolute maxima and minima values in a given interval.**

Step 1: Find all points when $\frac{dy}{dx} = 0$.

Step 2: Take end points of the interval like $x = a, b, c$ ...

Step 3: At all these points, calculate values of y.

Step 4: Take the maximum and minimum values out of these values that is $f(a)$ $f(b)$ and $f(c)$

(*ii*) If $f(a)$ is an **extreme value** of $f(x)$, then $f'(a) = 0$

(*iii*) Second Derivative Test.

Step 1: Put $y = f(x)$ and find $\frac{dy}{dx}$, i.e. $f'(x)$.

Step 2: Put $\frac{dy}{dx} = 0$, i.e. $f'(x) = 0$ and solve it for x giving: $x = a, b, c, ...$

Step 3: Select $x = a$. Find $\frac{d^2y}{dx^2}$, i.e. $f''(x)$ at $x = a$.

If $\left.\frac{d^2y}{dx^2}\right]_{x=a}$, i.e. $f''(x)$ is negative, then $f(x)$ is maxima at $x = a$.

If $\left.\frac{d^2y}{dx^2}\right]_{x=a}$, i.e. $f''(x)$ is positive, then $f(x)$ is minima at $x = a$.

13. INTEGRATION

Integration is the inverse process of differentiation. Here we are given the derivative of a function and we are to find the original function. Such process is known as **integration** or **anti-differentiation** or **anti-derivative.**

Let $y = f(x)$, then differential co-efficient of y w.r.t. x will be some other function of x. We may write,

$$\frac{dy}{dx} = f'(x) \qquad ...(i)$$

From (i), we can write $dy = f'(x)\,dx$.

Here 'dy' is the infinitesimal part of 'y' called the differential of y corresponding to a variation 'dx' in x.

If we sum up all the differentials (parts) we can obtain y. That is:

Sum of all "dy" = y = sum of all $[f'(x)\,dx]$

Mathematically, we use the symbol $\int$ for denoting the above sum, so we may write

$$y = \int dy = \int f'(x)dx \qquad ...(ii)$$

This process of summing up is called integration, and $\int$ is the mathematical symbol for integration. The expression $f'(x)$ is called integrand.

From the above, we find that in differentiation we obtain differential or an infinitesimal part of y corresponding to an infinitesimal variation in x and in integration we obtain summed up value of the variable y.

Thus, integration is inverse of differentiation.

That is if $y = f(x)$ and $\frac{dy}{dx} = f'(x)$

Then $y = f(x) = \int f'(x)dx$

Example, let $y = x^{n+1}$, then

$$\frac{dy}{dx} = (n+1)\,x^{n+1-1} = (n+1)\,x^n$$

$$\therefore \quad dy = (n+1)x^n\,dx$$

or $$\int dy = \int (n+1)x^n dx = (n+1)\int x^n dx$$

or $$\frac{x^{n+1}}{n+1} = \int x^n dx$$

Why do we need to integrate within limit ?

Note: Let $y = x^7 + c$, where c is a constant, then

$$\frac{dy}{dx} = 7x^6 + 0 = 7x^6$$

$\therefore \int 7x^6 dx$ should be equal to $x^7 + c$. However, according to the mathematical formulae described earlier, we have

$$\int 7\,x^6\,dx = x^7.$$

That is, during integration the constant 'c' does not re-appear. It is because of the reason that the differential co-efficient of the constant is zero.

Evidently, we cannot say definitely whether $\int 7x^6\,dx$ is x^7 or $x^7 + c$. Such integration is called **Indefinite Integration.** In fact, to all the integrals we have obtained so far, we should add 'c' a constant. However, c may turn out to be zero. The value of 'c' depends upon the problem under consideration or the boundary conditions (limit) of the problem.

13.1. Definite Integral

The integration may be carried out between definite limits.

If $y = f(x)$ and $\frac{dy}{dx} = f'(x)$

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Then the value of y within limits $x = a$ to $x = b$ is expressed as

$$[y]_{x=a}^{x=b} = \int_a^b dy = \int_a^b f'(x)\,dx$$

That is $[y]_{x=a}^{x=b} = \int_a^b f'(x)\,dx = [f(x)]_{x=a}^{x=b}$

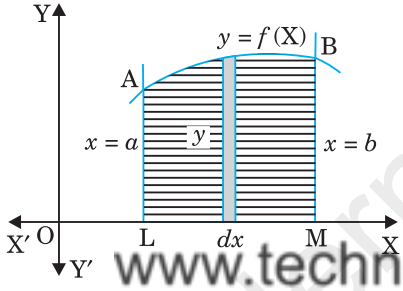
$$= f(b) - f(a)$$

13.1.1. Area Under Simple Curve

To find the area, bounded by the curve $y = f(x)$, X-axis the ordinates $x = a$ and $x = b$.

The required area is composed of a number of thin vertical strips. Let the arbitrary strip be of height y and width dx.

Then dA, area of elementary strip $= y\,dx$, where $y = f(x)$.



This area is known as **elementary area.**

$\therefore$ A is the total area of the region ALMB

$$A = \int_{x=a}^{b} dA.$$

Hence, $A = \int_{x=a}^{b} y\,dA = \int_a^b f(x)\,dx$

$$= [F(x)]_{x=a}^{x=b} = [F(b) - F(a)]$$

13.1.2. Theorems of Integration

(*i*) If $n \neq -1$, then

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}\left[\because \frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n\right]$$

(*ii*) $\int (u \pm v \pm w)\,dx = \int u dx \pm \int v dx \pm \int w dx$

That is, the integral of sum or difference of a number of functions is equal to the sum difference of their integrals.

(*iii*) $\int c dx = cx$, where c is a constant.

(*iv*) $\int cx^n\,dx = c\,\frac{x^{n+1}}{n+1}$ where c is constant.

(*v*) $\int x^{-1}\,dx = \log_e x\left[\because \frac{d}{dx}(\log_e x) = \frac{1}{x}\right]$

[Note that in theorem 1, we have assumed $n \neq -1$ for this reason]

(*vi*) $\int e^x\,dx = e^x$

(*vii*) $\int a^x\,dx = \frac{a^x}{\log_e a}$

(*viii*) $\int e^{ax+b}\,dx = \frac{e^{ax+b}}{a}$

SOLVED EXAMPLES

Example 1.

Find $\int x^7\,dx$.

Solution: Using theorem 1. Here $n = 7$

$$\int x^7\,dx = \frac{x^{7+1}}{7+1} = \frac{x^8}{8}.$$

Example 2.

Find $\int (x^6 + x^{-6})\,dx$

Solution: $\int (x^6 + x^{-6})\,dx = \int x^6\,dx + \int x^{-6}\,dx$

$$= \frac{x^{6+1}}{6+1} + \frac{x^{-6+1}}{-6+1}.$$

$$= \frac{x^7}{7} - \frac{x^{-5}}{5}$$

Example 3.

Integrate the following:

$\int (7e^{5x} + 7^x + 3)\,dx.$

Solution: $\int (7e^{5x} + 7^x + 3)\,dx$

$$= \int 7e^{5x}\,dx + \int 7^x\,dx + \int 3dx$$

$$= 7\,\frac{e^{5x}}{5} + \frac{7^x}{\log_e 7} + 3x.$$

Example 4.

Evaluate $\int (5^x + e^{5x})\,dx$

Solution: $\int (5^x + e^{5x})\,dx = \int 5^x\,dx + \int e^{5x}\,dx$

$$= \frac{5^x}{\log_e 5} + \frac{e^{5x}}{5}.$$

PRACTICE PROBLEMS 8

1. Obtain the following integrals:

(*i*) $\int x^{15}\,dx$ (*ii*) $\int t^7\,dt$ (*iii*) $\int x^{1/2}\,dx$

(*iv*) $\int x^{-15}\,dx$ (*v*) dx.

2. Integrate the following:

(*i*) $\int (3x^{-7} + x^{-1})\,dx$ (*ii*) $\int (8 + x)\,dx$.

3. Find $\int\left(x + \frac{1}{x}\right)dx$.

4. Evaluate $\int e^{3x+4}\,dx$.

5. Find $\int (e^{-5x} + 3)\,dx$.

6. Obtain the following integrals:

(*i*) $\int 6^t\,dt$ (*ii*) $\int (6^4 + 9^6)\,dx$.

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Answers to Practice Problems

1. (*i*) $\frac{x^{16}}{16}$ (*ii*) $\frac{t^8}{8}$ (*iii*) $\frac{2x^{3/2}}{3}$

(*iv*) $-\frac{x^{-14}}{14}$ (*v*) $-2x^{-\frac{1}{2}}$

2. (*i*) $-\frac{1}{2}x^{-6} + \log x$ (*ii*) $8x + \frac{x^2}{2}$

3. $\frac{x^2}{2} + \log x$

4. $\frac{e^{3x+4}}{3}$

5. $-\frac{e^{-5}}{5} + 3x$

6. (*i*) $\frac{6^t}{\log 6}$ (*ii*) $(6^4 + 9^6)x$

13.2. Integrals of Trigonometric Functions

(*i*) $\int \sin x\, dx = -\cos x$

(*ii*) $\int \sin(ax)\, dx = -\frac{\cos ax}{a}$

(*iii*) $\int \cos x\, dx = \sin x$

(*iv*) $\int \cos ax\, dx = \frac{\sin ax}{a}$

(*v*) $\int \sec^2 x\, dx = \tan x$

(*vi*) $\int \text{cosec}^2\, dx = +\cot x$

(*vii*) $\int \sec x \tan x\, dx = \sec x$

(*viii*) $\int \text{cosec}\, x \cot x\, dx = -\text{cosec}\, x.$

SOLVED EXAMPLES

Example 1.

Integrate the following:

$$\int (\sin 6x + \cos 5x + \sec^2 x)\, dx$$

Solution: $\int (\sin 6x + \cos 5x + \sec^2 x)\, dx$

$= \int \sin 6x\, dx + \int \cos 5x\, dx + \int \sec^2 x\, dx$

$= -\frac{\cos 6x}{6} + \frac{\sin 5x}{5} + \tan x.$

Example 2.

Integrate the following:

$$\int \frac{1}{1+\sin\theta}\, d\theta$$

Solution:

$$\int \frac{1}{1+\sin\theta}\, d\theta = \int \left[\frac{1}{1+\sin\theta}\right]\left[\frac{1-\sin\theta}{1-\sin\theta}\right] d\theta$$

$$= \int \frac{1-\sin\theta}{1-\sin^2\theta}\, d\theta = \int \frac{1-\sin\theta}{\cos^2\theta}\, d\theta$$

$= \int (\sec^2\theta - \sec\theta\tan\theta)\, d\theta$

$= \tan\theta - \sec\theta.$

13.3. Integration by Parts

Theorem: Integral of the product of the functions

= [1st function × integral of the second − ∫(differential coefficient of 1st function × integral of second) *dx*.]

That is, if $u = f(x)$ and $v = f(x)$, then

$$\int uv\, dx = u\int v\, dx - \int \left[\frac{du}{dx} \times \int v\, dx\right] dx$$

SOLVED EXAMPLE

Example 1.

$\int x \sin x\, dx.$

Solution: Let $u = x$ and $v = \sin x$

Then using $\int uv\, dx = u\int v\, dx - \int \left[\frac{du}{dx} \times \int v\, dx\right] dx$

$$\int x \sin x\, dx = x\int \sin x\, dx - \int \left[\frac{d(x)}{dx} \times \int \sin x\, dx\right] dx$$

$= x(-\cos x) - \int (-\cos x)\, dx$

$= -x\cos x + \int \cos x\, dx$

$= -x\cos x + \sin x.$

PRACTICE PROBLEMS 9

Integrate the following:

1. (*i*) $\int \cos^2\theta\, d\theta$ (*ii*) $\int \tan^2\theta\, d\theta.$

2. (*i*) $\int \frac{1}{1-\sin x}\, dx$ (*ii*) $\int \frac{1}{1-\cos x}\, dx$

3. $\int \left(t - \cos\omega t + \frac{1}{t}\right) dt$

4. Evaluate $\int \tan x\, dx.$

5. Find $\int \sin^3 x \cos x\, dx.$

6. Evaluate $\int \tan^2\theta \sec^2\theta\, d\theta.$

7. Integrate (*i*) $\frac{4x^3}{1+x^4}$ (*ii*) $\frac{x}{1+x^2}$.

8. Evaluate $\int x^3 \sin x^4\, dx.$

9. Integrate $\frac{1}{x^3}\cos\frac{1}{x^2}$.

10. Evaluate $\int \theta^{-\frac{1}{2}} \sin\theta^{\frac{1}{2}}\, d\theta.$

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Answers to Practice Problems

1. (*i*) $\frac{\theta}{2}+\frac{1}{4}\sin 2\theta$. **Hint :** $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

(*ii*) $\tan\theta - \theta$. [**Hint.** $\tan^2\theta = \sec^2\theta - 1$]

2. (*i*) $\tan x + \sec x$ (*ii*) $-(\cot x + \operatorname{cosec} x)$

[**Hint :** $\frac{1}{1-\sin x} = \frac{1}{1-\sin x}\times\frac{1+\sin x}{1+\sin x}$

$= \frac{1+\sin x}{1-\sin^2 x} = \frac{1+\sin x}{\cos^2 x}$]

3. $\frac{t^2}{2} - \frac{\sin\omega t}{\omega} + \log t$.

4. $-\log\cos x$.

5. $\frac{\sin^4 x}{4}$

6. $\frac{\tan^3\theta}{3}$

7. (*i*) $\log(1+x^4)$ (*ii*) $\log(1+x^2)$

8. $-\frac{\cos x^4}{4}$

9. $-\frac{1}{2}\sin\frac{1}{x^2}$

10. $-2\cos\theta^{\frac{1}{2}}$

13.4. Indefinite Integrals

Now the value of y between $x = a$ and $x = b$ is given by the value of y at $x = b$ **minus** value of y at $x = a$, that is:

$[y]_{x=a}^{x=b}$ = (value of y at $x = b$) – (value of y at $x = a$)

= $f(x)$ at 'b' – $f(x)$ at 'a'

$\therefore \quad \int f'(x)\,dx = [f(x)]_{x=a}^{x=b} = f(b) - f(a)$

Such an integral between definite limits is called definite integral.

SOLVED EXAMPLE

❑ **Example 1.**

Integrate the following:

$\int_3^6 (u+at)\,dt$ *where u and a are constants.*

Solution: $\int_3^6 (u+at)\,dt = \int_3^6 u\,dt + \int_3^6 at\,dt$

$= [ut]_3^6 + \left[\frac{1}{2}at^2\right]_3^6$

$= [u\times 6 - u\times 3] + \left[\frac{1}{2}a\times 36 - \frac{1}{2}a\times 9\right]$

$= 3u + \frac{27}{2}a$.

PRACTICE PROBLEMS 10

Evaluate the following:

1. (*i*) $\int_0^{\pi/2}\sin\theta\,d\theta$ (*ii*) $\int_0^{\infty} x^{-1/2}\,dx$

2. (*i*) $\int_5^6 e^x\,dx$ (*ii*) $\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta$

3. Evaluate $\int_R^{\infty}\frac{GMm}{x^2}\,dx$

4. Find the value of $\int_u^v Mv\,dv$

5. Evaluate $\int_{r_1}^{r_2}\left(r - \eta\frac{q_0 q}{r^2}\right)dr$

6. Find $\int_0^{\pi/2}(1-\cos\theta)^{1/2}\,d\theta$

[**Hint :** $1-\cos\theta = 2\cos^2\frac{\theta}{2}$.]

7. Evaluate $\int_0^{\infty}\frac{dt}{t}$

8. Evaluate (*i*) $\int_0^T E_0 I_0 \sin\omega t\sin(\omega t+\phi)\,dt$

(*ii*) $\int_0^T I_0\sin\omega t\,dt$

(*iii*) $\int_0^{T/2} I_0\sin\omega t\,dt$

Given that $\omega T = 2\pi$.

Answers to Practice Problems

1. (*i*) 1 (*ii*) ∞

2. (*i*) $e^6 - e^5$ (*ii*) 2

3. $\frac{GMm}{R}$

4. $\frac{1}{2}M(v^2-u^2)$

5. $\eta q q_0\left[\frac{1}{r_2}-\frac{1}{r_1}\right]$

6. 2

7. ∞

8. (*i*) $\frac{T}{2}E_0 I_0\cos\phi$ (*ii*) Zero (*iii*) $\frac{2I_0}{\omega}$.

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CHAPTER

1 PHYSICAL WORLD

1. WHAT IS PHYSICS?

Curious nature of humans about everything around them has led the world to the age of modern science and technology. The word Science is derived from Latin word Scientia which means "to know". Science is a systematic way to understand the natural phenomenon in detail so that the knowledge gained may be used to predict, modify and control the phenomenon. Science starts from the exploration of whatever we see around us. Exploration leads to the ideas to perform experiments and outcome of these experiments helps us in developing theories to explain the nature. It all started with careful observation of physical environment. People observed meaningful patterns in natural phenomenon so that tools may be developed to interact with the nature. Earlier progress in the field of science was very limited and slow. But by the middle of the twentieth century people from various countries started making significant contributions leading to rapid growth in the field of science. And today, science has completely changed the way we are leading our lives.

The word Physics is derived from a Greek word "fusis" which means nature. So we can say that physics is the part of natural sciences like geology, chemistry and biology. Defining physics precisely is difficult but in a broad sense we can say that ***physics is the study of basic laws that govern and control the nature and natural phenomenon***. We know that there is regular repetition of day and nights. We experience different kinds of seasons and their regular repetition. There are several other natural phenomena like rain, eclipse, volcanoes, earthquakes, storms, tornadoes, rainbow, tides, etc. Humans were always naturally curious about all these phenomena. This curiosity and prolonged observation led people to predict eclipses in those ancient times when we were deprived of technology.

There are two main aspects of science: one is theory and the other is experimentation. Interrelation between theory and experimental observations is key to the progress of science. No theory of science can be declared as final theory because science is always dynamic. Many times new experiments yield new results and existing theory may or may not be able to explain the new findings. Modification of an existing theory is often required when it is unable to explain the new findings. Sometimes modifications are minor and little changes and additions to the existing theory may be enough to include new observations but sometimes changes may be drastic. But often older theory exists

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KEY NOTE

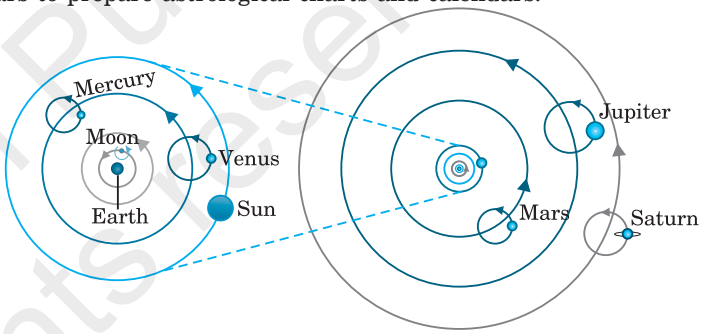
The exploration of whatever we see around us scientifically uses scientific methods which involve interdependent steps like systematic observations, controlled experimentation, qualitative and quantitative reasoning, mathematical modelling, further prediction, verification and falsification.

as a special case of new generalised theory. Let us take an example of propagation of sound waves in gaseous medium. At first, Newton proposed that propagation of sound waves in gaseous medium is an isothermal process but experimental findings proved it wrong. Later, another scientist named Laplace made correction to this idea and told that propagation of sound waves in gaseous medium is adiabatic process. Experimental results supported the Laplace correction.

Nicolaus Copernicus proposed a theory that the Sun is at the centre and planets are moving around the sun in circular orbits. But when Johannes Kepler analysed the data collected by Tycho Brahe, then he concluded that orbits are not circular rather planets move around the sun in elliptical orbits.

Competition Edge

- Ptolmey did lot of observations and mathematical calculations and was the first person to represent the most detailed geocentric model with earth as the centre of universe and other planets and the sun moving around it in circular orbits. The same model was used for over 1,500 years to prepare astrological charts and calendars.



Ptolmey's Geocentric model

- However in 1543, during scientific revolution Nicolaus Copernicus proposed another theory of heliocentrism stating that the sun is the centre of solar system with all the planets moving around it. This heliocentric theory replaced the geocentric model given by Ptolmey.

Light is a wave phenomenon and wave theory of light was established but discovery of photoelectric effect became a turning point. Wave theory of light was unable to explain some aspects related to photoelectric effect. Einstein explained photoelectric effect on the basis of particle theory of light and finally dual character of light was established. It was established that in certain situations light behaves as a wave but there are certain other situations when light behaves as particles. Light particles are called photons. Photons carry momentum just like some other moving matter particle and hence can apply force on the surface of incidence. Another scientist De-Broglie extended this concept of duality for matter. De-Broglie introduced the concept of matter waves. Nature is found to be symmetric in many cases. Here we can see that as light wave can behave like streams of particles in certain cases, similarly matter can also behave like wave in certain situations. But for normal day-to-day objects, this wave character of matter is not pronounced. Wave character of matter becomes more significant when we talk about atoms, molecules and subatomic particles. It was around the beginning of the twentieth century when it was realised that Newtonian mechanics is not able to explain some features of atomic phenomenon.

Learning & More

Ibn al-Haytham was a Muslim scientist, mathematician, philosopher and astronomer from Arab, who was also the first theoretical physicist. He was the first to give the concept of scientific methods to prove any hypothesis.

He was the first to explain the vision in humans and stated that we see the things due to reflection of light.

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Later on, Quantum Mechanics was developed to deal with atomic and molecular phenomenon. It is basically wave character of matter which becomes significant when we talk about such small particles. Prediction of behaviour of these particles on the basis of Newtonian mechanics only becomes difficult. Quantum mechanics takes wave character of matter also into account.

It is not always true that only experimental outcomes help in correcting theories but many times theoretical advancements can also help to develop the idea for some experiments. For example, concept of antiparticle was developed theoretically by Paul Dirac but it was confirmed two years later, when Carl Anderson made experimental discovery of positron which is an antiparticle of electron.

Unification is an important approach in physics because we want to explain diversity of physical phenomenon only in terms of a few rules and concepts. Basically, we believe that there are only a few universal laws which when applied in different domains and conditions can be used to explain everything. We can further understand this approach of unification by taking an example of gravitational force. Same law of gravitation can be used to explain the fall of an apple from a tree, motion of the moon around the earth and also the motion of planets around the sun. Similarly, there are some basic laws of electromagnetism and we can explain all electromagnetic phenomena using them. Further efforts are made to unify all fundamental forces that exist in nature.

Reductionism is another important approach used in physics. We know that ultimately there are only a few rules which create everything. When same rule is applied repeatedly at different levels then complex and bigger systems are created. So we can understand that by analysing the behaviour and properties of smaller parts of some bigger system, we can predict the behaviour and properties of bigger system also. Such kind of approach is called reductionism. Here is one good example to understand the approach of reductionism. In kinetic theory of gases we explain temperature, energy, etc. in terms of properties of molecular constituents of the bulk system and on the other hand in thermodynamics we use same quantities like temperature or internal energy to predict the behaviour of bulk system.

REMEMBER

Unification is an important approach in physics which allows us to explain the diversity of physical phenomenon only in terms of a few rules and concepts.

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2. SCOPE AND EXCITEMENT OF PHYSICS

Scope of physics is extended to all domains of life. Even biological and medical sciences depend on the knowledge of physics. We know that physics talks about nature and so everything under the sun and beyond the sun is directly or indirectly related to the physics. Scope of physics can be understood by looking at its various sub-disciplines. Broadly we can divide physics into two domains of interest. One is macroscopic and the other is microscopic. In macroscopic domain we talk about phenomenon at the laboratory, terrestrial and astronomical scales. However, in microscopic domain we talk about atoms, molecules and nuclear processes.

Macroscopic phenomena are studied in Classical Physics. Classical physics includes subjects like Mechanics, Thermodynamics, Optics and Electromagnetism. When we talk about particles having linear dimension greater than 10^{-6}m moving with speed less than the speed of light, then classical physics can be considered as good description of nature. Mechanics is based on Newton's Laws of motion and law of gravitation. In mechanics we talk about the motion or

REMEMBER

According to reductionism, by analysing the behaviour and properties of smaller parts of some bigger system, we can predict the behaviour and properties of bigger system also.

equilibrium of particles, general system of particles, rigid bodies and deformable bodies. In thermodynamics we do not talk about motion of bodies as whole but we are interested in the motion of microscopic constituents of the system which is related to internal energy, temperature, entropy, etc. In thermodynamics we talk about change in internal energy of the system through external work or heat transfer. In thermodynamics we talk about heat engines and refrigerators. Optics is the study of phenomenon involving light. We study about laws of reflection and refraction to predict the behaviour of light. We study about lenses and mirrors in optics which are building blocks of several optical instruments like telescope, microscope, etc. Electromagnetism talks about electrical and magnetic phenomenon. Electrons and protons are two charged particles of opposite nature, available in materials. Electric field is always present around the electric charge whether the charge is in the state of rest or motion. But when the charge moves then it also generates magnetic field around it. Basic laws of electromagnetism were given by Coulomb, Oersted, Ampere and Faraday. Maxwell formulated basic laws in terms of set of four equations collectively known as Maxwell's equations. Using Maxwell's equations we can explain any electromagnetic phenomenon.

In microscopic domain we study about phenomenon related to atoms, molecules, nucleus and subatomic particles like electron, proton and neutron. Basically we talk about constitution and structure of the matter in terms of atoms and molecules. In microscopic domain we talk about basic internal structure of atoms. We know that matter can exist like a wave or a particle. Wave character in matter is not appreciable when we deal with macroscopic scales but it becomes very significant when we deal with subatomic particles like electron, proton and neutron. Classical physics does not account for wave nature and hence is unable to explain phenomenon related to atomic scales. Quantum physics is accepted widely as one proper framework for explaining microscopic phenomenon.

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We can now understand that the scope of physics is really vast. At one end we learn about internal structure of an atom which involves approximately 10^{-30} kg particles like electron and on the other end we learn about the system of size of universe which involves a total mass 10^{55} kg, which is believed to be the mass of observable universe. We can also talk in terms of range of length that we deal in physics. At one end we study about phenomenon at atomic scale where we deal with lengths as small as 10^{-14} m or even less, whereas at the other end we talk about galaxies or entire universe which we have explored to the length 10^{26} m. Similarly, we can divide length by speed of light to get the range of time interval that we deal in physics and it turns out lying in between 10^{-22} s and 10^{18} s.

A new domain of interest has emerged in between microscopic and macroscopic, and the same is called mesoscopic. At the mesoscopic scale we deal with a few tens or hundreds of atoms.

KEY NOTE

Classical physics deals with the particles with linear dimension greater than micrometres and moving with speed much less than speed of light.

There are many reasons to say that physics is one of the most exciting subjects to study. Most exciting feature about physics is universality of its basic theories and concepts. We can use same concepts in different contexts to understand wide variety of phenomenon taking place around us. It is not only qualitative thinking but quantitative measurement is also possible since all the basic laws of nature can

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be expressed in the form of mathematical equations. Mathematics is the language of physics and it is really wonderful that such a vast realm of nature can be understood with the help of some mathematical equations. "**The most incomprehensible thing about the universe is that it is comprehensible**". The above statement was made by Einstein and he was correct because there is always some logic behind everything but we may or may not be aware of that logic. Many learners of physics find experimentation as an exciting feature of physics. They think about new experiments to unlock the hidden secrets of nature. Experiments are not only important for discovering new things but also to verify the existing laws. Regarding his proposed theories, Einstein said **"No number of experiments can prove me right, a single experiment can prove me wrong".** So we can say that an existing law of physics may not be an eternal truth and thus continuous verification is necessary with the help of new kinds of experiments. For example, Newton's laws of motion failed to agree for particles moving with the speeds comparable to that of light and it was discovered on the basis of experiments. Einstein's special theory of relativity is needed to explain the behaviour of a particle moving at such a high speed. Another exciting area of interest is Applied Physics. In this field we apply physical laws to make devices which are important and useful for humans but it requires a lot of ingenuity and persistence of efforts. Just take an example of washing machine. At early stage washing machines were developed as electromechanical device and human intervention was required to finish the task but with the advancement of technology people are able to develop fully automatic washing machines which require minimum human intervention. It is the hard work of people that today we are trying to build a driverless car. There is no limit to the progress and so there is no limit to the excitements of physics.

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We know that there are some basic laws in physics and these laws are universally applicable in many situations. We should understand that any phenomenon we observe in nature is complicated manifestation of the basic laws. But in a particular context everything may not be equally important. So to understand the phenomenon we should recognise essential features first. In first go, we can neglect the less important features to discover the basic rules and then we can add further corrections for the refinements in theory. We can take an example of free fall under gravity. Due to attraction of earth, all objects near the surface of earth fall with the same acceleration. Suppose you have to verify this by comparing free fall of two objects. One object is feather and the other object is a small stone. But in this case you will find that feather is taking more time than a stone to fall through the same height. Here we need to understand that presence of air resistance is complicating this situation of free fall. Air resistance in case of feather is significant and hence is also affecting its motion. If we can create a long evacuated tube so that there is no air resistance inside it then we can see that feather and stone are taking precisely the same time to fall through the same height. It is not practical to consider all complexities in one go. As next step we can study about air resistance to implement it along with the gravity so that theory of free fall may be used in more realistic situation. Hence we get to know that by separating important features of a phenomenon from less important features, we can easily understand the phenomenon and can manage the complexities involved in it.

Captivate Your Curiosity

Albert Einstein was a theoretical physicist born in Ulm, Germany in 1879. He is known as the greatest physicist of all time for his remarkable works in physics. He explained the concept of photoelectric effect using the concepts of light quanta, photons. He was given the Nobel Prize for the same in year 1921. He explained Brownian motion and proposed the convincing picture of matter on atomic level. He proposed his own version of theory of relativity, now known as *The General Theory of Relativity*. He further worked on his theory and proposed the matter-energy equivalence equation ($E = mc^2$). Some of his work also includes topic like Plank's body radiation, static model of universe, analysis of quantum mechanics, quantitative statistics of a gas of a massive boson, gravitational waves, etc. For his revolutionary scientific ideas published in 1905 in forms of papers, year 2005 was declared as International Year of Physics.

3. PHYSICS, TECHNOLOGY AND SOCIETY

Knowledge of physics gives rise to the technology and the technology is supposed to be used for the benefit and development of the society. Apart from a few bad examples like atom bomb, technology has always helped in improving human life. Today we are witnessing the era of high technology and we should know that any field of science and technology cannot sustain and make progress without using established concepts and laws of physics. Need is mother of inventions and it can be easily understood.

At the early stage of human civilisation, travelling from one place to another was very difficult and time-consuming process. People were using animals to commute from one place to other. Invention of wheel was the first major step to improve the travelling. But still animals were used to pull the carts with wheels. People were still not satisfied with the results and slowly realised the need of some machine to help move the vehicle with faster speeds and reliability. Heat engines were developed and branch of thermodynamics arose from this need. Laws of thermodynamics were used to develop steam engine in England in the eighteenth century. This development triggered industrial revolution in England and greatly affected human civilisation. Today we have all kinds of vehicles which can move on road, water or air. Today we have supersonic planes which can move with a speed greater than that of sound. No one can deny the positive effect of reduced travel time on the human society.

Next important aspect for human progress is information exchange. At an early stage written messages from one person to another were delivered physically by people. Process was again time consuming and could not be afforded by everyone. Slowly several types of vehicles were developed and travel time was reduced, so organised postal services were started by governments and common people also started using this facility. But this process of information exchange was still time consuming and people were not satisfied with it. Invention of telephone was like dream come true and slowly telephone reached the common people. It was really wonderful for the people to talk to the other person who is sitting far away. But there is no end to the progress. In the nineteenth century, laws of electricity and magnetism were discovered and the existence of electromagnetic waves was established. It is not always easy to foresee the applications of discovered physics. Very soon power of electromagnetic waves was realised and it triggered the development in the field of wireless communication. Earlier, telephone was a fixed device but today we have mobile phones and can talk to people while on move. Silicon chip is another great application of physics which triggered computer revolution. Computer technology has developed very fast in the last three decades of the twentieth century. Internet technology which is driven by computers connected across the globe has become a boon for the information exchange. Today we can find information on any subject at any time by surfing the Internet.

Nothing seems impossible if we have great determination to progress. It was during 1933 when great physicist Rutherford had dismissed the idea of producing energy from atoms. But a few years later in 1938, neutron-induced fission of uranium was discovered by Hahn and Meitner. Nuclear power reactors and nuclear weapons are based on this discovery. Still it is not very easy to handle a nuclear power reactor and we are using fossil fuels. We should be aware that there is limited

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amount of fossil fuels available and such resources are being consumed very fast so there is an urgent need to develop alternate sources of energy which are inexhaustible and cheap. There is a considerable progress in this direction. For example, solar energy, wind energy, geothermal energy, etc. are being used to produce electricity. But still a lot of scope is there for further improvements.

From the above discussion we can understand the connection between physics, technology and society. Physics is the study of nature and natural phenomenon. Based on observations, experimentation and analysis, physicists try to discover basic rules and laws which govern the natural world. To start with physics we must have an idea of fundamental forces existing in our nature.

Physics and Technology

Technology	Scientific principle
Refrigerator	Laws of Thermodynamics
Computer	Digital Logic
Television	Electromagnetic Waves
Radio	Electromagnetic Waves
Nuclear Power Generation	Controlled Nuclear Fusion
Sonar	Reflection of Ultrasonic Waves
Optical Fibres	Total Internal Reflection
Rocket Propulsion	Newton's Law of Motion
Ship	Archimedes Principle
Aeroplane	Bernoulli's Theorem
Electric Generator	Faraday's Law of Electromagnetic Induction
Microscope	Optics
Electron microscope	Wave Nature of Electrons
Electric Transformer	Electromagnetic Induction
Giant Metre Wave Radio Telescope (GMRT, Pune)	Detection of Radio Waves
Photocell	Photoelectric Effect
Hubble Space Telescope (HST)	Optics
Microwave	Electromagnetic Radiation
Cell Phone	Electromagnetic Waves
Global Positioning System (GPS)	Electromagnetic Waves
CD Player	Magnetism

4. FUNDAMENTAL FORCES IN NATURE

To understand the scientific concept of force we need to learn physics but we already have intuitive notion about force. We mechanically interact with surrounding objects by applying force on them and also experience forces applied by those objects on us. When we try to walk inside the swimming pool then we can feel the force applied by the water molecules on our skin. Similarly we can visualise the force applied on us by surrounding atmospheric air. We must have learned about pressure in previous classes and force applied by liquid and gases is visualised in terms of pressure. In mechanics we study about the scientific role of force in motion or equilibrium of different types of objects. But on intuitive level we are already expert in doing these things. Just watch yourself that you move different objects including your own bodies in a

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Learning & More

It is believed that gravitational force arises due to exchange of particles known as "graviton" between objects. For graviton, charge = 0, mass = 0, spin = 2. Gravitational force is very weak force and due to this fact there is no experimental evidence of this particle called graviton.

complicated fashion to live your daily life, by applying required force in a particular way. In later chapters of mechanics we shall learn about the science behind all these phenomena. In subsequent chapters we will learn about several types of forces. For example: Contact force between bodies along the normal to their surface of contact, Friction which is contact force between bodies parallel to their surface of contact, Tension applied by taut strings and ropes, Force of gravitation, Force applied by springs, Force due to surface tension in liquids, Force of buoyancy, Force of viscosity, Force due to pressure applied by liquids and gases, Electromagnetic forces between charged particles, Nuclear forces binding protons and neutrons together as nucleus, and so on.

Around the twentieth century it was realised that there are only some fundamental forces in nature and the rest of the forces can be explained in terms of these fundamental forces only. In this chapter we shall not discuss about exact quantitative description of these forces. These forces are introduced briefly in this chapter. According to present stage of our understanding there are only four fundamental forces in nature and the same are described here.

4.1. Gravitational Force

Everything around us is made up of some kind of matter and mass is one of the most important characteristic of matter. Gravitational force is the force of mutual attraction between any two objects due to their masses. Gravitational force is a universal force which means all objects inside universe experience gravitational force due to each other. Gravitational force acts over long distance and is unaffected by the intervening medium between objects. Magnitude of the gravitational force is directly proportional to the product of masses of the objects and inversely proportional to the square of the distance between the objects. But it is weakest among all fundamental forces. Magnitude of gravitational force is appreciable when at least one of the two objects has large mass. For example, in our near surroundings, earth is the heaviest object and hence force of gravitation applied by earth on all objects near its surface is very much significant and is known as weight of the objects. In our day-to-day life it is a good approximation to neglect gravitational force acting between any two objects except that applied by earth on objects. Role of gravitational force is observed in large scale phenomenon of the universe. For example, gravitational force plays central role in: motion of the moon around the earth, motion of man-made satellites orbiting around the earth, motion of earth and other planets around the sun, formation and evolution of stars and galaxies. We can see that gravitational force is although weakest among all fundamental forces but with a long range of application it is capable of holding the universe together.

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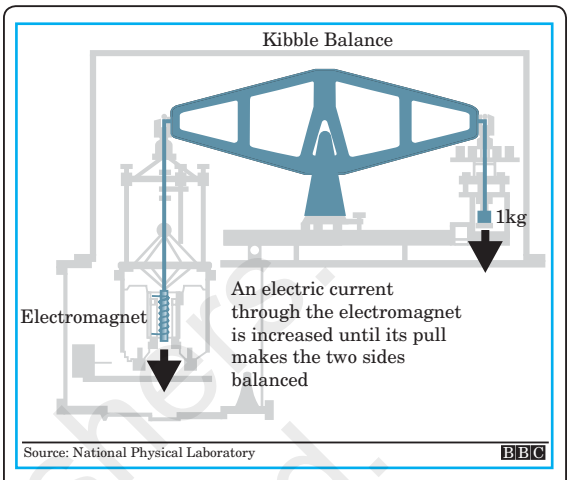
Captivate Your Curiosity

In space sciences, **gravity assist** is a boon for a travelling spacecraft. It is used as a manoeuvre to gain or reduce the speed of spacecraft crossing the orbits of planets or moons. Spacecraft Voyager 2 took two gravity assist from Jupiter and Saturn to escape the solar system.

4.2. Electromagnetic Force

Every material around us is made up of atoms. We have several types of atoms in our nature and the same are listed in periodic table. All atoms are assembly of three types of particles only which are known as electron, proton and neutron. Although internal structure of proton and neutron has also been discovered but for most of the discussions, we can still treat above stated three particles as fundamental particles behind all materials. Electron and proton are the two particles which have inherent property of charge. There is no charge in neutron. Charge is known as electrical property and we call neutron as an electrically neutral particle. As mass gives rise to gravitational force, charge gives

rise to the electromagnetic force. Hence, we can understand that neutron cannot apply or experience the electromagnetic force with other particles. In a simple situation when charges are at rest then they can apply only electric forces on each other. Nature of charge on electron and proton is opposite, although same in magnitude. This is something which makes charge different from mass because mass is only of one type. These two kinds of charges cancel the electrical effect of each other when put together and that is why, as a convention, electron is assigned negative charge and proton is assigned positive charge. Similar charges repel each other and opposite charges attract each other with this electric force. Note that gravitational force is always attractive by nature. Coulomb arrived at formula for electric force acting between charges. Same as gravitational force, magnitude of electric force is proportional to the product of the two charges and inversely proportional to the square of the distance between charges. When charged particles are in a state of motion then they apply another type of force also on each other which is called magnetic force. This is in addition to the electric force. Electrical effect is always there whether the charged particle is in a state of rest or motion but magnetic effects arise only when the charged particles are in a state of motion. We should know that subatomic particles are always in state of motion and hence both the forces are always in action when we talk about atoms. These two forces play an important role in the formation of several types of materials. Electric and magnetic effects are in general inseparable from each other and that is why we refer these forces as electromagnetic forces. Electromagnetic forces are also long range forces like gravitational force and do not require any intervening medium between objects. But unlike gravity, intervening medium affects the electromagnetic force acting between the objects. Mechanism of magnetic force is different but electric force is always compared with gravitational force. Electric force is much stronger than gravitational force and electric force between two protons is found to be 10^{36} times of gravitational force acting between them for the same distance. But here one important point should be noted that atom in natural state is neutral due to equal number of electrons and protons in it. So matter is electrically neutral in a normal situation and these electromagnetic forces are not usually observed and thus we can see that gravitational force dominates terrestrial phenomenon, in spite of the fact that electromagnetic forces are much stronger than gravitational forces. But electromagnetic forces are responsible for everything happening at the atomic or molecular level. After a certain height in our atmosphere the air particles get ionised and we can see electrical effects like lightening or auroras in the sky. We should be aware of the fact that all the forces that we experience and apply in our daily life are basically electromagnetic forces. Just take a simple example of someone holding the book against gravity. It is electromagnetic interaction between the particles of hand and book at the atomic and molecular level which manifests necessary force to hold the book. Similarly, we can take an example of frictional force, and this force also arises due to electromagnetic interaction at the atomic level between objects near their contact surfaces. A block of iron is solid and tough because there are strong electromagnetic forces operating between iron atoms which hold them together strongly. Electromagnetic forces between water molecules are small and hence it can flow and adapt the shape of a vessel in which it is poured. We can take an example of force of tension applied by tout strings and


Kibble Balance
1kg
Electromagnet
An electric current through the electromagnet is increased until its pull makes the two sides balanced
Source: National Physical Laboratory
BBC
How an electromagnet can calculate the kilogram

Captivate Your Curiosity

Kilogram gets a new definition

- Currently, it is defined by the weight of a platinum-based ingot called "Le Grand K" which is locked away in a safe in Paris.
- Researchers meeting in Versailles voted to get rid of it in favour of defining a kilogram in terms of an electric current.
- In a world where accurate measurement is now critical in many areas, such as in drug development, nanotechnology and precision engineering.

How does the new system work?

- Electromagnets generate a force. Scrap-yards use them on cranes to lift and move large metal objects, such as old cars. The pull of the electromagnet, the force it exerts, is directly related to the amount of electrical current going through its coils. There is, therefore, a direct relationship between electricity and weight.
- So, in principle, scientists can define a kilogram, or any other weight, in terms of the amount of electricity needed to counteract the weight (gravitational force acting on a mass).

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Captivate Your Curiosity

Here's the tricky part

- There is a quantity that relates weight to electrical current, called Planck's constant - named after the German physicist Max Planck and denoted by the symbol h.
- But h is an incredibly small number and to measure it, the research scientist Dr Bryan Kibble built a super-accurate set of scales. The Kibble balance, as it has become known, has an electromagnet that pulls down on one side of the scales and a weight - say, a kilogram - on the other.
- The electrical current going through the electromagnet is increased until the two sides are perfectly balanced.
- By measuring the current running through the electromagnet to incredible precision, the researchers are able to calculate h to an accuracy of 0.000001%.

this force of tension is also the result of electromagnetic interaction at the atomic levels inside the material of string. So we can understand that electromagnetic forces play a very important role in our life.

4.3. Strong Nuclear Force

We should be aware of the very rough model of atom. Nucleus is very tiny volume located at the centre of atom and is responsible for most of the mass of the atom because electrons orbiting around the nucleus are almost mass-less particles. Nucleus of hydrogen atom is just one proton or else for other atoms, nucleus contains protons and neutrons. Strong nuclear force binds protons and neutrons inside the nucleus. Strong nuclear force is the strongest among all fundamental forces but its range of application is extremely small which is of the order of 10^{-15}m. It is almost 100 times stronger than electromagnetic forces. One important characteristic about nuclear force is that it is charge independent. Hence nuclear force acts equally between a proton and a proton, a neutron and a neutron, and a proton and a neutron. And it is responsible for the stability of nucleus because there is strong electric repulsion between protons inside the nucleus but nuclear attraction between these particles is much stronger and holds them together. We must know that electrons do not experience nuclear force.

4.4. Weak Nuclear Force

Weak nuclear force was first revealed in radioactive beta decay. In beta decay a neutron is converted into a proton, an electron and an antineutrino. This force is much weaker than the strong nuclear force but is not as weak as gravitational force. According to present understanding, protons and neutrons are composite particles and are made up of Quarks. Quarks exist in six "flavours", which are known as up, down, top, bottom, strange and charm. Weak interaction allows these quarks to swap their flavours. This fundamental force is responsible for the decay process whereas other fundamental forces are binding things together. Range of weak nuclear force is of the order of 10^{-16} m. Hence, it acts only at subatomic level and it has important role in creating different kinds of elements and it also helps in the power generation in stars.

4.5. Unification of Fundamental Forces

We have already discussed about the basic idea of unification in physics. We strongly believe that there are only some fundamental laws which run the complete show of this universe. We can see that there are only three basic particles known as electron, proton and neutron, which create all kinds of materials we have. In 1687, Isaac Newton described terrestrial and celestial mechanics in a unified manner. He explained that same laws of motion and law of gravitation are applicable in both the domains. However, the same laws were not sufficient to explain atomic, molecular and subatomic phenomenon. Quantum mechanics is required for the same which accounts for wave nature of particles. In terrestrial and celestial domains, wave nature of matter is not at all significant and hence Newton's laws of motion and gravitation are enough to explain everything.

Many times new discoveries in physics have helped in unifying existing theories in several domains. In 1820, Oersted demonstrated that there is magnetic field around current carrying wire. In 1830,

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Faraday demonstrated that electric field can be created from magnetic field. Soon it was realised that Oersted and Faraday both gave two different aspects of the same entity. We got to know that Electric field and Magnetic field are two inseparable entities and this way we achieved unification of electricity and magnetism. Contribution of Maxwell was very significant to unify electricity and magnetism. After unification the term "Electromagnetism" came in use.

In 1873, James Clerk Maxwell proved that light is an electromagnetic wave and all kinds of electromagnetic waves move with same speed as that of light. Hence, we can say that work of Maxwell unified the field of electromagnetism and optics. Einstein tried to unify gravity and electromagnetic force but somehow could not succeed. But there are some recent developments towards unification of forces that we must be aware of. In 1979, Sheldon Glashow, Steven Weinberg and Abdus Salam showed that weak nuclear force and electromagnetic force are basically two different aspects of same force and they called it with a common name "electro-weak force". Carlo Rubia and Simon Vander Meer performed experiments based on the theory of electro-weak force. The theory was experimentally verified in 1984. People are still speculating about the unification of electro-weak force and strong nuclear force. People are also speculating about unifying force of gravitation with rest of the fundamental forces. May be one day we shall be able to say that there is only one fundamental force which appears in different forms in different contexts.

5. NATURE OF PHYSICAL LAWS

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There are many physical laws which can be expressed mathematically to understand and predict the phenomenon happening around us. But if we think carefully then we can understand that not everything can be proved with physics and mathematics. Because at the most basic stage, everything starts from certain hypothesis or assumptions which are strongly supported by the range of observations, experiments and data available. To understand this we can talk about law of gravitation. Universal law of gravitation given by Newton is an example of hypothesis or assumption. Newton observed several terrestrial and celestial phenomena before proposing the law of gravitation. Newton proposed that any two objects inside this universe attract each other with a force proportional to the product of their masses and inversely proportional to the square of the distance between the two. We cannot prove law of gravitation mathematically but can always verify and substantiate law of gravitation with the help of observations and experiments. Newton successfully explained several natural phenomena using his law of gravitation and finally it was established as universal law because it was applicable for all particles and objects inside this universe. There are many other physical laws which are based on assumptions or certain assumed postulates. For example, Bohr's model of hydrogen atom is based on certain rules or postulates. These postulates were visualised after analysis of large amount of spectroscopic data. Similarly, Einstein's special theory of relativity is also based on two assumptions. First assumption is that speed of light in vacuum remains constant and is independent of the source and observer. Second assumption is that physical laws are valid in all inertial frames of reference. Hypotheses are not only required in physics but are also applicable to mathematics. 'Two parallel lines never intersect each other' is one example of hypothesis used in geometry. So we can understand that physicists explore the

universe by analysing a wide range of phenomenon and then with the help of facts of observation and experimentations try to discover the laws which may summarise these observed facts. Established physical laws then can be used to predict the outcome of certain phenomenon.

When we observe certain phenomenon then we associate several physical entities to analyse them. In general, most of these associated physical entities may change with time but some of the physical entities remain constant with time and these entities are said to be conserved. There are conservation principles related to such physical entities. Principle of conservation of energy is one important example. Use of conservation of energy may be tricky in some cases since energy exists in many different forms and one form of energy can be converted into another form. When all forms of energy (heat, mechanical energy, electromagnetic energy, etc.) are taken into account then it can be demonstrated that energy is conserved. Let us take an example of some object falling towards the earth. In this case when object falls then gravitational potential energy of the system decreases and it results in increase of kinetic energy of the object. So here we can say that the total energy of the system is constant and the lost gravitational potential energy appears as kinetic energy of the object. But if we do precise measurements then we can verify that the kinetic energy gained by the object is less than the loss in potential energy. Because when some object moves in atmosphere then air particles resist its motion and some energy is lost as heat. Some part of this lost energy is absorbed by the air molecules and remaining becomes internal energy of the object which may change the temperature of the object. Loss of energy as heat may not be significant when object is falling near the surface of earth but just think about a meteor falling towards the earth. Meteors are burnt and many times get evaporated before hitting the earth's surface. In these examples total energy of the object may not be constant but total energy of the object plus surroundings remains constant. The general law of conservation of energy is true for all types of forces and is valid even when the energy gets transformed from one form into another. Most important is the fact that the conservation of energy is uniformly applicable in all domains of nature, whether it is microscopic or macroscopic. Conservation of energy is valid for all terrestrial and celestial phenomena in broad sense. If we assume universe as ideally isolated system then its total energy remains precisely constant in spite of the fact that different kinds of phenomenon are continuously taking place inside the universe. We should understand that concept of energy is one of the key concepts of physics and many times we write energy expressions to answer related questions.

Earlier it was believed that mass remains constant and law of conservation of mass was considered as one of the basic conservation law of nature. Einstein discovered that mass is not conserved and it can be converted into energy and vice versa. According to Einstein's theory, when mass m is annihilated completely then amount of energy released is $E = mc^2$, where c is speed of light in vacuum. In nuclear fission process some mass is lost and appears as energy. And this method is used to generate energy in nuclear power plants and atom bombs. Note that in nuclear processes only small change in mass is responsible for such a huge amount of energy. Mass energy conversion happens only in nuclear reactions and here energy involved is too large but when we talk about exothermic or endothermic chemical reactions then in such cases involved energy is too small and corresponding change in mass cannot be measured and thus we can safely assume that in chemical

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reactions total mass remains constant. Today we know that mass is not a conserved quantity but still it can be used as an important concept in many situations like in case of chemical reactions. In chemical reactions rearrangement of atoms takes place among different molecules. And total mass of the products is same as total mass of the reactants.

KEY NOTE

In nuclear processes, huge amount of energy is produced by small changes in mass.

According to Newton's laws of motion we can derive that total linear momentum and total angular momentum for an isolated system remains conserved. These are two important conservation principles which are valid even in situations where Newton's laws are not valid, for example in subatomic domain. In 1931, Wolfgang Pauli predicted the existence of a new particle (presently known as neutrino) emitted during β-decay, using conservation laws of energy and momentum.

Physical laws do not change with time. Hence, you get the same result from an experiment performed on two different times but under the same set of conditions. Similarly, space is homogenous and there is no preferred location in the universe; hence laws of nature remain same for every location in this universe. For example, same law of gravitation can be used to find acceleration due to gravity near the surface of earth or near the surface of the moon. However, acceleration due to gravity on surface of the moon is one-sixth of that on surface of earth. Hence, our observations may be different in different places and contexts but physical law remains same everywhere.

Name	Country of origin	Discovery
Archimedes	Greece	Principle of buoyancy and principle of lever
Torricelli	Italy	Barometer
Galileo Galilei	Italy	Law of inertia
G. Fahrenheit	Germany	Mercury thermometer
Christiaan Huygens	Holland	Wave nature of light
Becquerel	France	Natural radioactivity
William Stanley	USA	Electric transformer
Isaac Newton	UK	Laws of motion, Law of gravitation
Michael Faraday	UK	Laws of electromagnetic induction
James Clerk Maxwell	UK	Theory of electromagnetism, Proved that light is an electromagnetic wave
William Sturgeon	UK	Electromagnet
Heinrich Rudolf Hertz	Germany	Production of electromagnetic waves
J. C. Bose	India	Ultra short radio waves
W. K. Roentgen	Germany	X-rays
J. J. Thomson	UK	Electron
Marie Curie	Poland	Studies on natural radioactivity; Discovery of radium and polonium
Werner Heisenberg	Germany	Quantum mechanics; Uncertainty principle
Albert Einstein	Germany	Theory of relativity; Explanation of photoelectric effect
Victor Francis Hess	Austria	Cosmic radiation
R. A. Millikan	USA	Measurement of charge on electron
Ernest Rutherford	New Zealand	Nuclear model of atom
Niels Bohr	Denmark	Quantum model of hydrogen atom
C.V. Raman	India	Scattering of light by molecules
Louis Victor de Broglie	France	Wave nature of matter

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M.N. Saha	India	Thermal ionisation
S.N. Bose	India	Quantum Statistics
Wolfgang Pauli	Austria	Exclusion principle
Enrico Fermi	Italy	Controlled nuclear fission
E. Lenoir	France	Internal Combustion Engine
Paul Dirac	UK	Quantum statistics; Relativistic theory of electron
Edwin Hubble	USA	Expanding universe
Ernest Orlando Lawrence	USA	Cyclotron
James Chadwick	UK	Neutron
Hideki Yukawa	Japan	Theory of nuclear forces
Homi Jehangir Bhabha	India	Cascade process of cosmic radiation
Lev Davidovich Landau	Russia	Theory of condensed matter; Liquid helium
S. Chandrasekhar	India	Chandrasekhar limit, Structure and evolution of stars
John Bardeen	USA	Theory of superconductivity, Transistors
C.H. Townes	USA	Maser; Laser
Abdus Salam	Pakistan	Unification of weak and electromagnetic interactions
Kepler	Germany	Laws of planetary motion
James Watt	Scotland	Steam engine
Graham bell	Scotland	Telephone
G. Marconi	Italy	Wireless telegraphy
W. D. Wright	USA	Airplane
Max Planck	Germany	Quantum theory
John Bair	Scotland	Television
Jacques Charles	France	Hydrogen balloon, Charles law of ideal gas
Robert Boyle	Ireland	Boyle's law of ideal gas
Alessandro Volta	Italy	Electric cell, methane, volt, voltage, voltmeter
Christian Doppler	Austria	Doppler Effect

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Chapter Summary

Key Terms & Laws

- **Science:** A systematic attempt to understand natural phenomena around us and use the gained knowledge to predict, control and modify the phenomena.
- **Scientific Method:** It involves interconnected steps - Systematic observations, controlled experimentation, qualitative and quantitative reasoning, mathematical modelling, prediction and verification of theories.
- **Physics:** A basic discipline in the category of Natural Sciences, it is the study of nature and basic laws of nature and their manifestation in different conditions and domains. In physics, wide variety of physical phenomena, events and occurrences can be explained in terms of a few universal concepts and laws.
- **Unification:** It is an attempt to describe all the physical forces as a single (unified) set of mathematical relations. Four forces governing all the phenomena are – Gravitational force, electromagnetic force, weak nuclear force and strong nuclear force.

- **Reductionism:** It is an attempt to analyse and describe complex phenomena in terms of its simple and fundamental parts.
- **Conserved Quantities:** Physical quantities that remain invariant with time in a process or phenomena are known as conserved quantities.
- **Hypothesis:** It is a hypothesis based on observations and experiments, which cannot be proved but can be verified and disprove by experimentation.

FACTS

- Unification and reductionism are two principal thrusts that can be applied to all domains of life – from macroscopic level to microscopic level.
- Classical physics deals with macroscopic phenomena, including subjects like Mechanics, Electrodynamics, Optics and Themodynamics.
- Quantum theory is applicable to understand microscopic domains of physics, which includes structure of matter at minute scales of atoms and nuclei and their interaction with elementary particles.
- The scope of physics is very vast, covering tremendous range of magnitudes of various physical quantities like length, mass, time, etc.
- Physics and Technology are closely related, both giving rise to new ideas and concepts in each other's field. Both have direct impact on the society.
- Conservation of energy, momentum, angular momentum, charge, mass, etc are considered as fundamental laws in physics. These laws are strong connections with symmetries of nature. Some conservation laws are true for one fundamental force but not for the other.
- Phenomena may differ from place to place in universe but laws of nature are same everywhere.

Conceptual Questions

Q.1. Explain the scientific methods involved in deducing any scientific theory.

Ans. Scientific method involves following steps:

- Systematic observation
- Controlled experiments
- Quantitative and qualitative reasoning
- Mathematical modelling
- Prediction
- Verification or classification of theory

Q.2. How do we know that Earth was not flat in early days?

Ans. There were so many observations which led to the conclusion that Earth is not flat. One among them was when ship sails, its deck and then flag disappears one by one as if it is sinking and then suddenly whole ship disappears from the shore. This is only possible when earth is curved. Since this phenomenon of ship sinking was observed in all the directions, it is evident that earth is a sphere and not flat.

Q.3. How are microscopic domain and macroscopic domain in physics different from each other?

Ans. Microscopic domain in physics deals with the study of phenomena involving elementary particles like electrons, atoms, molecules, etc. It contributes majorly to Modern physics. However, macroscopic domain deals with the study of objects with finite size to large sizes in universe. For example, the span of galaxies, distance to stars, etc. This domain majorly contributes to the classical physics.

Q.4. Mention a few inventions in physics which brought a revolution in industry and technology.

Ans. Invention of steam engine, communication, printers, wireless communication, radio broadcast, computers, nuclear reactors, new modes of transportation like air bus, metro, trams, bullet trains, etc. are some of the few inventions which brought the revolution.

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Q.5. How is angular momentum of a gaseous nebula contracting under its own gravity is conserved?

Ans. The rotation inertia of the gaseous nebula which is contacting under its own gravity decreases and since its angular momentum is conserved, its angular speed increases giving it a spiral shape.

Q.6. Arrange the four fundamental forces in physics in decreasing order of their strength.

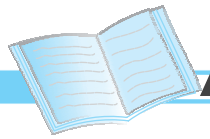
Ans. Nuclear forces, electromagnetic forces, weak forces and gravitational forces.

Q.7. How is a hypothesis different from a model?

Ans. A hypothesis is basically a proposed theory in favour of an observation in absence of in-depth knowledge of the content mainly based on assumptions. It is a part of initial stage of a scientific method. However, a model is a proposed theory explaining the observations with more knowledge and logical explanation.

Q.8. There are many scientific models proposed and failed. The corpuscular theory is an example of such case. The corpuscular theory given by Newton was later replaced by Huygen's wave theory of light. This was further replaced by Plank's quantum theory and finally by De-Broglie's hypothesis. Comment on the fact that despite failures in the above theories, we stress on the study of all the above given theories.

Ans. The in-depth study of all the above theories and their failure gives us a detailed idea about the scientific methods involved. This also proves that new discoveries lead to the change in the theories or the new theories.



NCERT FILE Solved

NCERT Textbook Exercises

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1.1. Some of the most profound statements on the nature of science have come from Albert Einstein, one of the greatest scientists of all time. What do you think did Einstein mean when he said: "The most incomprehensible thing about the world is that it is comprehensible"?

Ans. Millions of phenomena are occurring in our universe. As seen by a layman, physical world seems to be incomprehensible (it cannot be understood). But after many scientific discoveries it is concluded that all the physical phenomena varying from atomic to astronomic ranges can be understood in terms of certain basic laws of physics and the physical world can be understood. Hence the most incomprehensible thing in the physical world becomes comprehensible.

1.2. "Every great physical theory starts as a heresy and ends as a dogma." Give some examples from the history of science of the validity of this incisive remark.

Ans. The statement is true.

For example:

(*i*) Ptolemy postulated the existence of geocentric theory. According to him earth is at the centre of universe and heavenly bodies revolve around the earth. It is a heresy. But when Galileo proved the heliocentric theory then Ptolemy's geocentric theory was considered as a dogma.

(*ii*) Initially wave theory of light was given by Young's double slit experiment, it was a heresy. But when quantum theory was given by Einstein then wave theory was considered as a dogma.

1.3. "Politics is the art of the possible." Similarly, "Science is the art of the soluble." Explain this beautiful aphorism on the nature and practice of science.

Ans. To win over the votes, a politician does anything and everything possible without knowing anything about the problem. Therefore, Politics is the art of the possible. However a scientist after making a large number of observations comes to a conclusion based upon certain facts. Thus Science is the art of the soluble because a large number of processes can be explained in terms of a few concepts.

1.4. Though India now has large base in science and technology, which is fast expanding, it is still a long way from realising its potential of becoming a world leader in science. Name some important factors which in your view have hindered the advancement of science in India.

Ans. The important factors which hindered the advancement of science in India are:

(*i*) Poverty, due to which there is lack of resources in India.

(*ii*) Insufficient funds for science and technology.

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(*iii*) There is a lack of coordination between the industrialists and the scientists. Our industrialists have to raise their confidence in the ability of our scientists so there is no need to import technology from developed countries.

(*iv*) Lack of academic freedom given to the young scientists and technologists for developments in the fields of science and technology.

1.5. No physicist has ever "seen" an electron, yet, all physicists believe in existence of electrons. An intelligent but superstitious man advances this analog to argue that 'ghosts' exist even though no one has "seen" one. How will you refute his argument?

Ans. It is absolutely true that no physicist has ever seen an electron but its existence is proved by large number of experiments For example, flow of current due to electrons, structure of atom consisting of protons, electrons and neutrons, but there is hardly any evidence for the existence of "Ghosts". Hence, these two statements are entirely different from each other.

1.6. The shells of crabs found around a particular coastal location in Japan seem mostly to resemble the legendary face of Samurai. Given below are two explanations of this observed fact. Which of these strikes you as a scientific explanation?

(*a*) A tragic sea accident several centuries ago drowned a young Samurai. As a tribute to his bravery, nature through its inscrutable ways immortalised his face by imprinting it on the crab shells in that area.

(*b*) After the sea tragedy, fisherman in that area, in a gesture of honour to their dead hero, let free any crab shell caught by them which accidentally had a shape resembling the face of a Samurai. Consequently, the particular shape of the crab shell survived longer and therefore in course of time the shape was genetically propagated. This is an example of evolution by artificial selection.

[**Note:** This interesting illustration taken from Carl Sagan's 'The Cosmos' highlights the fact that often strange and inexplicable facts which on the first sight appear supernatural actually turn out to have simple scientific explanations. Try to think out other examples of this kind.]

Ans. The explanation (*b*) seems more as a scientific explanation.

1.7. The industrial revolution in England and Western Europe more than two centuries ago was triggered by some key scientific and technological advances. What were these advances?

Ans. Following were the key scientific and technological advances which brought scientific revolution:

(*i*) One of the major discoveries was the discovery of electricity by designing dynamos and motors.

(*ii*) The discovery of blast furnace to convert iron into steel.

(*iii*) Cotton gin which was used to separate the seeds from the cotton many times faster than by the hand.

(*iv*) The discovery of explosives, steam engines, power looms brought the industrial revolution.

1.8. It is often said that the world is witnessing now a second industrial revolution, which will transform the society as radically as did the first. List some key contemporary areas of sciences and technology, which are responsible for this revolution.

Ans. Following areas are responsible for this revolution:

(*i*) The development of information technology for Internet, robots, satellite communications, etc.

(*ii*) The development of superconducting materials at room temperature.

(*iii*) The development of IR detectors for remote sensing, night vision.

(*iv*) Modern techniques for the development of agriculture.

(*v*) The development in biotechnology.

(*vi*) The development in nanotechnology will change the face of the world in coming years.

(*vii*) The development of genetic engineering.

1.9. Write in about 1000 words a fiction piece based on your speculation on the science and technology of the twenty-second century.

Ans. There was a time when we used animals to cover large distances and it used to take significant amount of time to move from one place to another. But soon with the development of motorised vehicles we were able to move faster. Today we have airplanes and can cover very large distances in very less amount of time. Development never stops. Today we are using supersonic planes for defense purposes only but with the advancement of technology appropriately high speed planes will also be available for public usage. Breakthrough discovery or invention is about to happen in the field of electricity production and by the twenty-second century cost of electricity will go down drastically. People will stop using petrol or diesel to drive the vehicles. Because at that time all vehicles including airplanes and ships will be electricity driven. By the twenty-second century we would be able to make super-efficient rechargeable batteries to store large amount of electricity to be used for vehicles. Charging time for these batteries will be very small. People would be able to charge their vehicles at home and at the same time charging points will also be available for public like we have petrol pumps at present. We already know that displacement current is a kind of wireless electricity which flows between the plates of capacitor during charging in spite of the fact that both the plates of capacitor are kept apart. So we can today visualise electricity flowing without

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the metallic wires from one place to another. Wireless form of electricity will save lot of energy that is lost during transmission from power station to the point of use. It will also enable us to set up charging points in air so that air vehicles may also be charged on the way. By the twenty-second century we shall be able to develop super-efficient materials to absorb solar energy. Transportation vehicles will be made in such a way that it will be able to tap solar energy also to charge its battery. These super-efficient solar panels will be able to fulfil most of the electricity requirement of vehicles. Alternate way of charging will help during rainy season. Today we have bullet trains based on magnetic levitation and by the twenty-second century we shall get rid of existing railway network to replace it with advanced bullet trains. These trains will be moving at very high speed and at the same time it will be safe for the humans. So we can understand that travel time is going to be reduced very drastically.

Today we are witnessing that machines are slowly entering in our lives to ease our lives. Washing machine, dish washer, vacuum cleaner are some of the examples. By the twenty-second century people will be able to develop much more advanced and sophisticated machines to take care of humans. For example, wireless vacuum cleaner will be able to clean your house without human intervention. We know that today people are developing driverless cars and by the twenty-second century it will be highly advanced and nowhere humans will be driving the cars. Complete traffic will consist of driverless vehicles and with the advancement of technology, accidents will be a thing of past attributed to human drivers. Vehicles will be able to understand your command just like a human driver. What I am talking about is called machine learning. Artificial intelligence is one of the most important parts of machine learning. So ultimately it is the appropriately intelligent computers in various forms which will control the machines. By the twenty-second century we shall be able to control intelligent household equipment even if we are somewhere else. We are heading towards completely mechanised life where all our works will be handled by computerised machines. In production factories all works will be done by machines and humans will be just monitoring and taking key decisions.

There was a time when we were forced to go to the market to buy something needed. But Internet-based e-commerce is totally changing the way we used to shop. Today we can get almost everything we need to be delivered at our door step. Today we may not be satisfied with the delivery time for the product we order. Experiments have already started to use drones for delivery. By the twenty-second century such systems will be fully developed and functional. Just think about one fact that everything around us is made up of three elementary particles only which are known as electron, proton and neutron. Anything we can visualise is only special arrangement of these particles. In future computer software will develop digital copy of many of the products available in the market. When you select a certain product then attached hardware will be able to make that product on the spot using particles from the atmosphere.

Discovery of electromagnetic waves was a major breakthrough in the history of science. Result of this discovery is wireless communication we are using today. Mobile telephony has completely revolutionised the way we communicate with each other. By the twenty-second century we might be able to develop a machine which will help to transfer holographic projection of a person to a remote location. Holographic projection will be perfectly synchronised with the brain of the person. Hence the person will be able to control the holographic projection at remote location as he or she can control his or her own body using the brain. Hence we can attend the meetings and interact with the people around the globe without physically going there. Note that these holographic projections will be just optical image which can talk and walk like a real person but would not be able to interact physically with the people otherwise it will create law and order problems in the society.

Now we can understand that our material life will become far easier than it is today. But there may be some downsides of these technical developments. Due to lack of physical activities many humans might turn lazy and it will affect their health in undesirable way. Equipment of war will be highly developed. Even today we have already developed some air vehicles which do not require any human pilot. These airplanes can carry large ammunition to destroy the target. By the twenty-second century we shall develop war equipment which will be controlled remotely. War will be just like a computer game but never forget that killings would be real.

1.10. Attempt to formulate your 'moral' views on the practice of science. Imagine yourself stumbling upon a discovery, which has great academic interest but is certain to have nothing but dangerous consequences for the human society. How, if at all, will you resolve your dilemma?

Ans. Science is a journey towards truth. A scientific discovery may have both good or adverse effects for human society. If any discovery has great academic interest but dangerous consequences for the human society, the information must be made public. It is the responsibility of the scientist to spread awareness about its truth and the ways to stop its misuse. For example, discovery of nuclear fission reaction to produce tremendous energy is of great interest for the society but nuclear energy should be used for peaceful purposes only.

1.11. Science, like any knowledge, can be put to good or bad use, depending on the user. Given below are some of the applications of science. Formulate your views on whether the

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particular application is good, bad or something that cannot be so clearly categorised:

(*a*) Mass vaccination against small pox to curb and finally eradicate this disease from the population. (This has already been successfully done in India.)

(*b*) Television for eradication of illiteracy and for mass communication of news and ideas.

(*c*) Prenatal sex determination

(*d*) Computers for increase in work efficiency

(*e*) Putting artificial satellites into orbits around the Earth

(*f*) Development of nuclear weapons

(*g*) Development of new and powerful techniques of chemical and biological warfare.

(*h*) Purification of water for drinking

(*i*) Plastic surgery

(*j*) Cloning.

Ans. (*a*) good (*b*) good (*c*) bad (*d*) good (*e*) good (*f*) bad (*g*) bad (*h*) good (*i*) good (*j*) good, only if done for healthy reasons.

1.12. India has had a long and unbroken tradition of great scholarship—in mathematics, astronomy, linguistics, logic and ethics. Yet, in parallel with this, several superstitious and obscurantistic attitudes and practices flourished in our society and unfortunately continue even today—among many educated people too. How will you use your knowledge of science to develop strategies to counter these attitudes?

Ans. In spite of so much development, there are several superstitious and obscurantistic attitudes flourished in our Indian society. Education is the best way to get rid of these superstitious attitudes and practices. Factors responsible for it are illiteracy, lack of scientific knowledge, poverty, etc.

To get rid of these, parents should be encouraged to send their wards to school where teachers can take the responsibility to counter these attitudes. The media like newspapers, television, radio, Internet can play a major role to spread awareness among society to eradicate these superstitions.

1.13. Though the law gives women equal status in India, many people hold unscientific views on a woman's innate nature, capacity and intelligence, and in practice give them a secondary status and role. Demolish this view using scientific arguments, and by quoting examples of great women in science and other spheres and persuade yourself and others that, given equal opportunity, women are on par with men.

Ans. A woman is not inferior to a man in any sphere of life. Although there is a small difference in the anatomy of man and woman but there is no difference in the capability of a woman in work, intelligence and best use of mind. There are large numbers of successful women in our country who excelled in their fields. Sarojini Naidu, Lata Mangeshker, Margret Thatcher, Indira Gandhi, Kalpna Chawla had done great jobs in their respective fields. Hence we can say that women are on par with men.

1.14. "It is more important to have beauty in the equations of physics than to have them agree with experiments." The great British physicist P.A.M. Dirac held this view. Criticise this statement. Look out for some equations and results in this book which strike you as beautiful.

Ans. The given statement is partially true. Any equation in physics which agrees with experiment is not always simple and beautiful. For example, the mass-energy relation $E = mc^2$, Newton's equation $F = ma$ are simple and beautiful. But the equations involved in the theory of relativity or in higher physics are not simple and beautiful.

1.15. Though the statement quoted above may be disputed, most physicists do have a feeling that the great laws of physics are at once simple and beautiful. Some of the notable physicists, besides Dirac, who have articulated this feeling, are: Einstein, Bohr, Heisenberg, Chandrasekhar and Feynman. You are urged to make special efforts to get access to the general books and writings by these and other great masters of physics. Their writings are truly inspiring !

Ans. The books written by these scientists are interesting and inspiring. Students should go to a good library and consult these books of great masters of physics. G. Gamow book of 'Foundations and Frontiers', Feynman book 'The Pleasure of Finding Things Out', EM Rogers book 'Physics for the inquiring mind'. These writings are truly inspiring and having immortal values.

1.16. Textbooks on science may give you a wrong impression that studying science is dry and all too serious and that scientists are absent-minded introverts who never laugh or grin. This image of science and scientists is patently false. Scientists like any other group of humans have their share of humorists, and many have led their lives with a great sense of fun and adventure, even as they seriously pursued their scientific work. Two great physicists of this genre are Gamow and Feynman.

Ans. The given statement is true. There are many scientists like any other group of humans who have led their lives with a great sense of fun and adventure. There are many great scientists who have their share of humorists alongwith being physicists, e.g. Feynman, Gamow, Bohr, Einstein, Dr. C.V. Raman, APJ Kalam, Homi Jahangir Bhabha are great scientists as well as jovial, fun loving.

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Revision Exercises

Very Short Answer Questions *Carrying 1 mark*

1. What is the principle behind working of **sonar?**
2. What is the scientific principle behind production of ultra high magnetic fields?
3. Who discovered the neutron?
4. Define Science.
5. Define Physics.
6. What do we mean by unification?
7. Explain the reduction as one of the principal thrusts in physics.
8. Name the five branches of physics.
9. What do we study in electromagnetism?
10. What is the role of physics in Astronomy?
11. What is the contribution of physics to our society?
12. What is the basic law behind the rocket launch?
13. Who is credited for giving the law of inertia?
14. Who discovered the electron?
15. Name the Indian physicist who gave the theory of inelastic scattering of light by molecules.
16. Who proposed the theory of expanding universe?
17. Which among the fundamental forces is weakest in nature?
18. Define the gravitational force.
19. What are electromagnetic forces?
20. Arrange the fundamental forces in decreasing order of their magnitude of strength.
21. Who is credited for the discovery of absolute temperature?
22. Define the law of conservation of energy.
23. Define the law of conservation of angular momentum.
24. What is law of conservation of charge?
25. Who received the Nobel prize for photo electricity?

Additional Questions *Carrying 1 mark*

26. Computers work on the principal of
    (*a*) Superconductivity
    (*b*) Digital logic
    (*c*) Magnetic confinement of plasma
    (*d*) Electromagnetic induction
27. Which of the following statements is correct regarding the strong nuclear forces?
    (*a*) It is a central force
    (*b*) It is a conservative force
    (*c*) It obeys the inverse square law
    (*d*) It has shortest range
28. Which of the following statements is false regarding the gravitational force?
    (*a*) It is conservative force
    (*b*) It is central force
    (*c*) It obeys inverse square law
    (*d*) It is the strongest force in nature
29. Which of the following concepts has no contributions by Einstein?
    (*a*) General Theory of relativity
    (*b*) Photoelectric effect
    (*c*) Quantum model of hydrogen atom
    (*d*) Quantum theory
30. Fusion test reactor (Tokamak) is based on
    (*a*) Interference
    (*b*) Photoelectric effect
    (*c*) Magnetic confinement of plasma
    (*d*) Motion of charged particles in electric and magnetic field

Fill in the Blanks *Carrying 1 mark*

1. The country of origin of Archimedes is ..... .
2. The field particle of gravitational forces is known as ..... .
3. ..... are based upon the controlled nuclear chain reaction.
4. The basic principle behind the computers is ..... .
5. The country to which Niels Bohr belongs is ..... .
6. The law of force between two charges is given by ..... .

Short Answer Questions *Carrying 2 or 3 marks*

1. Name some basic scientific methods.
2. Name the fundamental forces in nature.
3. What is the role of physics in biological sciences?
4. How is physics and technology related?
5. Write the expression for (*i*) Newton's law of gravitational force (*ii*) Coulomb's law.
6. How can we say that charge is quantised?
7. Write a brief note on electromagnetic forces.
8. Give an example in support of law of conservation of momentum.
9. Write a short note on Satyendranath Bose.
10. Write about the life of C.V. Raman and his contribution towards science.
11. Arrange all the fundamental forces in increasing order of their range.

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Long Answer Questions *Carrying 5 marks*

1. Write some main features of gravitational force.
2. Write a short note on different branches of physics.
3. What are the fundamental forces? Give example of each force.
4. Write some salient features of electromagnetic forces.
5. What are the important conservation laws in classical physics? Explain them.
6. How is physics related to other branches of science, i.e. chemistry, medicine, astronomy and mathematics?

Hints & Answers for Revision Exercises

Very Short Answer Questions

1. Sonar works on the principle of reflection of ultrasonic waves.
2. It is based on concept of superconductivity.
3. James Chadwick discovered the neutron.
4. Science is the understanding of natural phenomena around us in depth and to learn predicting them.
5. Physics is the study of basic laws of nature and its manifestation in different laws. It gives the basic laws which can be implemented universally.
6. Unification is the method of explaining diverse physical phenomenon using some basic laws.
7. Reductionism is one of the principal thrusts in science. It is the method of explaining or understanding more complex phenomena in physics using the science involved in its simplified subpart phenomena.
8. Mechanics, Heat and thermodynamics, Theory of relativity, Electromagnetism and Quantum mechanics are five branches of physics.
9. Electromagnetism is the study of behaviour of charged particles and magnetic materials under the influence of magnetic and electric field.
10. In astronomy, telescopes, binoculars, solar telescopes and radio telescopes, etc. are used to observe and study space in detail.
11. Physics is involved in a lot of development including the invention of telephone, telegraph, Internet, satellite communication, radio, televisions, etc. It is involved in development of every field like transportation, communication, safety, defense, medical equipments, etc. There are numerous contributions of physics in welfare of society.
12. Rocket launch is based on Newton's laws of motion.
13. Galileo Galilei is credited for Law of inertia.
14. J.J. Thomson discovered electron.
15. C.V. Raman gave the theory of inelastic scattering of light by molecules.
16. Edwin Hubble proposed the theory of expanding universe.
17. Gravitational force is the weakest of all the fundamental forces.
18. Gravitational force is the force of attraction between any two objects by the virtue of their masses.
19. Electromagnetic forces are forces associated with stationary or moving charged particles.
20. Nuclear force, electromagnetic force, weak nuclear force and gravitational force.
21. Lord Kelvin is credited for the discovery of absolute temperature.
22. Law of conservation of energy states that energy can neither be created nor can be destroyed. It is totally conserved and can be only transformed from one form to another.
23. According to the law of conservation of angular momentum, the net angular momentum of a system remains same if there is no external torque acting on the system.
24. Law of conservation of charge states that charges can never be created or destroyed. It can be only transferred from one body to another.
25. Einstein

Additional Questions

26. The correct answer is option (*b*)
27. The correct answer is option (*d*)
28. The correct answer is option (*d*)
29. Quantum model of hydrogen is explained by Neils Bohr. Thus the correct option is (*c*).
30. Fusion test reactor (Tokamak) is based on magnetic confinement of plasma. Thus the correct option is (*c*).

Fill in the Blanks

1. Greece
2. graviton
3. Nuclear reactors
4. digital logic
5. Denmark
6. coulomb

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Short Answer Questions

1. Some basic scientific methods are

(*a*) Careful observation

(*b*) Experiments

(*c*) Qualitative and quantitative reasoning

(*d*) Mathematical modelling

(*e*) Prediction

(*f*) Verification of a theory

2. The fundamental forces in nature are gravitational forces, weak nuclear forces, electromagnetic force and strong nuclear forces.

3. From treatment of diseases like cancer with radio isotopes to basic microscope, electron microscope, X-rays and other basic tools used in medical sciences involve some or more physics in it.

4. Physics is the study of nature and natural phenomena. Technology is the implying these laws of physics for practical purposes. For example, optical fibres are based on total internal reflection. Rocket launches can be explained by Newton's law of motion.

5. (*i*) According to Newton's law of gravitation, the force of attraction between two objects with mass m_1 and m_2 kept at a distance r *m* can be expressed as: $F = \frac{Gm_1m_2}{r^2}$, here G is gravitational constant.

(*ii*) Coulomb's law states the force between two charges q_1 and q_2 kept at a distance r is directly proportional to the product of charges and is indirectly proportional to the square of distance between them and is given as:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

6. Charge present on a body is always integral multiple of e. or $q = ne$, where n is an integer, and $-e$ is charge of an electron and $+e$ is charge of proton.

7. Electromagnetic forces are associated with stationary or moving charged particles. They can be repulsive or attracting in nature depending on the nature of charge. They follow inverse square law and are conservative in nature.

8. Recoiling of gun on firing involves conservation of linear momentum. The linear momentum of the system is zero before firing. When gun is fired the momentum of bullet is in forward direction and gun gains equal momentum in opposite direction. Thus the net momentum of the combination is zero.

Long Answer Questions

1. Some features of gravitation forces are:

(*a*) It is always attractive in nature

(*b*) Weakest force in nature

(*c*) It also acts on large distances.

(*d*) It is central force.

(*e*) It is conservative in nature.

(*f*) It obeys the inverse square law of distance. Thus gravitational force between two bodies is inversely proportional to the square of distance between them.

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CHAPTER

UNITS AND MEASUREMENT

1. INTRODUCTION

Physics is the science of measurement and whenever we define a physical quantity then there has to be a way to determine the magnitude of that physical quantity. To begin the chapter let us take an example of distance between two points. Suppose your friend's home is near your home and you have to tell the distance of your friend's home from your home. How will you describe this distance to others? What you can do is just walk from your home to friend's home, observing uniform steps and count the number of steps. Now you can tell others that it is 300 steps (say) away from your house. Now try to understand what you have done. You were aware of the fact that you cover almost same distance in every step you take forward. You have used distance covered in each step as standard unit distance and described the required distance as 300 times of this unit distance. Hence, for measurement we need to first define unit for a physical quantity and then we can compare the given amount of physical quantity with this unit to know how many times of the unit is total amount. And that is how we can determine the magnitude of given amount of physical quantity in terms of described unit. To describe the magnitude of a physical entity we write a number followed by unit to measure it ($Q = nu$, Here n is the number and u is unit of measurement). Let us now again take the same example of distance and now we can ask some other person to measure the number of steps between your home and your friend's home. This time we might get different number of steps as the distance between these two points. Hence, unit to express the distance here is subjective and it changes from person to person. But in physics we need consistent and invariable units for measurements. These units must be objectively defined so that these remain constant. One more important point to be noted here is that we can have more than one type of unit for a particular physical quantity and when we measure a given amount of the same physical quantity then in different units we shall get different numbers to describe the magnitude but we know that actual magnitude is same ($Q = n_1u_1 = n_2u_2$).

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REMEMBER

- A measurable quantity in terms of which general laws of physics can be described is a physical quantity.
- An internationally accepted arbitrary standard measurement of any physical quantity is called unit.

If different people are using different units for the measurement of a physical quantity then understanding one another will become a great problem. Hence, we can say that measurement of a physical quantity requires comparison with certain basic, arbitrarily chosen, internationally accepted reference standard which is known as unit. And magnitude of the physical quantity is expressed as number followed by unit of measurement.

In physics we define many physical quantities but these physical quantities are interrelated with one another. And we need only a limited number of units to express all the physical quantities. There are a few fundamental or base physical entities and corresponding units are called fundamental or base units. Units of remaining physical entities can be expressed in terms of these fundamental units. Units of these physical entities are known as derived units. The complete set of fundamental units and derived units is called system of units. It is essential to develop common system of units based on international agreements for global progress.

2. THE INTERNATIONAL SYSTEM OF UNITS

If we talk about the early stage of development then scientists of different countries were using different systems of units for measurement. But when people across the globe started communicating with each other, in terms of technology exchange then soon it was realised that there has to be some common agreement on international level for units of measurement that we use to describe something.

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Internationally accepted system of units for measurement is the International System of Units abbreviated as SI. The SI was developed in 1971 by General Conference on Weights and Measures for international usage in scientific, technical, industrial and commercial work. SI is prevalent in today's time and we use decimal system in it and hence conversions within the system have become convenient.

There are seven base units (refer to Table 1) and two supplementary units in SI to measure corresponding physical entities.

Table 1. SI Units

Base quantity	SI units		Year
	Name	Symbol	
Length	Metre	m	1983
Mass	Kilogram	kg	1889
Time	Second	s	1967
Electric Current	Ampere	A	1948
Thermodynamic temperature	Kelvin	K	1967
Amount of substance	Mole	mol	1971
Luminous Intensity	Candela	cd	1979

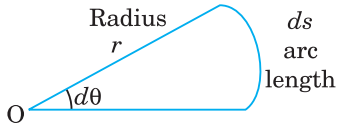

Fig. 2.1. Plane angle.

Two supplementary units are made for measuring plane angle and solid angle. Plane angle ($d\theta$) as shown in Fig. 2.1 in geometry is defined as arc length (ds) divided by radius. SI unit for the plane angle is radian and is written as rad in symbolic form. Solid angle is defined

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as area divided by square of the radius. If r is the radius of a spherical surface and dA is its surface area, then solid angle subtended by area dA will be $d\Omega = \frac{dA}{r^2}$ steradian (refer to Fig. 2.2). SI unit for the solid angle is steradian and is written as sr in symbolic form. Note that plane angle and solid angle both are dimensionless quantities.

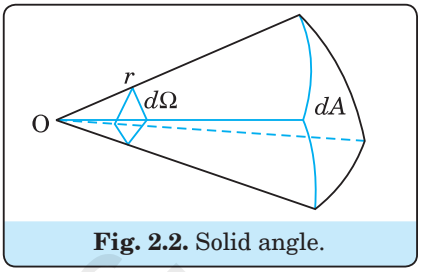

Fig. 2.2. Solid angle.

There are standard definitions for seven base units and some of these are improved over the period of time. Latest definitions of these standard physical units are given as follows:

One **metre** is the distance travelled by light in vacuum during the time interval of 1/299,792,458 of a second.

One **kilogram** is equal to the mass of the international prototype of the kilogram. This prototype is a cylinder made of platinum-iridium alloy, kept at international bureau of Weights and Measures, at Sevres, near Paris, France.

One **second** is the duration of 9,192,631,770 cycles of electromagnetic radiation emitted due to the transition between the two hyperfine levels corresponding to ground state of cesium-133 atom.

One **ampere** is the amount of constant current flowing in two parallel conducting wires of negligible cross section and infinite length which would produce a force of 2×10^{-7}N per metre of length, between wires, when placed in vacuum.

One **kelvin** is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

One **mole** is the amount of substance which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. These elementary entities may be atoms, molecules, ions, electrons, etc.

One **candela** is the luminous intensity of a source in a given direction that emits monochromatic electromagnetic radiation of frequency 540×10^{12} Hz having radiant intensity of 1/683 watt per steradian, in the same direction.

Before the advent of SI, scientists from different countries were using different systems of units for measurement. Three most popular systems in use were the CGS, the FPS and the MKS. FPS is also known as British system. Base units for length, mass and time in these systems are as follows:

CGS: centimetre, gram and second.

FPS: foot, pound and second.

MKS: metre, kilogram and second

In Table 2 metric prefixes are given that are commonly used.

Captivate Your Curiosity

Triple point of water is the temperature with value 273.16 K at which water co-exists in form of ice, water and water vapours simultaneously.

Table 2. Metric Prefixes

Prefix	Power	Symbol
yocto	10^{-24}	y
zepto	10^{-21}	z
atto	10^{-18}	a
femto	10^{-15}	f
pico	10^{-12}	p
nano	10^{-9}	n
micro	10^{-6}	μ
milli	10^{-3}	m
centi	10^{-2}	c
deci	10^{-1}	d
deca	10^{1}	da
hecto	10^{2}	h
kilo	10^{3}	k
mega	10^{6}	M
giga	10^{9}	G
tera	10^{12}	T
peta	10^{15}	P
exa	10^{18}	E
zetta	10^{21}	Z
yotta	10^{24}	Y

SOLVED EXAMPLE

❑ **Example 1:**

The density of iron is 7.87 g/cm^3. Express it in SI units.

Solution: $1 \text{ g cm}^{-3} = 10^3 \text{ kg m}^{-3}$

$\therefore \quad 7.87 \text{ g cm}^{-3} = 7.87 \times 1000 \text{ kg m}^{-3}$

$= \mathbf{7870 \text{ kg m}^{-3}}$

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3. MEASUREMENT OF LENGTH

There are several ways to measure distance and the device needed to make the measurement depends on the range of length to be measured and requirement of the level of accuracy. If we take an example of a metre scale then divisions on scale are made at a separation of 1 mm. Hence, the minimum distance it can measure accurately is 1 mm or 10^{-3} m. Similarly, minimum length that vernier calipers can measure accurately is 0.1 mm or 10^{-4} m. Minimum length that the screw gauge and spherometer can measure accurately is 0.01 mm or 10^{-5} m. You must have seen that a metre scale is used to measure the cloth and tailor uses measuring tape to measure the body dimensions hence many times choice of measuring device depends on what you want to measure. For example, vernier caliper is suitable to measure the diameter of rod or small ball, etc. If we want to measure the diameter of a thin wire then screw gauge is an appropriate choice. In all above cases we are talking about direct measurement where object is available for the direct measurement. But think about situations where you want to measure distance of the moon from the surface of earth or diameter of the moon, then obviously direct measurement is not possible. Similarly, think about measuring the size of atom of certain material or measuring the size of proton then in such cases also direct measurement through some device is not possible. Hence, we need to develop special indirect methods to measure the distances beyond the normal range.

3.1. Measurement of Large Distances

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As we know that it is not possible to measure distance of a distant star or a planet using normal methods. Some special methods are required to estimate such a large distance. Several methods are developed over a period of time and accuracy in measurements has also improved with the development of science and technology. In today's time, one of the most commonly used approach is transit-time method. In this method signals are sent to distant object and then same signals are received back after reflection from that object. Time gap (Δt) is measured between sending and receiving of the signal. Let x be the distance of the object then total distance covered by the signal in round trip becomes $2x$. If v is the speed of the signal, we can write the following equation to calculate distance of object from the point of measurement:

$$\Delta t = \frac{2x}{v} \Rightarrow x = \frac{v\Delta t}{2}$$

Here v is the speed of light. Transit-time method is also used in radio navigation systems for aircrafts and ships, etc. Radar and GPS are examples where transit-time method is used to calculate the distances.

Parallax method is an important method for measuring large distances. To understand parallax, hold one pencil vertically in front of you facing towards a plane wall. Try to hold it at one place and now look at it through left eye, by closing the right eye. After some time look at it through right eye, by closing the left eye. When you switch over between left eye and right eye then you will observe an apparent shift in the position of pencil with respect to the wall. Both of our eyes are located at different positions and hence both of them need to look at different angles to see the same object and due to this we observe the apparent shift in the position of object. This is

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called parallax. When we observe some object normally with both eyes open then these two streams of information received from two eyes are processed through brain and we get rough idea of the relative distance of objects from us and thus both eyes are needed for correct 3d vision of surroundings. When we talk about parallax method to measure distance of distant object, then we need to select two points of observation to look at the same distant object. Distance between these two points of observation is called **basis**. For example, separation between our eyes is basis when we look at same object through left and right eyes separately. Using some arrangements we measure the angle between two directions along which the same object is observed from two different points of observation. This angle is called **parallax angle** or **parallactic angle**.

Refer to Fig. 3.1 to understand parallax method. In this figure S is one distant star. P and Q are two locations on earth from where the same star is observed. As we know that distance between these two points in parallax method is called basis. Let a be the distance between points P and Q and so a is the basis for this calculation. Let D be the distance of star and it is same for both the points P and Q. We can see that star is observed along two different directions from points P and Q. Angle between these two directions is called parallax angle and here θ is assumed as parallax angle. With the help of optical devices we can measure θ. Basis a is already known to us. Thus we can calculate the distance of star D.

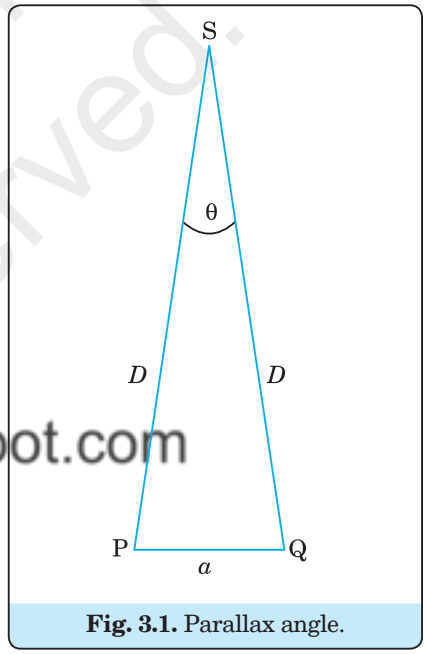

Fig. 3.1. Parallax angle.

Note that $D >> a$, and hence we can treat a as the length of arc and assume D as the radius to apply formula as follows:

$$\text{Angle} = \frac{\text{arc}}{\text{radius}}$$

$$\Rightarrow \qquad \theta = \frac{a}{D} \Rightarrow D = \frac{a}{\theta}$$

We can use the above method to measure the distance of the moon or any planet from earth (D) and then using the same distance we can measure the diameter of the moon or that planet also. We look at two diametrically opposite points of planet or the moon from same location on earth. Telescope is needed to get aligned along two different directions to observe those two diametrically opposite points. Let α be the angle between these two directions. We have already measured the distance D of planet or the moon and let d be its diameter then we can use the same formula as follows:

$$\text{Angle} = \frac{\text{arc}}{\text{radius}}$$

$$\Rightarrow \qquad \alpha = \frac{d}{D} \Rightarrow d = \alpha D$$

We should be aware of the fact that parallax method is suitable for measuring distances less than 100 light years only. We have Copernicus method to measure the distance of inferior planets from the sun. Inferior planets are Venus and Mercury where radius of orbit around the sun is less than that for the earth. Planets with orbital radius around the sun, more than that for the earth are called superior planets. Kepler's third law method is used to measure the distance of superior planets from the sun.

FORMULAE USED

$$\text{Angle} = \frac{\text{arc}}{\text{radius}}$$

$$\theta = \frac{a}{D},$$

Here a is the length of arc and D is the radius.

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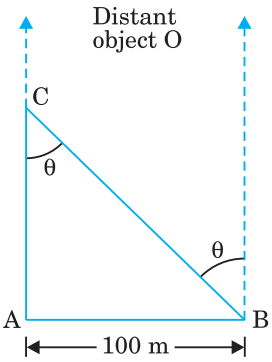
SOLVED EXAMPLES

Example 1.

A man wishes to estimate the distance of a nearby tower from him. He stands at a point A in front of the tower C and spots a very distant object O in line with AC. He then walks perpendicular to AC up to B, a distance of 100 m, and looks at O and C again. Since O is very distant, the direction BO is practically the same as AO; but he finds the line of sight of C shifted from the original line of sight by an angle θ = 40° (θ is known as parallax. Estimate the distance of the tower C from his original position A.)

[*N.C.E.R.T. Solved Example*]



Solution: Given, parallax angle, θ = 40°

From the figure,

$$\tan\theta = \frac{AB}{AC}$$

$$\Rightarrow \quad AC = \frac{AB}{\tan\theta} = \frac{100\text{ m}}{\tan 40^\circ} = \frac{100\text{ m}}{0.839} \approx \mathbf{119\text{ m}}$$

Example 2.

The moon subtends an angle of 57 minutes at two diametrically opposite points on earth. Calculate the distance of the moon from the earth.

Given, diameter of earth = 1.276 × 10⁷ m

Solution: Given, parallactic angle,

$$\theta = 57' = \frac{57}{60} \times \frac{\pi}{180}\text{ rad}$$

Basis, b = diameter of earth = 1.276×10^7 m

The distance of the moon from the earth is

$$d = \frac{b}{\theta} = \frac{1.276 \times 10^7}{57 \times \pi} \times 60 \times 180$$

$$= \mathbf{7.696 \times 10^8\text{ m}}$$

Example 3.

The sun's angular diameter is measured to be 1920″. The distance D of the sun from earth is 1.496×10^{11} m. What is the diameter of the sun? [N.C.E.R.T. Solved Example]

Solution: Given, Angular diameter of the sun,

$$\theta = 1920'' = \frac{1920}{60 \times 60} \times \frac{\pi}{180} = 9.3 \times 10^{-3}\text{ rad}$$

Distance of the sun from the earth,

$$d = 1.496 \times 10^{11}\text{ m}$$

Linear diameter of the sun is

$$D = \theta \times d = 9.3 \times 10^{-3} \times 1.496 \times 10^{11}$$

$$\approx \mathbf{1.39 \times 10^9\text{ m}}$$

Example 4.

A planet revolves around the sun with half as fast as the earth revolves around the sun. Determine its orbital size as compared to that of the earth.

Solution: Let T_p and T_e are periods of revolution of the planet and the earth, respectively. Then,

$$T_p = 2T_e$$

Orbital size of earth, a_e = 1 AU

Orbital size of planet, a_p = ?

From Kepler's law of periods, we have

$$\frac{T_p^2}{T_e^2} = \frac{a_p^3}{a_e^3}$$

$$\therefore \quad a_p = \left[\frac{T_p}{T_e}\right]^{\frac{2}{3}} \times a_e = \left[\frac{2T_e}{T_e}\right]^{\frac{2}{3}} \times \text{AU}$$

$$= \mathbf{1.59\text{ AU}}$$

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3.2. Estimation of Very Small Distances

As we are aware that measuring large distances like in astronomy is not possible with conventional measuring instruments, similarly measurement of distance at the atomic or molecular level is also not possible with these measuring instruments. We might think about employing high resolution microscopes to help in measuring small distances but there is a limitation to resolution to which an optical microscope can be used. The minimum size which can be resolved by the microscope is comparable to wavelength of light used in microscope. In normal cases we use visible light for optical microscopes and wavelength of the visible light ranges from 4000 Å to 7000 Å. Hence, we can understand that optical microscope cannot resolve particles of sizes smaller than 4000 Å. Limit imposed by light to the resolution of microscope is due to the wave nature of light. We can enhance resolution of microscopes to a great extent by using electron beam in place of light beam. In electron microscopes beam of accelerated electrons is used as a source of illumination. We should know that wave character becomes very much prominent in particles like electrons and moreover wavelength of an electron as a matter wave can be as small as some fraction of an angstrom

and this is much shorter than the wavelength of visible light. Hence, electron microscopes have a much higher resolving power than optical microscopes which use visible light. Electron microscopes with a resolution of 0.6 Å are available and such electron microscopes can almost resolve atoms and molecules in a sample of some material. A transmission electron microscope has been developed which has resolution better than 0.5 Å. Electrostatic and electromagnetic lenses have been developed to control the electron beam just like light is focused by glass lenses in optical light microscope to form the image of object. We know that electron is a charged particle and can be controlled by electromagnetic fields. Electrostatic and electromagnetic lenses are designed analogous to glass lenses that we use in optical light microscopes. Hence, we can understand that the electron beam is used to illuminate object and electron beam reflected from that object is focused with special electromagnetic lenses to form highly magnified and well-resolved image. It helps in revealing detailed structure of smaller objects. Limit of resolution is further enhanced with the development of scanning tunneling microscopes (STM). A very sharp metal wire tip is used to scan the surface. This metal tip is brought very close to the surface to be scanned and electric voltage is applied to the tip. Further arrangements are there to form image of the surface up to an extremely large scale. We can resolve individual atoms using this method. Quantum mechanical effect of tunneling and piezoelectric effect is used to develop scanning tunneling microscopes. Hence, in recent time resolution limit has become better than an angstrom and today it is possible to estimate the sizes of small particles like molecules.

Several other special methods are also possible to determine the distances at molecular level. Let us take an example of determining the molecular size of oleic acid. Oleic acid looks like some soap dissolved in liquid. Size of the molecule of oleic acid is of the order of 10^{-9}m. There is a very simple method to estimate the size of a molecule of oleic acid. In this method we form a mono-molecular layer of oleic acid on the surface of water. With the help of simple measurements and calculations we can get thickness of this layer which is nothing but the diameter of molecule of oleic acid. Following steps are observed while forming mono-molecular layer of oleic acid on water surface.

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First we take 1 cm^3 of oleic acid and dissolve it in sufficient amount of alcohol to make total volume of 20 cm^3. From this solution we take only 1 cm^3 and then again dissolve it in sufficient alcohol to make total volume of 20 cm^3. Hence, in this final solution we have $\frac{1}{20 \times 20}$ cm^3 of oleic acid per cm^3 volume of solution.

Now, we put water in a large trough and sprinkle some lycopodium powder on the water surface. A few drops of prepared solution of oleic acid is poured on water surface and oleic acid present in this solution spreads into a thin large and nearly circular film of molecular thickness. We then measure diameter of this oleic acid film to calculate area (A) of the deposited film. Using some precise method we need to determine volume of each drop of solution. Let V cm^3 be the volume of each drop and n is number of drops of solution used to form the film on the surface of water.

Total volume of solution used to make the film = nV cm^3

Total volume of oleic acid present in this solution = $nV\frac{1}{20 \times 20}$ cm^3.

Now we can divide this volume by calculated area of film to get thickness t of the film.

$$t = \frac{nV}{20 \times 20\, A} \text{cm}$$

We know that deposited film is mono-molecular layer of oleic acid and hence above calculated thickness of film is equal to diameter of molecule of oleic acid. If we make careful measurements then we get the thickness of film to be of the order of 10^{-9} m.

SOLVED EXAMPLE

Example 1. *A drop of olive oil of radius 0.20 mm spreads as a film on the water surface. Calculate the molecular size of the olive oil if the film is*

(i) circular, with a radius of 10 cm

(ii) rectangular, with size 20 cm × 10 cm

Solution: Radius of oil drop = 0.20 mm

(*i*) Thickness of olive oil film = $\frac{\text{Volume of oil drop}}{\text{Area of the film}}$

$$= \frac{\frac{4}{3}\pi \times (0.020)^3 \text{ cm}^3}{\pi \times (10)^2 \text{cm}^2}$$

$$= \frac{4}{3} \times (20)^3 \times 10^{-11} \text{cm}$$

$$= 1.067 \times 10^{-7} \text{ cm}$$

Assuming that the film has one molecular thickness, molecular size of olive oil = **1.067×10^{-7} cm.**

(*ii*) Thickness of olive oil film = $\frac{\text{Volume of oil drop}}{\text{Area of the film}}$

$$= \frac{\frac{4}{3}\pi \times (0.020)^3 \text{ cm}^3}{(20 \times 10)\text{cm}^2}$$

$$= \frac{4}{3}\pi \times \frac{(20)^3}{2} \times 10^{-11} \text{cm}$$

$$= 1.674 \times 10^{-7} \text{ cm}$$

$\therefore$ Molecular size of olive oil = **1.674×10^{-7} cm**

Table 3. Range of Lengths

Object Size or Distance	Order of Length (m)
Diameter of neutron	10^{-15}
Diameter of nucleus	10^{-14}
Diameter of hydrogen atom	10^{-10}
Length of a virus	10^{-6}
Wavelength of visible light	10^{-7}
Size of white blood cells	10^{-5}
Thickness of a tracing paper	10^{-4}
Height of a man	10^{0}
Height of Kanchenjunga mountain above sea level	10^{4}
Diameter of the Earth	10^{7}
Diameter of Sun	10^{9}
Distance between Moon and Earth	10^{8}
Distance between Sun and Earth	10^{11}
Distance between Sun and Pluto	10^{13}
Span of Milky way galaxy	10^{21}
Distance between Andromeda and Milky way galaxies	10^{22}
Farthest observed astronomical object	10^{26}

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3.3. Range of Lengths

At one end size of the proton is of the order of 10^{-15} m and at the other end boundaries of the observable universe are extended up to a distance of 10^{26} m. So in physics we come across a wide range of lengths. Along with the standard SI units we often require different units to be used in different contexts for convenience. Some of such special units are as follows:

Astronomical Unit (AU): The average distance between the earth and the sun is called 1 AU. 1 AU = 1.496×10^{11} m.

Light Year (ly): One light year is the distance travelled by light in one year with a speed of 3×10^8 m/s. 1 ly = 9.46×10^{15} m.

Parallactic second (parsec): One parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second. 1 parsec = 3.08×10^{16} m.

1 fermi (f) = 1 f = 10^{-15} m. It is often used to describe the radius of nucleus.

1 angstrom = 1 Å = 10^{-10} m.

1 micron = 1 μm = 10^{-6} m.

Table 3 depicts the order of the lengths and distances between some objects.

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SOLVED EXAMPLES

Example 1.

How many parsec are there in one metre?

Solution: 1 parsec = 3.08×10^{16} m

$$\therefore \quad 1 \text{ m} = \frac{1}{3.08 \times 10^{16}} = 0.325 \times 10^{-16} \text{ parsec}$$

$$= \mathbf{3.25 \times 10^{-17}\ parsec}$$

Example 2.

What is the average distance of earth from the sun, expressed in (i) light year (ii) parsec.

Solution:

Average distance of earth from the sun = 1 AU

$$= 1.496 \times 10^{11} \text{ m}$$

(*i*) 1 light year = 9.461×10^{15} m

$$\therefore \quad R = \frac{1.496 \times 10^{11}}{9.461 \times 10^{15}} \text{ ly} = 0.158 \times 10^{-4} \text{ ly}$$

$$= \mathbf{1.58 \times 10^{-5}\ ly}$$

(*ii*) 1 parsec = 3.08×10^{16} m

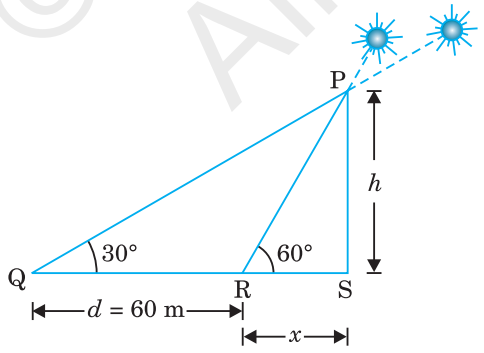
$$R = \frac{1.496 \times 10^{11}}{3.08 \times 10^{16}} \text{ parsec} = 0.486 \times 10^{-5} \text{ parsec}$$

$$= \mathbf{4.86 \times 10^{-6}\ parsec}$$

Example 3.

The difference in length of shadow for a tower standing on a level ground in 60 m when the sun's altitude is 30° and 60°. Calculate the height of the tower.

Solution:



For ΔPRS, $\cot 60° = \frac{x}{h}$

For ΔPQS, $\cot 30° = \frac{d + x}{h}$

$$\cot 30° - \cot 60° = \frac{d + x - x}{h} = \frac{d}{h}$$

$$\therefore \quad h = \frac{d}{\cot 30° - \cot 60°} = \frac{60}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$= \frac{60\sqrt{3}}{3 - 1} = \frac{60\sqrt{3}}{2}$$

$$= 30\sqrt{3}$$

$$= \mathbf{51.96\ m}$$

Example 4.

If the size of the nucleus (in the range of 10^{-15} to 10^{-14} m) is scaled upto the tip of a sharp pin ($\simeq 10^{-5}$ m), what roughly is the size of the atom? Assume tip of the pin to be in the range 10^{-5} m to 10^{-4} m.

[N.C.E.R.T. Solved Example]

Solution: Magnification = $\frac{\text{Size of the tip of sharp pin}}{\text{Size of nucleus}}$

$$= \frac{10^{-5}}{10^{-15}} = 10^{10}$$

We know that, actual size of atom = 1 Å = 10^{-10} m

This is now scaled upto a factor of 10^{10}

$\therefore$ Apparent size of atom = $10^{-10} \times 10^{10} = \mathbf{1\ m}$

Example 5.

If the observable universe ($\simeq 10^{26}$ m) is shrunk to the size of the earth ($\simeq 10^{7}$ m), how large would be milky way on this scale?

Solution: Magnification = $\frac{\text{Size of earth}}{\text{Size of observable universe}}$

$$= \frac{10^{7}}{10^{26}} = 10^{-19}$$

Actual size of Milkyway = 10^{21} m

Apparent size of Milkyway = 10^{21} m $\times 10^{-19}$

$$= \mathbf{100\ m}$$

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Table 4. Range of Mass

Object	Order of mass (kg)
Electron	10^{-30}
Neutron	10^{-27}
Nucleus	10^{-27}
Hydrogen atom	10^{-27}
White blood cell	10^{-13}
Dust particle	10^{-9}
Dew drop	10^{-6}
Mosquito	10^{-5}
Raspberry	10^{-3}
Human	10^{2}
Car	10^{3}
Aircraft	10^{8}
Moon	10^{23}
Earth	10^{25}
Jupiter	10^{27}
Sun	10^{30}
Milky way galaxy	10^{41}
Observable Universe	10^{55}

4. MEASUREMENT OF MASS

In physics we talk about objects and systems whose masses may vary over a wide range. In physics at one side we discuss about particles like electrons with mass of the order of 10^{-30} kg and at another side about the known universe with total estimated mass of about 10^{55} kg. Mass of the moon is of the order of 10^{23} kg, mass of earth is of the order of 10^{25} kg and mass of the sun is of the order or 10^{30} kg. Mass is a basic property of matter and is independent of location of object in space and physical conditions like temperature and pressure. SI unit for measurement of mass is kilogram (kg). Prototypes of international standard kilogram are developed by the International Bureau of Weights and Measures. These prototypes are made available in different laboratories belonging to different countries. In India, prototype of kilogram is kept at National Physical Laboratory (NPL), New Delhi. Mass of most of the objects of our daily life can be measured using a common balance that we can see in several shops. Special balances are used by jewellers to measure the mass of valuables like gold more precisely. Mass spectrograph is used to measure the mass of atomic and subatomic charged particles. When a charged particle is projected inside uniform magnetic field then it follows circular path. Radius of the trajectory followed by such charged particles in electromagnetic field is proportional to the mass of particle. And thus by measuring the radius of trajectory for a given speed of particles, we can estimate the mass of these particles. We can understand that direct measurement of mass is not possible at atomic or subatomic level and similarly direct methods cannot be used to measure the mass of some planet or the moon. We can use gravitational methods to measure the mass of celestial objects. For example, time period of revolution of a planet around the sun depends on mass of the sun and thus we can estimate mass of the sun by using time periods of planets. Similarly, estimate of the mass of the planet can be done with the help of time period of the moon orbiting around it. Table 4 depicts the order of masses.

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SOLVED EXAMPLES

❑ **Example 1.**

If mass of an electron is 9.11×10^{-31} kg, how many electrons will weigh 2 kg?

Solution: Number of electrons in 2 kg

$$= \frac{\text{Total mass}}{\text{Mass of one electron}}$$

$$= \frac{2 \text{ kg}}{9.11 \times 10^{-31} \text{ kg}} = \mathbf{2.2 \times 10^{30}}$$

❑ **Example 2.**

Density of a neutron star is 2.7×10^{17} kg m^{-3}. Assume the star to be spherical, calculate the radius of the neutron star if its mass is twice the mass of the sun.

Take mass of the sun = 2.0×10^{30} kg

Solution:

Given mass of the neutron star, $M = 2 \times 2.0 \times 10^{30}$ kg

$$= 4.0 \times 10^{30} \text{ kg}$$

$$\text{Density of neutron star} = \frac{M}{\frac{4}{3}\pi r^3} = 2.7 \times 10^{17} \text{ kg m}^{-3}$$

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{M}{2.7 \times 10^{17}}$$

$$= \frac{4 \times 10^{30}}{2.7 \times 10^{17}} \text{ m}^3$$

$$\Rightarrow \quad r^3 = 3.527 \times 10^{12} \text{ m}^3$$

$$\Rightarrow \quad r = 1.52 \times 10^4 \text{ m} = \mathbf{15.2 \text{ km}}$$

5. MEASUREMENT OF TIME

There are many natural periodic phenomena taking place around us. For example, day and night, seasons, etc. these events give us a feeling of passing time. We have developed so many types of clocks and watches to keep a track of time. At present we have achieved tremendous accuracy

in the measurement of time using atomic standard of time. This atomic standard of time is based on periodic electromagnetic vibrations produced in a cesium atom. These cesium atomic clocks are used in various countries to maintain national standards of time. In case of quartz wristwatch we use small quartz crystal and watch is driven by the vibrations produced by quartz crystal. Similarly, cesium clocks are regulated by the vibrations produced by caesium atom. In a caesium atomic clock, second is defined as the time interval needed to complete 9,192,631,770 vibrations of the electromagnetic radiation corresponding to transition between two hyperfine levels of the ground state of caesium-133 atom. These clocks are very accurate and portable standard of time. We have such clocks in National Physical Laboratory (NPL), New Delhi, which has the responsibility for the maintenance and improvements of several physical standards which includes time and frequency also. Four caesium atomic clocks are used to maintain the standard of time and frequency. Indian Standard Time (IST) is linked to this set of atomic clocks. These atomic clocks are so accurate that they lose or gain not more than 3 μs in one year.

In physics we deal with wide range of time. Life span of most unstable particle is found to be 10^{-24} s, which is such a small period of time. Similarly, at the other end estimated age of universe is believed to be 10^{17} s, and this time interval is so large. Table 5 shows the order of average time interval for various phenomena.

Table 5. Range of Time

Event	Order of time interval (s)
Life-span of Higgs particle	10^{-22}
Time period of X-rays	10^{-19}
Time period of atomic vibrations	10^{-15}
Time period of light wave	10^{-15}
Life time of an excited state of an atom	10^{-8}
Time period of radio wave	10^{-6}
Period of a sound wave	10^{-3}
Wink of eye	10^{-1}
Time between successive human heart beats	10^{0}
Time taken by light from moon to reach earth	10^{0}
Time taken by sunlight to reach earth	10^{2}
Time period of a satellite	10^{4}
Rotation period of the earth	10^{5}
Rotation period of the moon	10^{6}
Revolution period of the moon	10^{6}
Revolution period of the earth	10^{7}
Travel time for light from proxima centauri star	10^{8}
Average human life	10^{9}
Age of Egyptian pyramids	10^{11}
Time since extinction of dinosaurs	10^{15}
Age of the earth	10^{17}
Age of the universe	10^{17}

KEY NOTE

Information of order of magnitude of measured quantities is required when we make comparison between them. We can determine the order of magnitude of a measured quantity in following ways:

A. Firstly, the measured quantity is expressed in the scientific notation, i.e. in the form $a \times 10^{b}$, where $1 \le a < 10$.

B. (*i*) If $a < \sqrt{10}$ (= 3.162), the order of magnitude is equal to b.

(*ii*) If $a > \sqrt{10}$ (= 3.162), the order of magnitude of the measured quantity is equal to $b + 1$.

FORMULAE USED

Fractional error in measurement of time = $\dfrac{\text{Difference in time } (\Delta t)}{\text{time interval } (t)}$

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SOLVED EXAMPLES

Example 1.

Write the order of magnitude of the given measurements:

(i) 57

(ii) 1001

(iii) 879000

(iv) 0.04

(v) 38750000

(vi) 0.00000681

Solution: Expressing each number in terms of nearest power of 10, we get

(*i*) $57 = 0.57 \times 10^2$ m

$0.5 < 0.57 < 5$

Order of magnitude = **2**

(*ii*) $1001 = 1.001 \times 10^3$

$0.5 < 1.001 < 5$

Order of magnitude = **3**

(*iii*) $879000 = 0.879 \times 10^6$

$0.5 < 0.879 < 5$

Order of magnitude = **6**

(*iv*) $0.04 = 4 \times 10^{-2}$

$0.5 < 4 < 5$

Order of magnitude = **–2**

(*v*) $38750000 = 3.875 \times 10^7$

$0.5 < 3.875 \times 5$

Order of magnitude = **7**

(*vi*) $0.00000681 = 0.681 \times 10^{-5}$

$0.5 < 0.681 < 5$

Order of magnitude = **–5**

Example 2.

A human heart beats once in 0.8 s. How many times it will beat in the life of 70 years of a man?

Solution: Life span of man = 70 years

Period of heart beat = 0.8 s

$$\text{Total number of heart beats} = \frac{70 \times 365.25 \times 24 \times 60 \times 60}{0.8}$$

$$= \mathbf{2.76 \times 10^9 \text{ times}}$$

Example 3.

Two atomic clocks are allowed to run for 100 years and they differ by 0.1 s only. What will be the accuracy of standard atomic clock in measuring 1 s?

Solution:

Given, time, $t = 100$ years

$= (100 \times 365 + 24)$ days

(24 days added due to 24 leap years in 100 years)

$\therefore \quad t = 36524 \text{ days} = 36524 \times 86400$ s

Also, $\Delta t = 0.1$ s

Fractional error in measurement of time

$$= \frac{\Delta t}{t} = \frac{0.1}{36524 \times 86400} = \mathbf{0.32 \times 10^{-10}}$$

Accuracy shown by atomic clock is 0.32×10^{-10} part in 1 s.

Example 4.

The age of the universe is about 10^{10} years whereas the mean life for human existence is 10^6 years. Find the physical meaning of the time interval which is approximately half way between these two on a logarithmic scale.

Solution: Mean life of human existence

$$t_1 = 10^6 \text{ years}$$

Mean life of universe, $t_2 = 10^{10}$ years

If t is the time interval, which is half way between t_1 and t_2, then

$$\log t = \frac{1}{2}\left[\log t_1 + \log t_2\right]$$

$$= \frac{1}{2}\left[\log 10^6 + \log 10^{10}\right]$$

$$= \frac{1}{2}[6 + 10] = 8$$

$$t = \mathbf{10^8 \text{ years}}$$

which is equal to the travel time for light from the nearest star.

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Answers to Practice Problems

1. (*a*) 8.71×10^{-2} rad
   (*b*) 1.45×10^{-3} rad
   (*c*) 2.4×10^{-5} rad.
2. 10^3 kg/m^3, 1 g/cc
3. 3×10^{27}
4. 353.57 cm^2

PRACTICE PROBLEMS 1

1. Convert the angle of (*a*) 5° (degree) (*b*) 5′ (arc min) (*c*) 5″ (arc min) in radian.
2. Find the density of water at 4°C (in g/cc and in kg/m^3).
3. How many protons weigh 5 kg? The mass of a proton is 1.67×10^{-27} kg.
4. Calculate the surface area of a cylinder of diameter 5 cm and height 20 cm.

5. What is the average distance of the sun from earth in (*i*) light years (*ii*) in parsec?

6. A huge circular container is observed from two diametrically opposite position X and Y on the earth. The angle subtended by the two directions of observations is 1°26′4″. The diameter of earth is about 1.27×10^7 m. What is the distance of container from earth?

7. What will be the diameter of an oil molecule if twenty drops of olive oil of radius 0.12 mm are spread on the surface of water, so that they form a circular film of radius 25 cm.

8. Calculate the linear magnification of a square of side 2 mm, which when projected on a screen, gives an image of a square of side 2 cm.

9. How much is the empty space in 1 mole of Helium gas at STP if the radius of helium molecule is about 0.98 Å?

10. A pole is standing on a levelled ground.

The shadow of a pole is 30 m longer when the sun's altitude is 45° than when it was 60°. Calculate the height of the pole.

Answers to Practice Problems

5. (*i*) 0.16×10^{-4} light year
(*ii*) 0.49×10^{-5} parsec
6. 5.08×10^8 m
7. 73.7×10^{-9} m
8. 10
9. 99.98%
10. 231 m

6. ACCURACY, PRECISION OF INSTRUMENTS AND ERRORS IN MEASUREMENT

Physics is the science of measurement and in case of such experimental science, measurement is most important. We have several measurement techniques which involve direct or indirect measurement of a certain physical quantity. There are several measuring instruments which are used to measure certain physical quantities. Some uncertainty is always associated with the result that we get from the measuring instruments. Uncertainty in the result of measurement is called error. Many times some physical quantity depends on some other measured physical quantities and thus error is propagated in the final result of calculations.

Accuracy and precision are two terms which are often used. We should be clear about the difference between the two. If measured result is close to the true value of the physical quantity then it is said to be more accurate. Hence, accuracy is a measure of how close the measured value is to the true value of the quantity. However, precision is to what resolution or limit the quantity is measured. Vernier caliper can measure a minimum distance of 0.1 mm accurately and a screw gauge can measure a minimum distance 0.01 mm accurately. Hence, screw gauge is more precise in measurement than vernier caliper. Note that although screw gauge is more precise in measurement but measurement taken from vernier caliper may be more accurate because accuracy of result of measurement may depend on several other factors including resolution of the measuring instrument. Let the true value of a certain physical quantity be 4.589 mm. When it is measured by an instrument of resolution 0.1 mm then the measured value is found to be 4.4 mm. Again when the same quantity is measured by another instrument of resolution 0.01 mm then the measured value is found to be 4.23 mm. Here, first measurement is closer to the true value of quantity and hence the first result is more accurate although it is less precise. However, second measurement is less accurate but more precise. We can easily understand that precision in the measurement depends on the instrument but accuracy may depend on the person performing measurement and several other factors. We can understand now that error is always an integral part of the measurement. Error analysis is very important part of the experiment. We can classify errors as systematic errors and random errors.

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Systematic errors

Systematic errors always tend to deviate the result in one particular direction either positive or negative. In other words measured result will be greater than or less than the actual value. Systematic errors may be generated from the number of sources and some examples are as follows:

1. Instrumental errors

Faulty design, calibration error and zero error in the instrument can be a source of systematic error. Simplest example we can take is one metre scale whose ends are worn off, it will always read less than the actual value. Similarly, think about one barometer which has its scale little bit shifted and it reads 74 cm when actual height is 76 cm. Similarly zero mark on vernier scale may not coincide with zero mark of main scale and this initial shift will be reflected in the readings. Similar zero error can appear in screw gauge which can be either positive or negative and hence in other words it may increase or decrease the final result. The systematic error can be minimised by zero correction and improving the instruments.

2. Imperfection in experimental procedure

Some experiments are affected by external physical conditions like temperature, pressure, humidity, etc. and these factors may affect the result of the experiment systematically. For example, we want to measure the speed of sound using some standard experimental procedure then we should know that increase in humidity increases the speed of sound. Hence, result of the experiment can be affected by the changing humidity level of the environment. Similarly, procedure of measurement can also affect the result. For example, we normally place the thermometer under the armpit to measure the temperature of the person. But we should know that temperature measured in such a manner will always be less than the actual temperature of the body of the person. Another example in this category may be the measurement of potential difference between two points in an electric circuit. Voltmeter is connected in parallel between those two points and to perform measurement voltmeter draws a little current from the circuit. And due to this potential difference between those two points changes. Hence reading of the voltmeter will be different from the actual reading. Again to measure current in a circuit we connect an ammeter in series with the branch where current is to be measured. But we know that there is some resistance of the ammeter and it affects the current in the circuit and hence the measured value of current becomes slightly different from the actual value.

3. Personal errors

This is very subjective kind of error and depends on the person performing the experiment. Person may be careless about following the proper procedure while performing the experiment. Person may not observe proper precautions while performing experiment. If the apparatus is not properly set for the experiment then it may become a source of error in the experiment. So we can understand that there is an endless scope for the number of reasons behind the personal errors in the experiment. While reading the position of needle on scale of some metre, person might look at it a little far from left side or right side and then there are chances for the error in the assessment due to parallax. Similarly, suppose you want to measure the time period for

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40 oscillations using some stopwatch, then there are chances to make slight errors in timing to start and stop the stopwatch.

From the given description about systematic errors we can understand that systematic error is a kind of error whose source and outcome in the result are predictable. Hence, it is possible to minimise systematic errors in the measurements. Selection of better quality instruments for the measurement is first step towards minimising systematic error due to faulty instruments which are not properly calibrated for the measurement. If you want to measure some dimension using vernier caliper or screw gauge then you should first check its zero error. Sometimes it is possible to remove the zero error but sometimes it is not. If it is not possible to remove the zero error then we should first note down the zero error to be adjusted later in the final result. For better measurements we need to think about ways to minimise personal bias as far as possible.

Random errors

We know that everything cannot be predicted. For example, there are random and small fluctuations in temperature and pressure of the surroundings. If certain experiment is too sensitive for these parameters then these changes will affect the measured result randomly. Suppose you are performing some electrical experiment. Voltage supply is one important part of the experiment. Random fluctuations in voltage can affect the experimental results. Person can also be a source of random error in the readings. Suppose one person is assigned to note down temperature shown by thermometer every one minute, in a certain temperature-sensitive process. Then every time person notes down the temperature, he or she is likely to make some mistake in the reading, hence person can also add random error in the measurement. Hence random errors are those errors which affect the result irregularly in terms of sign and size. The random errors can be reduced by taking more sets of readings of the measurement. The mean of these results will be closer to the actual reading. Thus to avoid random errors repeated measurements or readings should be taken.

Least count error

Least count of a certain measuring instrument is the smallest value of the corresponding physical entity that can be measured by the measuring instrument. It is often referred to as the resolution of that instrument. Smaller least count of an instrument means that instrument is more precise. For example, least count of vernier caliper is 0.1 mm and that for screw gauge is 0.01 mm; hence screw gauge is more precise than vernier caliper in measuring length. If some value is less than the least count of instrument then given instrument cannot measure it correctly. Let us take an example of metre scale in which graduations are made at a regular spacing of 1 mm. Hence, 1 mm is least count of metre scale. Now assume some length is 10.1 mm and you are using metre scale to measure it then you can measure it correctly only up to 10 mm. For the remaining you cannot say anything for sure. Without guessing anything, what best you can say is that given length is between 10 mm and 11 mm. So you can imagine that least count of an instrument is maximum possible uncertainty in the results obtained from that instrument. Thus, it can be said that the least count error can be minimised by using an instrument with higher resolution or smaller value of least count.

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6.1. Error Analysis – Absolute Error, Relative Error and Percentage Error

To obtain the best possible result from an experiment we often repeat the same experiment several times and list the results obtained in every attempt. We have already discussed about several types of sources to the errors in an experiment. Sometimes we may overestimate the result and some other time we may underestimate the result, so mean value is often considered as final result of the experiment. But a little error analysis can be done to estimate uncertainty or possible errors in the outcome of the experimental result.

Let x_1, x_2, x_3,....., x_n be the measured values of given amount of physical entity when measurement is repeated n times. As explained earlier we take arithmetic mean value as the best possible value of given physical entity under given conditions of measurement and it can be calculated as follows:

$$x_{mean} = \frac{x_1 + x_2 + x_3 + \cdots\cdots + x_n}{n}$$

Or,

$$x_{mean} = \frac{\sum_{i=1}^{i=n} x_i}{n}$$

We can now treat x_{mean} as the true value of given physical quantity. Magnitude of the difference between true value and the individual measured value is called **absolute error** for that measurement. Absolute errors in measurements can be written as follows:

$$|\Delta x_1| = |x_1 - x_{mean}|$$
$$|\Delta x_2| = |x_2 - x_{mean}|$$
$$|\Delta x_3| = |x_3 - x_{mean}|$$
.
.
.
$$|\Delta x_n| = |x_n - x_{mean}|$$

Note that the difference of individual measurement from the mean value may be positive or negative because mean value is somewhere in the middle of those values but absolute error is always taken as positive; hence just the magnitude of difference of individual measurement with the mean or true value of that physical quantity.

Arithmetic mean of these absolute errors is called **mean absolute error** for the measured physical quantity. Mean absolute error can be calculated as follows:

$$\Delta x_{mean} = \frac{|\Delta x_1| + |\Delta x_2| + |\Delta x_3| + \cdots\cdots + |\Delta x_n|}{n}$$

Or,

$$\Delta x_{mean} = \frac{\sum_{i=1}^{i=n} |\Delta x_i|}{n}$$

FORMULAE USED

(*i*) $x_{mean} = \dfrac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$

$$x_{mean} = \frac{\sum_{i=1}^{i=n} x_i}{n}$$

(*ii*) Mean absolute error Δx_{mean}

$$= \frac{|\Delta x_1| + |\Delta x_2| + \cdots + |\Delta x_n|}{n}$$

$$= \frac{\sum_{i=1}^{i=n} \Delta x_i}{n}$$

where $|\Delta x_1| = |x_1 - x_{mean}|$, $|\Delta x_2| = |x_2 - x_{mean}|$ and so on.

(*iii*) Fractional error/Relative error

$$= \frac{\Delta x_{mean}}{x_{mean}}$$

(*iv*) Percentage error $= \dfrac{\Delta x_{mean}}{x_{mean}} \times 100$

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Now the final result of measurement can be written as: $x_{mean} \pm \Delta x_{mean}$

If x is some measured value of the given physical quantity then it will lie in an interval as follows:

$$x_{mean} - \Delta x_{mean} \le x \le x_{mean} + \Delta x_{mean}$$

Here Δx_{mean} is the mean absolute error but sometimes we write error as relative error and percentage error.

The ratio of mean absolute error to that with mean value of the measured physical quantity is called **relative error or fractional error.**

$$\text{Relative error} = \frac{\Delta x_{mean}}{x_{mean}}$$

When relative error is expressed as percentage then it is called percentage error.

$$\text{Percentage error} = \frac{\Delta x_{mean}}{x_{mean}} \times 100$$

SOLVED EXAMPLES

Example 1.

The diameter of a wire measured in an experiment was 0.022 cm, 0.023 cm, 0.026 cm, 0.025 cm, 0.024 cm and 0.025 cm find

(i) the mean value of diameter

(ii) absolute error in a each measurement

(iii) Percentage error

Solution: (*i*) Mean diameter

$$d_m = \frac{0.022 + 0.023 + 0.026 + 0.025 + 0.024 + 0.025}{6}$$

$$= 0.02416 \text{ cm}$$

d_m = **0.024 cm** [Rounded off to 2 significant figures]

(*ii*) Absolute errors in the different measurement are

$\Delta d_1 = d_m - d_1$
$= 0.024 - 0.022$
$=$ **0.002 cm**

$\Delta d_2 = d_m - d_2$
$= 0.024 - 0.023$
$=$ **0.001 cm**

$\Delta d_3 = d_m - d_3$
$= 0.024 - 0.026$
$=$ **–0.002 cm**

$\Delta d_4 = d_m - d_4$
$= 0.024 - 0.025$
$=$ **–0.001 cm**

$\Delta d_5 = d_m - d_5$
$= 0.024 - 0.024 =$ **0**

$\Delta d_6 = d_m - d_6$
$= 0.024 - 0.025$
$=$ **–0.001 cm**

(*iii*) Absolute errors $= \frac{\sum |\Delta d_i|}{6}$

$$= \frac{0.002 + 0.001 + 0.002 + 0.001 + 0 + 0.001}{6}$$

$= 0.0012 = 0.001$
(Rounded off to 1 significant figure)

$$\text{Percentage error} = \frac{\text{Absolute error}}{d_m} \times 100$$

$$= \frac{0.001}{0.024} \times 100$$

$= 4.2\% \approx$ **4%**
(Rounded off to 1 significant figure)

Example 2.

Refractive index of a flint glass (μ) was measured in an experiment and was found to be 1.655, 1.667, 1.655, 1.659, 1.669 and 1.654. Find

(i) the mean value of μ

(ii) mean absolute error

(iii) Relative error

(iv) Percentage error

Solution: Given, $\mu_1 = 1.655$, $\mu_2 = 1.667$, $\mu_3 = 1.655$
$\mu_4 = 1.659$, $\mu_5 = 1.669$, $\mu_6 = 1.654$

(*i*) Mean value of μ is

$$\mu_m = \frac{\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6}{6}$$

$$= \frac{1.655 + 1.667 + 1.655 + 1.659 + 1.669 + 1.654}{6}$$

$= 1.6598$
$=$ **1.660** (Rounded off to 4 significant figures)

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(*ii*) Absolute errors in the different measurements are

$\Delta\mu_1 = \mu_m - \mu_1$

$= 1.660 - 1.655$

$= 0.005$

$\Delta\mu_2 = 1.660 - 1.667 = -0.007$

$\Delta\mu_3 = 1.660 - 1.655 = 0.005$

$\Delta\mu_4 = 1.660 - 1.659 = 0.001$

$\Delta\mu_5 = 1.660 - 1.669 = -0.009$

$\Delta\mu_6 = 1.660 - 1.654 = 0.006$

Mean absolute error

$$\overline{\Delta\mu} = \frac{\sum|\Delta\mu_i|}{6}$$

$$= \frac{0.005 + 0.007 + 0.005 + 0.001 + 0.009 + 0.006}{6}$$

$= 0.0055 \approx \mathbf{0.006}$

(Rounded off to 1 significant figure)

(*iii*) Relative error

$$\Delta\mu = \frac{\overline{\Delta\mu}}{\mu_m} = \frac{0.006}{1.660} = 0.0036$$

$= \mathbf{0.004}$

(*iv*) Percentage error $= \dfrac{\overline{\Delta\mu}}{\mu_m} \times 100\% = \mathbf{0.4\%}$

6.2. Combination of Errors

When there is involvement of measurements of several entities in an experiment then several calculations may be required to perform among these measured values. So it is essential to learn how errors are propagated in the final results. We can take an example of determining acceleration due to gravity at some location using simple pendulum. Acceleration due to gravity in terms of time period (T) and length of thread (l) of pendulum can be written as follows:

$$g = \frac{4\pi^2 l}{T^2}$$

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Here, we measure the length of thread and the time period of oscillation. Error is associated with measurement of both the quantities. We need to learn to evaluate combined error in calculated value of g. One point that we need to keep in mind is that we are required to calculate maximum possible error in the final result.

a) Error of sum or a difference

Let A and B be two physical quantities with measured values $A \pm \Delta A$ and $B \pm \Delta B$ respectively. Here ΔA and ΔB are absolute errors in the measurements of A and B respectively. Let Z represent the sum of A and B and ΔZ represent absolute error in Z.

$$Z = A + B$$

$$\Rightarrow \quad Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$$

$$\Rightarrow \quad Z \pm \Delta Z = (A + B) \pm \Delta A \pm \Delta B$$

$$\Rightarrow \quad \pm \Delta Z = \pm \Delta A \pm \Delta B$$

From above we can see that maximum possible value of

$$\Delta Z = \Delta A + \Delta B$$

Similarly assume Z represents the difference of A and B and ΔZ represents the absolute error in Z.

$$Z = A - B$$

$$\Rightarrow \quad Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$$

$$\Rightarrow \quad Z \pm \Delta Z = (A - B) \pm \Delta A \pm \Delta B$$

$$\Rightarrow \quad \pm \Delta Z = \pm \Delta A \pm \Delta B$$

From above we can see that maximum possible value of

$$\Delta Z = \Delta A + \Delta B$$

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Hence we have got the same result as we got in case of finding the sum of A and B. Hence, the rule can be written as follows:

When two similar physical quantities are added or subtracted then absolute error in the final result is equal to the sum of absolute errors in the individual physical quantities.

For example, let temperature $\theta_1 = 20\ °C \pm 0.5\ °C$ and temperature $\theta_2 = 30\ °C \pm 0.25\ °C$ then difference of temperature can be written as $\Delta\theta = \theta_2 - \theta_1 = 10\ °C \pm 0.75\ °C$. Similarly, let time $t_1 = 2\ s \pm 0.1\ s$ and time $t_2 = 3\ s \pm 0.05\ s$ then total time can be written as $t = t_1 + t_2 = 5\ s \pm 0.15\ s$. Hence, in both the cases of sum or difference, absolute errors are always added for the final result.

b) Error of a product or a quotient

Let A and B be two physical quantities with measured values $A \pm \Delta A$ and $B \pm \Delta B$. Let Z be equal to AB and ΔZ is absolute error in Z.

$$Z = AB$$

$$\Rightarrow \quad Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$$

$$\Rightarrow \quad Z \pm \Delta Z = AB \pm \Delta AB \pm A\Delta B \pm \Delta A\Delta B$$

Since ΔA and ΔB are small numbers hence we can ignore their product to rewrite the above equation as follows:

$$\Rightarrow \quad Z \pm \Delta Z = AB \pm \Delta AB \pm A\Delta B$$

Dividing the above equation by $Z = AB$ we get the following:

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

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Hence, maximum possible relative error in the value of Z can be written as follows:

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Now for the division also we can proceed as follows:

Let A and B be two physical quantities with measured values $A \pm \Delta A$ and $B \pm \Delta B$. Let Z be equal to A/B and ΔZ is absolute error in Z.

$$Z = \frac{A}{B}$$

$$\Rightarrow \quad Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B} = \frac{A\left(1 \pm \frac{\Delta A}{A}\right)}{B\left(1 \pm \frac{\Delta B}{B}\right)}$$

$$\Rightarrow \quad Z \pm \Delta Z = \frac{A}{B}\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

Here $\Delta B/B$ is a small number and hence we can use binomial approximation to rewrite the expression as follows:

$$\Rightarrow \quad Z \pm \Delta Z = \frac{A}{B}\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)$$

$$\Rightarrow \quad Z \pm \Delta Z = \frac{A}{B}\left(1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A}\frac{\Delta B}{B}\right)$$

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Since ΔA and ΔB are small numbers and hence we can neglect the term with product of both of them to rewrite the equation as follows:

$$\Rightarrow \quad Z \pm \Delta Z = \frac{A}{B}\left(1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}\right)$$

Dividing the above equation by $Z = A/B$ we get the following:

$$\Rightarrow \quad 1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

$$\Rightarrow \quad \pm \frac{\Delta Z}{Z} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

We can see that the maximum value of relative error in Z can be written as follows:

$$\Rightarrow \quad \frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Note that we are getting the same result for multiplication and division both. Hence the rule for error in product and division can be written as follows:

When two physical entities are multiplied or divided then relative error of the final result is equal to sum of the relative errors in individual physical entities.

c) Error due to power raised on measured quantity

Let A be a physical quantity with measured value $A \pm \Delta A$ and another entity Z is equal to A^n. Let ΔZ be absolute error in Z.

$$Z = A^n$$

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$$\Rightarrow \quad Z \pm \Delta Z = (A \pm \Delta A)^n$$

$$\Rightarrow \quad Z \pm \Delta Z = A^n\left(1 \pm \frac{\Delta A}{A}\right)^n$$

Since $\Delta A/A$ is small number and hence we can use binomial approximation to rewrite the above equation as follows:

$$\Rightarrow \quad Z \pm \Delta Z = A^n\left(1 \pm n\frac{\Delta A}{A}\right)$$

Dividing the above equation by $Z = A^n$, we get the following:

$$\Rightarrow \quad 1 \pm \frac{\Delta Z}{Z} = 1 \pm n\frac{\Delta A}{A}$$

$$\Rightarrow \quad \frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$$

Hence relative error in Z is equal to n times of relative error in A.

d) Error due to combination of product, division and power

We can combine the rules learned above. Relative errors of all measured quantities in case of product and division are added to calculate the relative error in the final result. If in an equation a physical quantity is raised to some power then that power is multiplied to the relative error for the quantity before adding it with others for final result.

Let us randomly assume following relation for a hypothetical physical entity Z in terms of measured quantities A, B and C.

$$Z = \frac{A^l B^m}{C^n} = A^l B^m C^{-n}$$

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Combining all rules we can write relative error of Z as follows:

$$\frac{\Delta Z}{Z} = l\frac{\Delta A}{A} + m\frac{\Delta B}{B} + n\frac{\Delta C}{C}$$

Note that there is always positive sign between the terms irrespective of the fact that raised power is negative or positive. Remember that we always calculate maximum possible error in the measurement so errors are always added together.

Let us take another hypothetical relation for some physical quantity Z as follows:

$$Z = \frac{A^2B^{2/3}}{CD^{1/4}}$$

Relative error in Z can be written as follows:

$$\frac{\Delta Z}{Z} = 2\left(\frac{\Delta A}{A}\right) + \frac{2}{3}\left(\frac{\Delta B}{B}\right) + \left(\frac{\Delta C}{C}\right) + \frac{1}{4}\left(\frac{\Delta D}{D}\right)$$

We can take another example of acceleration due to gravity at point that can be written in terms of time period and length of thread of simple pendulum as follows:

$$g = \frac{4\pi^2 l}{T^2}$$

Here $4\pi^2$ is constant and let Δl and ΔT are absolute errors in the measurement of length and time period of pendulum. Relative error in g can be written as follows:

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

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FORMULAE USED

(*i*) If $Z = A + B$ or $Z = A - B$
then $\Delta Z = \pm\Delta A \pm \Delta B$
Maximum value of
$\Delta Z = \Delta A + \Delta B$

(*ii*) If $Z = AB$

$$\frac{\Delta Z}{Z} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

Also for $Z = \frac{A}{B}$

$$\frac{\Delta Z}{Z} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

(*iii*) For $Z = A^n$

$$\frac{\Delta Z}{Z} = \pm\frac{n\Delta A}{A}$$

SOLVED EXAMPLES

Example 1.

The temperature of two bodies measured by a thermometer are $t_1 = 20\ °C \pm 0.5\ °C$ and $50\ °C \pm 0.5\ °C$. Calculate the temperature difference and the error therein. *[N.C.E.R.T. Solved Example]*

Solution: Temperature difference:

$$\Delta t = t_2 - t_1$$
$$= (50\ °C \pm 0.5\ °C) - (20\ °C \pm 0.5\ °C)$$
$$= \mathbf{30\ °C \pm 1\ °C}$$

Example 2.

Two resistances of values $150 \pm 2\ \Omega$ and $250 \pm 5\ \Omega$ are connected in series. Find their equivalent resistance.

Solution: Given, $R_1 = 150 \pm 2\ \Omega$

$$R_2 = 250 \pm 5\ \Omega$$

Equivalent resistance

$$R = R_1 + R_2 = (150 \pm 2\ \Omega) + (250 \pm 5\ \Omega)$$
$$R = (150 + 250) \pm (2 + 5)$$
$$= \mathbf{(400 \pm 7)\ \Omega}$$

Example 3.

The resistance $R = \frac{V}{I}$, where $V = 100 \pm 5V$ and $I = 10 \pm 0.2$ A. Find the percentage error in R. *(N.C.E.R.T. Solved Example)*

Solution: Percentage error in $V = \frac{5}{100} \times 100 = 5\%$

Percentage error in $I = \frac{0.2}{10} \times 100 = 2\%$

$$R = \frac{V}{I}$$

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$

$\Rightarrow$ Total percentage error in R is

$$\frac{\Delta R}{R} \times 100 = 5\% + 2\%$$
$$= \mathbf{7\%}$$

Example 4.

The error in measurement of side of a cube is 2%. Find the error in the volume of the cube?

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Solution: Given, percentage error in side length a is

$$\frac{\Delta a}{a} \times 100 = 2\%$$

Volume of cube is, $V = a^3$

$\therefore$ Percentage error in volume of cube is:

$$\frac{\Delta V}{V} \times 100 = 3\left(\frac{\Delta a}{a}\right) \times 100 = 3 \times 2\% = \mathbf{6\%}$$

Example 5.

The length, breadth and height of a rectangular block are found to be 15.12 ± 0.02 cm, 7.86 ± 0.01 cm and 4.16 ± 0.02 cm, respectively. Compute the percentage error in the volume of the block.

Solution: Given, length, $l = 15.12 \pm 0.02$ cm

Breadth, $b = 7.86 \pm 0.01$ cm

Height, $h = 4.16 \pm 0.02$ cm

Volume of rectangular block is

$$V = l \times b \times h$$

The percentage error in the volume is given by

$$\frac{\Delta V}{V} \times 100 = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}\right) \times 100$$

$$= \left(\frac{0.02}{15.12} + \frac{0.01}{7.86} + \frac{0.02}{4.16}\right) \times 100$$

$$= \left(\frac{2}{1512} + \frac{1}{786} + \frac{2}{416}\right) \times 100$$

$$= 0.1323 + 0.1272 + 0.4808 = 0.7403\%$$

$$= \mathbf{0.74\%}$$

(Rounded off to 2 significant figures)

Example 6.

Find the relative error in Z, if

$$Z = A^4 B^{1/3} / CD^{3/2}$$

(N.C.E.R.T. Solved Example)

Solution: The relative error in Z is

$$\frac{\Delta Z}{Z} = 4\left(\frac{\Delta A}{A}\right) + \frac{1}{3}\left(\frac{\Delta B}{B}\right) + \left(\frac{\Delta C}{C}\right) + \frac{3}{2}\left(\frac{\Delta D}{D}\right)$$

Example 7.

Two resistors of resistance $R_1 = 100 \pm 3$ ohm and $R_2 = 200 \pm 4$ ohm are connected (a) in series, (b) in parallel.

Find the equivalent resistance of the

(a) series combination

(b) parallel combination

Use for (a) the relation $R = R_1 + R_2$ and for (b)

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} \text{ and } \frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

(N.C.E.R.T. Solved Example)

Solution: (a) For the series combination, equivalent resistance is $R = R_1 + R_2$

$$= (100 \pm 3)\ \Omega + (200 \pm 4)\ \Omega$$

$$= (100 + 200) \pm (3 + 4)$$

$$= \mathbf{(300 \pm 7)\ \Omega}$$

(b) For the parallel combination, equivalent resistance of the combination is:

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} \quad \ldots(i)$$

$$\Rightarrow \quad R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{100 \times 200}{100 + 200} = \frac{200}{3} = 66.7\ \Omega$$

On differentiating equation (i) we get

Also, $$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Rightarrow \quad \Delta R' = (R'^2)\frac{\Delta R_1}{R_1^2} + (R'^2)\frac{\Delta R_2}{R_2^2}$$

$$\Rightarrow \quad \Delta R' = \left(\frac{R'}{R_1}\right)^2 \Delta R_1 + \left(\frac{R'}{R_2}\right)^2 \Delta R_2$$

$$= \left(\frac{66.7}{100}\right)^2 \times 3 + \left(\frac{66.7}{200}\right)^2 \times 4$$

$$= 1.8\ \Omega$$

$\therefore \quad R' = \mathbf{66.7 \pm 1.8\ \Omega}$

Example 8.

A physical quantity Y is expressed as

$$Y = \frac{p^2}{r}\left(\frac{q}{\sqrt{s}}\right)^3$$

If the percentage error in the measurement of p, q, r and s are 4%, 2%, 1% and 3% respectively, calculate the percentage error in Y.

Solution: Given, $Y = \frac{p^2}{r}\left(\frac{q}{\sqrt{s}}\right)^3$

$$\frac{\Delta p}{p} \times 100 = 4\%; \quad \frac{\Delta q}{q} \times 100 = 2\%$$

$$\frac{\Delta r}{r} \times 100 = 1\% \text{; and } \frac{\Delta s}{s} \times 100 = 3\%$$

$\therefore$ The percentage error in Y is

$$\frac{\Delta Y}{Y} \times 100$$

$$= 2\left(\frac{\Delta p}{p} \times 100\right) + 3\left(\frac{\Delta q}{q} \times 100\right) + \frac{\Delta r}{r} \times 100 + \frac{3}{2}\left(\frac{\Delta s}{s} \times 100\right)$$

$$= 2 \times 4\% + 3 \times 2\% + 1\% + \frac{3}{2} \times 3\%$$

$$= (8 + 6 + 1 + 4.5)\% = \mathbf{19.5\%}$$

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7. SIGNIFICANT FIGURES

From the previous sections we have understood that every measurement involves errors due to finite resolution of instrument and several other factors. Results of scientific measurements are always written in a way that it may indicate accuracy involved in the measurement. When we measure certain physical entity then certain digits in the measured value are reliably correct. While reporting the measured value we should include all digits which are reliably correct plus one digit which is uncertain. In the measured value, all reliable digits plus first uncertain digit are known as significant digits or significant figures. If we include more uncertain digits while reporting the measured value then it will give false impression about the precision of measurement. So it is important to include only one uncertain digit. If someone measures the time interval of certain phenomenon as 2.57 s then here 2 and 5 are reliable digits whereas 7 is first uncertain digit and hence there are total three significant digits in this measured value. Similarly if someone writes length of object as 256.7 cm then here 2, 5 and 6 are reliable digits but 7 is uncertain digit and there are total four significant digits in the reported value.

Finally, we know that the significant figures in reported value indicate precision of measurement and which in turn depends on the least count of the measuring instrument. We can refer to the following rules to determine the number of significant digits in a reported value of measurement.

1. All the non-zero digits are significant. For example, 52.49 has four significant figures.
2. All the zeros between two non-zero digits are significant. If there is decimal point in the reported value even then this rule is not affected. For example, 2006 has four significant digits and 3050.08 has six significant figures.
3. If reported value is less than 1, then zeros to the right side of the decimal point up to first non-zero digit are not considered as significant. For example, in 0.003048, first three zeros are not significant and hence total number of significant digits is four in this case.
4. If there is no decimal point in the reported value then all terminal or trailing zeros are not considered as significant digits. For example, in 12800, there are only three significant digits.
5. If there is decimal point in the reported value then all trailing zeros are counted as significant figures. For example, 2.300 has four significant figures. Similarly, combining rule-3, we can understand that 0.05400 has four significant digits.
6. If there is some exact number which is not a measured value then it has infinite number of significant digits. For example, p = 3.1415926…. has infinite significant digits.
7. We know that physical quantities are measured in terms of some standard units. If we change the units of reported value then it has no effect on number of significant digits. For example, some length is reported to be equal to 1.207 cm, then we can understand that there are four significant figures in this reported value. We can write the same result as 0.01207 m, then also we can see that there are four significant figures. We can write the same result as 12.07 mm, then also we can see that there are four significant figures. Let us try even smaller unit like μm then we can write the same result as

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12070 μm, and number of significant digits is still four because last zero we know is not counted as significant figure as per rule-4.

Let us take another example of length which is reported as 1.500 m. We can use rule-5 to understand that number of significant digits reported in this case is four. Now let us write it in mm, then the measured value becomes 1,500 mm. Now rule-4 can mislead us to believe that there are only two significant digits, but we know that there are four significant digits in the reported value. We know that by changing units, number of significant digits are not supposed to be affected. In order to avoid this kind of confusion in some cases, it is better to write the value in scientific notation. In scientific notation we write the number as $a \times 10^b$, here a is a number between 1 and 10, and b is any positive or negative power of 10. For example, above measured value can be written as follows:

$$1.500 \text{ m} = 1.500 \times 10^2 \text{ cm} = 1.500 \times 10^3 \text{ mm}$$

Clearly, we can see that in all cases number of significant figures is four. Power of 10 does not affect the number of significant figures. But note that all zeros in base number of scientific notation are significant. Hence, there is no confusion related to the trailing zeros in base number because these zeros are always significant.

8. If the number is less than 1 then we conventionally put a zero to the left of decimal point and this zero is never considered as significant figure, but zeros at the end of such numbers are considered as significant. For example, 0.1760 has four significant digits.

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SOLVED EXAMPLES

❑ **Example 1.**

Specify the number of significant figures in the following measurements:

(i) 5.000 kg

(ii) 3500 m

(iii) 0.070 s

Solution: (*i*) Four: 5, 0, 0, 0 are all significant

(*ii*) Four: 3, 5, 0, 0 are all significant

(*iii*) Two, only 7 and 0 after it, are significant

❑ **Example 2.**

State the number of significant figures in the following measurements:

(i) 1.60×10^9 *kg*

(ii) 7.036 J

(iii) 0.0003 s

Solution: (*i*) Three: 1, 6 and 0

(*ii*) Four: 7, 0, 3 and 6

(*iii*) One: 3 only

7.1. Rules for Arithmetic Operations with Significant Numbers

Many times final result of some experiment is dependent on more than one measured quantities and in such cases calculations are involved to arrive at the final result. We can easily understand that accuracy in measured quantities will affect the accuracy of final result. Hence, now the question is that how to decide accuracy for the final result. We should understand one fact that final result of calculation can never be more accurate than the measured values from which it is calculated. So generally we can say that number of significant digits in the final result cannot be more than number of significant digits in the measured values from which it is calculated. Suppose we have measured time period of 40 oscillations and the measured value is

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89.75 s. Now the time period for single oscillation can be written as $T = \frac{89.75}{40}\text{s} = 2.24375\text{s}$, but time period reported this way will indicate superficial accuracy in measurement with more number of significant digits than the original measured value from which it is calculated. While reporting the final result we must ensure that accuracy of the final result is consistent with the accuracy in measured values from which it is calculated. To ensure this consistency we can observe the following rules:

1. In case of multiplication and division, number of significant digits in the final result should be equal to the minimum number of significant digits in all measured values which are involved.

   Example: Suppose we have measured mass and volume of a given sample of material and it is required to calculate density of material. Given mass = 3.126 g and Volume = 1.75 cm^3.

   $$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{3.126\text{ g}}{1.75\text{ cm}^3} = 1.78628\text{ g/cm}^3$$

   But we know that significant digits in one of the measured values are four and for the second measured value it is three; hence final result of density must be reported up to three significant digits only, and the number must be rounded off accordingly.

   Hence, Density = 1.79 g/cm^3.

2. In case of addition and subtraction rule is stated in terms of decimal places instead of number of significant figures. When two or more numbers are added or subtracted then number of decimal places in the final result should be same as the number with the least decimal places.

   Example: What should be the result of addition of numbers 536.16 g, 284.2 g and 0.107 g?

   If we follow the simple arithmetic then:

   $$536.16\text{ g} + 284.2\text{ g} + 0.107\text{ g} = 820.467\text{ g}.$$

   We can see that one of the numbers 284.2 g is correct only up to one decimal place; hence final result must also contain only one decimal place and must be rounded off accordingly. In above example final result should be rounded off as 820.5 g.

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SOLVED EXAMPLES

Example 1. *Add the numbers 8.31, 13.151 and 0.0039 and express the result to an appropriate number of significant figures.*

Solution:

$$\begin{array}{r} 8.31 \\ 13.151 \\ 0.0039 \\ \hline 21.4649 \end{array}$$

$$= \mathbf{21.47}$$

Here, 8.31 has least number of decimal place (two). So, the result is also rounded off up to second place of decimal.

Example 2. *Subtract 4.5×10^4 from 7.9×10^5 and express the result to an appropriate number of significant figures.*

Solution:

$$7.9 \times 10^5 - 0.45 \times 10^5 = 7.45 \times 10^5$$

$$= \mathbf{7.5 \times 10^5}$$

7.2. Rounding Off the Uncertain Digits

As we have seen above that final result which is based on some measured values must be consistent in terms of accuracy with these measured values, and in this process many times rounding off the digits is required. By convention if the insignificant digit to be dropped is more than 5 then preceding digit is raised by 1 and in case it is less than 5 then preceding digit is left unchanged. For example, 2.157 can be rounded off to 2.16 whereas 2.153 can be rounded off to 2.15.

If insignificant digit to be dropped is equal to 5 then we follow the following convention.

If the preceding digit is even then it is left unchanged and if it is odd then preceding digit is raised by 1.

For example, 5.665 can be rounded off to 5.66 whereas 5.655 can also be rounded off to 5.66 because preceding digit is odd in this case.

In scientific calculations we retain one digit more than significant digits for intermediate steps so that at the end it may be rounded off to proper significant digits.

SOLVED EXAMPLES

Example 1.
Round off the following numbers up to 3 digits:

(i) 17.65

(ii) 14,958

(iii) 3,49,338

(iv) 11.652

Solution: (*i*) 17.6

(*ii*) 15,000

(*iii*) 3,49,000

(*iv*) 11.6

Example 2.
Subtract 2.5×10^{-5} from 8.0×10^{-3} with regards to significant figure.

Solution: Here, $a = 2.5 \times 10^{-5}$

$= 0.000025$ (2 significant figures)

$b = 8.0 \times 10^{-3} = 0.0080$ (2 significant figures)

$b - a = 0.0080 - 0.000025$

$= 0.007975$

$= 7.975 \times 10^{-3}$

$= \mathbf{8.0 \times 10^{-3}}$

(rounded off to two significant figures)

Example 3.
Solve the following expressions and express the result to an appropriate number of significant figures:

(i) $65.5 \times 135.1 \times 0.61$

(ii) $\dfrac{4.23 \times 25.68}{658.2}$

(iii) $\dfrac{3.51 \times 10^{-4} \times 2.71 \times 10^{7}}{0.5562}$

Solution: (*i*) $65.5 \times 135.1 \times 0.61 = 5397.9205$

$= \mathbf{5400}$

[Rounded up to two significant figures]

(*ii*) $\dfrac{4.23 \times 25.68}{658.2} = 0.165040 = \mathbf{0.165}$

[Rounded off to 3 significant figures]

(*iii*) $\dfrac{3.51 \times 10^{-4} \times 2.71 \times 10^{7}}{0.5562} = 17.102 \times 10^{3}$

$= \mathbf{17.1 \times 10^{3}}$

[Rounded off to 3 significant figures]

Example 4.
Calculate the area of a circle of diameter 1.05 m and express the result to an appropriate number of significant figures.

Take $\pi = 3.14$

Solution: Given, diameter of the circle, $d = 1.05$ m

Area of the circle, $A = \pi\left(\dfrac{d}{2}\right)^2$

$= \dfrac{\pi d^2}{4} = \dfrac{3.14 \times (1.05)^2}{4}$

$= 0.8655$

$= \mathbf{0.866\ m^2}$

[Rounded off to 3 significant figures]

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❑ **Example 5.**

Each side of a cube is measured to be 7.203 m. What are the total surface area and the volume of the cube to appropriate significant figures?

[N.C.E.R.T. Solved Example]

Solution: Given, side of the cube,

$$a = 7.203 \text{ m}$$

Total surface area is

$$S = 6a^2$$
$$= 6 \times (7.203)^2 \text{ m}^2$$
$$= 311.299254 \text{ m}^2$$
$$= \mathbf{311.3\ m^2}$$

[Rounded off to 4 significant figures]

Volume, $V = a^3$

$$= (7.203)^3 = 373.714754 \text{ m}^3$$
$$= \mathbf{373.7\ m^3}$$

[Rounded off to 4 significant figures]

PRACTICE PROBLEMS 2

1. Add 2.535×10^2 cm to 1.415×10^3 cm (with regard to significant figures)

2. Which of the following mass measurements is most accurate and why?

(*i*) 5.00 kg (*ii*) 5.004 kg (*iii*) 5.01 kg

3. Solve $\sqrt{7.92 - 4.6}$ with regard to significant figures.

4. Calculate the perimeter of a rectangle within error limits. The sides of the rectangle are (7.5 ± 0.2) cm and (3.5 ± 0.2) cm.

5. While determining the time period of oscillation of a simple pendulum, the readings from various measurements are 1.73 s, 1.62 s, 1.52 s, 1.45 s and 1.83 s. Calculate the values of mean value of time period, absolute error, mean absolute error and relative error.

6. A potential of $V = (10 \pm 0.1)$ volt is applied across a resistance of (5 ± 0.2) ohm. Find the value of current within error limits.

7. Calculate the percentage error in value of R if $V = (50 \pm 5)$V and $I = (5 \pm 1)$A

8. A physical quantity P can be calculated from the relation.

$P = \dfrac{a^3 b^3}{\sqrt{cd}}$. What will be the percentage error in P when percentage errors in measurement of a, b, c, d are 2, 3, 1, 2 respectively.

9. What will be uncertainty in the density of a cube if uncertainty in mass and length is 2% and 3% respectively?

10. While performing Searle's experiment, a weight of 50 N is suspended from a wire. The extension produced is 0.121 cm and is measured by a micrometer of least count 0.001 cm. The diameter of the wire is 0.700 cm, measured by a screw gauge of least count 0.001 cm. The length is 100 cm measured with the help of scale of least count 0.1 cm. Calculate the Young's modulus of material of wire, given by $Y = \dfrac{FL}{Al}$ where F is weight, L is length of wire, A is area of wire and l is extension produced.

11. Two resistances $R_1 = (15 \pm 0.5)\Omega$ and $(20 \pm 0.7)\Omega$ are connected in parallel. Calculate the total resistance of the combination and maximum percentage error.

12. What will be focal length of a spherical mirror if distance of object from the mirror is (30 ± 0.5) cm and distance of image from the mirror is (10 ± 0.2) cm?

13. What will be the error in the measurement of kinetic energy, if the error in measuring momentum is 100%?

Answers to Practice Problems

1. 1.668×10^3 cm

2. The observation (*ii*) has least fractional error and hence, more accurate

3. 1.8

4. (22 ± 0.8) cm

5. 1.63 s, 0.12 s, ± 0.074

6. (2 ± 0.1) A

7. ±30%

8. 16.5%

9. ±11%

10. $Y = (1.07 \times 10^9 \pm 0.013 \times 10^9)\ \text{N/m}^2$

11. $R_P = \dfrac{60}{7}\Omega$, 3.5%

12. (7.5 ± 0.10) cm

13. 200%

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8. DIMENSIONS OF PHYSICAL QUANTITIES

In several chapters of physics we shall learn about many physical quantities. We have already learned that there are seven fundamental or base quantities and there are seven corresponding fundamental units for their measurements. When unit of a certain physical quantity is written in terms of these fundamental units then that is called a derived unit. So we can understand that physical quantities which are described by derived units can be represented as combination of seven fundamental or base quantities. Usually the combination of fundamental quantities, which represents a physical quantity, is expressed in symbolic form and is called **dimensional formula** for given physical quantity. To write dimensional formula of a physical quantity, we represent fundamental quantities with standard symbols which are listed as follows:

Length – L, Mass – M, Time – T, Electric current – A, Thermodynamic Temperature – K, Luminous intensity – cd, Amount of substance – mol.

To write dimensional formula conventionally, these above described symbols are written inside square brackets [ ]. When we use square brackets around a physical quantity then it means we are talking about dimensions of that physical quantity.

Dimensions of a physical quantity are the powers which are raised to fundamental quantities in order to represent that physical quantity. Note that all fundamental quantities may not be involved to represent a given physical quantity. For example, in mechanics all the physical quantities can be expressed in terms of Length, Mass and Time only. When some fundamental quantity does not appear in dimensional formula of a certain physical quantity then we can say that corresponding dimension is zero for that physical quantity.

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Let us take an example of area.

$$\text{Area} = \text{Length} \times \text{breadth}$$

We know that both are length hence dimensional formula of area can be written as follows:

$$[\text{Area}] = [\text{L}] \times [\text{L}] = [\text{L}^2]$$

Hence, we can see that area has 2 dimensions in length. We already know that area is independent of mass and time; hence it has 0 dimension in mass and time. Dimensional formula of area can also be written as follows:

$$[\text{Area}] = [\text{M}^0\text{L}^2\text{T}^0]$$

Similarly we can write the following dimensional formula for the volume:

$$[\text{Volume}] = [\text{L}^3]$$

Hence, volume has three dimensions in Length.

To express a given physical quantity in terms of fundamental quantities we can take help of unit of given physical quantity. For example, SI unit of velocity is m/s; hence velocity can be expressed as length divided by time:

$$\text{Velocity} = \text{Length/Time}$$

$$[\text{Velocity}] = \frac{[\text{L}]}{[\text{T}]}$$

$$\Rightarrow \quad [\text{Velocity}] = [\text{LT}^{-1}]$$

Hence we can see that velocity has dimension of length equal to 1, and dimension of time is –1. Note that speed and velocity are equivalent in this context.

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We know that unit of acceleration is m/s^2; hence acceleration can be expressed as follows:

$$\text{Acceleration} = \text{Length}/(\text{Time})^2$$

$$\Rightarrow \quad [\text{Acceleration}] = \frac{[L]}{[T^2]}$$

$$\Rightarrow \quad [\text{Acceleration}] = [LT^{-2}]$$

Hence acceleration has 1 dimension in Length and –2 dimensions in Time.

Let us now take an example of force, which is defined as mass multiplied with acceleration of object.

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$\Rightarrow \quad \text{Force} = \text{Mass} \times \text{Length}/(\text{Time})^2$$

$$\Rightarrow \quad [\text{Force}] = [M]\frac{[L]}{[T^2]}$$

$$\Rightarrow \quad [\text{Force}] = [MLT^{-2}]$$

Hence, force has 1 dimension in Mass, 1 dimension in Length and –2 dimensions in Time.

We know that work done and hence energy can be represented as force multiplied with the displacement and displacement has dimension of length. So dimensional formula for energy can be written as:

$$[\text{Energy}] = [ML^2T^{-2}]$$

Hence energy has 1 dimension in Mass, 2 dimensions in Length and –2 dimensions in Time.

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In this way we can write dimensional formula for several other physical quantities as shown in Table 6.

Table 6. Dimensional Formulae and SI units of Physical Quantities

S.No.	Physical quantity	Relation with other physical quantities	Dimensional formula	SI unit
1	Length	–	[L]	m
2	Mass	–	[M]	kg
3	Time	–	[T]	s
4	Electric current	–	[A]	A
5	Thermodynamic Temperature	–	[K]	K
6	Amount of Substance	–	[mol]	mol
7	Luminous intensity	–	[cd]	cd
8	Angle	$\frac{\text{arc}}{\text{radius}}$	Dimensionless	rad
9	Area	length × breadth	$[M^0 L^2 T^0]$	m^2
10	Volume	length × breadth × height	$[M^0 L^3 T^0]$	m^3
11	Frequency	$\frac{1}{\text{time period}}$	$[M^0L^0 T^{-1}]$	s^{-1} or Hz (hertz)
12	Specific volume	$\frac{\text{volume}}{\text{mass}}$	$[M^{-1}L^3 T^0]$	$m^3 kg^{-1}$

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S.No.	Physical quantity	Relation with other physical quantities	Dimensional formula	SI unit
13	Density	$\frac{\text{mass}}{\text{volume}}$	$[M\ L^{-3}\ T^{0}]$	kg m^{-3}
14	Specific gravity (Relative density)	$\frac{\text{density of a material}}{\text{density of water at 4 °C}}$	Dimensionless	—
15	Linear velocity or speed	$\frac{\text{displacement or distance}}{\text{time}}$	$[M^{0}\ L^{1}\ T^{-1}]$	m s^{-1}
16	Linear acceleration	$\frac{\text{velocity}}{\text{time}}$	$[M^{0}\ L\ T^{-2}]$	m s^{-2}
17	Angular velocity	$\frac{\text{angle}}{\text{time}}$	$[M^{0}\ L^{0}\ T^{-1}]$	rad s^{-1}
18	Angular acceleration	$\frac{\text{angular velocity}}{\text{time}}$	$[M^{0}\ L^{0}\ T^{-2}]$	rad s^{-2}
19	Centripetal acceleration	$\frac{\text{linear velocity}}{\text{radius}^2}$	$[M^{0}\ L\ T^{-2}]$	m s^{-2}
20	Linear momentum	mass × velocity	$[M\ L\ T^{-1}]$	kg m s^{-1}
21	Force	mass × acceleration	$[M\ L\ T^{-2}]$	N (newton)
22	Tension	force	$[M\ L\ T^{-2}]$	N
23	Impulse	force × time	$[M\ L\ T^{-1}]$	N s
24	Work	force × distance	$[\ M\ L^{2}\ T^{-2}]$	J (joule)
25	Energy (mechanical, heat, light, etc)	work	$[M\ L^{2}\ T^{-2}]$	J
26	Kinetic energy	$\frac{1}{2}\times(\text{mass})\times(\text{velocity})^2$	$[M\ L^{2}\ T^{-2}]$	J
27	Power	$\frac{\text{work}}{\text{time}}$	$[M\ L^{2}\ T^{-3}]$	W (watt)
28	Moment of force	force × perpendicular distance	$[M\ L^{2}\ T^{-2}]$	N m
29	Torque or couple	force × perpendicular distance	$[M\ L^{2}\ T^{-2}]$	N m
30	Angular frequency	2π × frequency	$[M^{0}\ L^{0}\ T^{-1}]$	rad s^{-1}
31	Angular momentum	Moment of inertia × angular velocity	$[M\ L^{2}\ T^{-1}]$	kg m^2 s^{-1}
32	Angular impulse	torque × time	$[ML^{2}T^{-1}]$	kg $m^2 s^{-1}$
33	Radius of gyration	Distance	$[M^{0}\ L\ T^{0}\ ]$	m
34	Moment of Inertia	mass × (radius of gyration)^2	$[M\ L^{2}\ T^{0}]$	kg m^2
35	Rotational kinetic energy	$\frac{1}{2}\times(\text{moment of inertia})\times(\text{angular velocity})^2$	$[ML^{2}\ T^{-2}\ ]$	J
36	Gravitational constant	$\frac{\text{force}\times(\text{distance})^2}{\text{mass}\times\text{mass}}$	$[M^{-1}\ L^{3}\ T^{-2}]$	N m^2 kg^{-2}
37	Acceleration due to gravity	$\frac{(\text{Gravitational constant})\times(\text{mass of earth})}{(\text{distance from the centre of earth})^2}$	$[M^{0}\ L\ T^{-2}]$	m s^{-2}
38	Gravitational potential energy	mass × (acceleration due to gravity) × height	$[\ M\ L^{2}\ T^{-2}]$	J
39	Escape velocity	$\sqrt{2gR}$	$[M^{0}\ L^{1}\ T^{-1}]$	m s^{-2}

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S.No.	Physical quantity	Relation with other physical quantities	Dimensional formula	SI unit
40	Hubble constant	$\frac{\text{velocity of recession}}{\text{distance}}$	$[M^0 L^0 T^{-1}]$	s^{-1}
41	Stress	$\frac{\text{force}}{\text{area}}$	$[M L^{-1} T^{-2}]$	$N m^{-2}$
42	Strain	$\frac{\text{change in dimension}}{\text{original dimension}}$	Dimensionless	—
43	Coefficient of elasticity	$\frac{\text{stress}}{\text{strain}}$	$[M L^{-1} T^{-2}]$	$N m^{-2}$
44	Force constant	$\frac{\text{force}}{\text{increase in length}}$	$[ML^0 T^{-2}]$	$N m^{-1}$
45	Thrust	force	$[M L T^{-2}]$	N
46	Pressure	$\frac{\text{force}}{\text{area}}$	$[M L^{-1} T^{-2}]$	$N m^{-2}$ or Pa (pascal)
47	Pressure gradient	$\frac{\text{pressure}}{\text{distance}}$	$[M L^{-2} T^{-2}]$	$N m^{-3}$
48	Velocity gradient	$\frac{\text{velocity}}{\text{distance}}$	$[M^0 L^0 T^{-1}]$	s^{-1}
49	Reynold number	$\frac{\text{density} \times \text{diameter} \times \text{velocity}}{\text{coefficient of viscosity}}$	$[M^0L^0T^0]$	—
50	Rate of flow	$\frac{\text{volume}}{\text{time}}$	$[M^0L^3T^{-1}]$	m^3s^{-1}
51	Surface Tension	$\frac{\text{force}}{\text{length}}$	$[M L^0 T^{-2}]$	$N m^{-1}$
52	Surface energy	$\frac{\text{energy}}{\text{area}}$	$[M L^0 T^{-2}]$	$J m^{-2}$
53	Coefficient of viscosity	$\frac{\text{force}}{\text{area} \times \text{velocity gradient}}$	$[M L^{-1} T^{-1}]$	daP (decapoise)
54	Temperature gradient	$\frac{\text{temperature}}{\text{distance}}$	$[M^0 L^{-1} T^0 K]$	$K m^{-1}$
55	Coefficient of thermal expansion	$\frac{\text{change in dimension}}{\text{dimension} \times \text{temperature}}$	$[M^0 L^0 T^0K^{-1}]$	K^{-1}
56	Specific heat capacity	$\frac{\text{quantity of heat}}{\text{mass} \times \text{temperature}}$	$[M^0 L^2 T^{-2}K^{-1}]$	$J kg^{-1} K^{-1}$
57	Latent heat	$\frac{\text{quantity of heat}}{\text{mass}}$	$[M^0 L^2 T^{-2}]$	$J kg^{-1}$
58	Wavelength	length of a wave	$[M^0 L T^0]$	m
59	Wavenumber	$\frac{1}{\text{wavelength}}$	$[M^0 L^{-1} T^0]$	m^{-1}
60	Electric charge	current × time	$[M^0 L^0 T A]$	C (coulomb)
61	Surface charge density	$\frac{\text{charge}}{\text{area}}$	$[M^0 L^{-2} TA]$	$C m^{-2}$
62	Volume charge density	$\frac{\text{charge}}{\text{volume}}$	$[M^0 L^{-3} TA]$	$C m^{-3}$

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S.No.	Physical quantity	Relation with other physical quantities	Dimensional formula	SI unit
63	Electric potential	$\frac{\text{work}}{\text{charge}}$	$[ML^2 T^{-3} A^{-1}]$	V (volt)
64	Electric field intensity	$\frac{\text{force}}{\text{charge}}$	$[MLT^{-3} A^{-1}]$	N C^{-1}
65	Electric flux	electric field × area	$[ML^3T^{-3}A^{-1}]$	N m^2 C^{-1}
66	Electric capacitance	$\frac{\text{charge}}{\text{potential difference}}$	$[M^{-1}L^{-2} T^4A^2]$	F (farad)
67	Electric dipole moment	charge × length	$[M^0L\ TA]$	C m
68	Conductivity	$\frac{1}{\text{resistivity}}$	$[M^{-1} L^{-3}T^3A^2]$	S m^{-1} or $\Omega^{-1}m^{-1}$
69	Resistivity	$\frac{\text{resistance} \times \text{area}}{\text{length}}$	$[ML^3 T^{-3}A^{-2}]$	Ω m
70	Electric resistance	$\frac{\text{potential difference}}{\text{current}}$	$[ML^2T^{-3}A^{-2}]$	Ω(ohm)
71	Electric conductance	$\frac{1}{\text{resistance}}$	$[M^{-1}L^{-2} T^3A^2]$	S(siemen) or Ω^{-1}(mho)
72	Faraday constant	Avogadro number × e	$[M^0L^0\ TA\ mol^{-1}]$	C mol^{-1}
73	Electric current density	$\frac{\text{current}}{\text{area}}$	$[M^0\ L^{-2}\ T^0\ A]$	A m^{-2}
74	Inductive reactance	ωL	$[ML^2\ T^{-3}A^{-2}]$	Ω
75	Intensity of magnetisation	$\frac{\text{magnetic moment}}{\text{volume}}$	$[M^0L^{-1}\ T^0A]$	A m^{-1} or N m^{-2} T^{-1}
76	Magnetic pole strength	$\sqrt{\frac{4\pi \times \text{force} \times \text{distance}^2}{\mu_0}}$	$[M^0\ L\ T^0A]$	A m
77	Magnetic dipole moment	pole strength × distance	$[M^0\ L^2\ T^0A]$	A m^2
78	Magnetic induction	$\frac{\mu_0 \times \text{current}}{2\pi \times \text{distance}}$	$[M\ L^0\ T^{-2}\ A^{-1}]$	N m^{-1} A^{-1} or tesla (T)
79	Magnetic flux	B × area	$[M\ L^2\ T^{-2}\ A^{-1}]$	N m A^{-1} or weber (Wb)
80	Permittivity of free space	$\frac{\text{charge} \times \text{charge}}{\text{force} \times \text{distance}^2}$	$[M^{-1}\ L^{-3}\ T^4A^2]$	C^2 N m^{-2}
81	Permeability of free space	$\frac{2\pi \times \text{force} \times \text{distance}}{\text{current}^2 \times \text{length}}$	$[ML\ T^{-2}A^{-2}]$	N A^{-2} or Wb A^{-1} m^{-1}
82	Coercivity	H (opposing)	$[M^0L^{-1}T^0A]$	A m^{-1} or N m^{-2} T^{-1}
83	Retentivity	I (residual)	$[M^0L^{-1}\ T^0\ A]$	A m^{-1} or N m^{-2} T^{-1}
84	Coefficient of self induction (L) or mutual induction (M)	$\frac{\text{emf} \times \text{time}}{\text{current}}$	$[ML^2T^{-2}A^{-2}]$	H (henry)
85	Capacitive reactance (X_C)	$\frac{1}{\omega C}$	$[ML^2T^{-3}A^{-2}]$	Ω

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S.No.	Physical quantity	Relation with other physical quantities	Dimensional formula	SI unit
86	Coefficient of self inductance (L) or mutual inductance (M)	$\frac{\text{magnetic flux}}{\text{current}}$	$[ML^2T^{-2}A^{-2}]$	H (henry)
87	Quality factor (Q)	$\frac{\omega_0 L}{R}$	$[M^0L^0T^0]$	No unit
88	Power factor ($\cos\phi$)	Trigonometric ratio	Dimensionless	No unit
89	Planck's constant	$\frac{\text{energy}}{\text{frequency}}$	$[ML^2T^{-1}]$	J s
90	Resonant angular frequency (ω_0)	$\frac{1}{\sqrt{LC}}$	$[M^0L^0T^{-1}]$	Hz
91	Refractive index	$\frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$	Dimensionless	No limit
92	Rydberg's constant	$\frac{\text{mass} \times \text{charge}^4}{8\varepsilon_0^2 \times h^3 \times \text{velocity}}$	$[M^0L^{-1}T^0]$	m^{-1}
93	Solar constant	$\frac{\text{energy emitted by the sun}}{\text{area} \times \text{time}}$	$[ML^0T^{-3}]$	$W\,m^{-2}$
94	Work function	energy	$[ML^2T^{-2}]$	J
95	Decay constant	$\frac{0.693}{\text{half-life}}$	$[M^0L^0T^{-1}]$	s^{-1}
96	Packing fraction	$\frac{\text{mass defect}}{\text{atomic number}}$	$[ML^0T^0]$	kg nucleon^{-1}

9. DIMENSIONAL FORMULA AND DIMENSIONAL EQUATION

We have already learned that all physical quantities can be expressed in terms of seven fundamental quantities. The expression which represents physical quantity in terms of fundamental quantities in symbolic form is called dimensional formula of that physical quantity. We know that fundamental quantities are represented using standard symbols and dimensional formula is written inside square brackets. For example, dimensional formula of area is $[M^0L^2T^0]$, dimensional formula for acceleration is $[M^0LT^{-2}]$, dimensional formula for force is $[MLT^{-2}]$, and dimensional formula for the energy is $[ML^2T^{-2}]$ and so on.

We can also write dimensional equations by equating physical quantity with its dimensional formula. And these dimensional equations can be obtained from equation which represents relationship between physical quantity and other physical quantities. We can write dimensional equation for above described physical quantities as follows:

$$[A] = [M^0L^2T^0]$$
$$[a] = [M^0LT^{-2}]$$
$$[F] = [MLT^{-2}]$$
$$[E] = [ML^2T^{-2}]$$

There is a wide variety of physical quantities which are used in physics. You may already be aware of some of them but there are many physical quantities which are to be introduced in their corresponding chapters. Hence, at this stage you may not be able to determine the dimensional formulae for all quantities but at a later stage when you

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will learn about these physical quantities then you can easily determine dimensional formula for that quantity with the concepts learned in this chapter.

Dimensional analysis can be helpful in many ways. And for illustration purpose we shall refer some formulae of later chapters from Table 6. But still you can understand the concepts that are given here.

From the analysis of dimensional formulae of physical quantities in given table, these physical quantities can be categorised as:

Dimensional constants: These are the physical quantities which have constant value and have dimensions. Speed of light in vacuum, Planck's constant and universal gravitational constant are some dimensional constants.

Dimensionless constants: These are the physical quantities which have constant values but are dimensionless. Mathematical constants like π, 1, 2, etc. are all dimensionless constants.

Dimensional variables: These are the physical quantities which have variable values and have dimensions. Physical quantities like velocity, acceleration, force, etc. are dimensional variables.

Dimensionless variables: These are the physical quantities which have variable values but are dimensionless. Physical quantities like angle, strain, etc. are dimensionless variables.

SOLVED EXAMPLES

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Example 1. *Name the physical quantities which have dimensional formulae as given below:*

(i) ML^2T^{-2} (iii) ML^2

(ii) ML^2T^{-3} (iv) MLT^{-1}

Solution:

(i) $ML^2T^{-2} = MLT^{-2}\,L$

$= \text{Force} \times \text{distance}$

$= \textbf{Work}$

(ii) $ML^2T^{-3} = ML^2T^{-2}\,T^{-1}$

$= \dfrac{\text{Energy}}{\text{Time}}$

$= \textbf{Power}$

(iii) $ML^2 = \text{Mass} \times (\text{Distance})^2$

$= \textbf{Moment of Inertia}$

(iv) $MLT^{-1} = M\,LT^{-1}$

$= \text{Mass} \times \text{Velocity}$

$= \textbf{Linear momentum or Impulse}$

Example 2. *Deduce the dimensional formula for the following physical quantities:*

(i) *Young's modulus*

(ii) *Co-efficient of viscosity*

(iii) *Surface Tension*

(iv) *Bulk modulus*

(v) *Force constant*

Solution:

(i) Young's modulus $= \dfrac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

$= \dfrac{F/A}{\Delta l / l} = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$

$\therefore \quad [Y] = \dfrac{[MLT^{-2}]\,[L]}{[L^2]\cdot[L]} = \textbf{[ML}^{-1}\textbf{T}^{-2}\textbf{]}$

(ii) Coefficient of viscosity,

$\eta = \dfrac{\text{Force}}{\text{Area} \times \text{Velocity gradient}}$

$= \dfrac{\text{Force}}{\text{Area}} \times \dfrac{\text{Distance}}{\text{Velocity}}$

$\therefore \quad [\eta] = \dfrac{[MLT^{-2}]\cdot[L]}{[L^2]\cdot[LT^{-1}]} = \textbf{[ML}^{-1}\textbf{T}^{-1}\textbf{]}$

(iii) Surface tension, $\sigma = \dfrac{\text{Force}}{\text{Length}}$

$\therefore \quad [\sigma] = \dfrac{[MLT^{-2}]}{[L]} = \textbf{[ML}^{0}\textbf{T}^{-2}\textbf{]}$

(iv) Bulk modulus $= \dfrac{\text{Pressure applied}}{\text{Relative decrease in volume}}$

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$$= \frac{P}{\Delta V/V} = \frac{P}{\Delta V} \cdot V$$

$$= \frac{\text{Force}}{\text{Area}} \cdot \frac{V}{\Delta V} = \frac{[MLT^{-2}] \cdot [L^3]}{[L^2] \cdot [L^3]}$$

$$= \mathbf{[ML^{-1}T^{-2}]}$$

(*v*) $\text{Force constant} = \dfrac{\text{Force}}{\text{Displacement}}$

$$= \frac{[MLT^{-2}]}{[L]} = \mathbf{[ML^0T^{-2}]}$$

Example 3.

Deduce the dimensional formulae for the following physical quantities:

(*i*) *Specific heat*

(*ii*) *Entropy*

(*iii*) *thermal conductivity*

(*iv*) *Stefan's constant*

(*v*) *Boltzmann's constant*

(*vi*) *Universal gas constant*

Solution: (*i*)

$$\text{Specific heat} = \frac{\text{Heat}}{\text{Mass} \times \text{Temperature}}$$

$$= \frac{\text{Work}}{\text{Mass} \times \text{Temperature}}$$

$$= \frac{\text{Force} \times \text{distance}}{\text{Mass} \times \text{Temperature}} = \frac{[MLT^{-2}][L]}{[M][K]}$$

$$= \mathbf{[M^0L^2T^{-2}K^{-1}]}$$

(*ii*) $\text{Entropy} = \dfrac{\text{Heat}}{\text{Temperature}} = \dfrac{[ML^2T^{-2}]}{[K]}$

$$= \mathbf{[ML^2T^{-2}K^{-1}]}$$

(*iii*) Thermal conductivity

$$= \frac{\text{Heat} \times \text{Distance}}{\text{Area} \times \text{Temperature} \times \text{Time}}$$

$$= \frac{[ML^2T^{-2}] \cdot [L]}{[L^2] \cdot [K] \cdot [T]}$$

$$= \mathbf{[MLT^{-3}K^{-1}]}$$

(*iv*) $\text{Stefan's constant} = \dfrac{\text{Energy}}{\text{Area} \times \text{time} \times (\text{temperature})^4}$

$$= \frac{[ML^2T^{-2}]}{[L^2] \cdot [T] \cdot [K]^4}$$

$$= \mathbf{[ML^0T^{-3}K^{-4}]}$$

(*v*) $\text{Boltzmann's constant} = \dfrac{\text{Energy}}{\text{Temperature}}$

$$= \frac{[ML^2T^{-2}]}{[K]}$$

$$= \mathbf{[ML^2T^{-2}K^{-1}]}$$

(*vi*) Universal gas constant

$$= \frac{\text{Pressure} \times \text{Volume}}{\text{No. of moles} \times \text{Temperature}}$$

$$= \frac{PV}{nT}$$

$$= \frac{[ML^{-1}T^{-2}][L^3]}{[\text{mol.}] \cdot [K]}$$

$$= \mathbf{[ML^2T^{-2}K^{-1}\ mol^{-1}]}$$

Example 4.

By the use of dimensions, show that angular momentum has the same physical units as the Planck's constant.

Solution: Angular momentum, $L = mvr$

$$\therefore \quad [L] = [M]\ [LT^{-1}]\ [L]$$

$$= [ML^2T^{-1}]$$

Now, Planck's constant, $h = \dfrac{E}{\nu}$

$$\therefore \quad [h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = \mathbf{[ML^2T^{-1}]}$$

Hence proved.

Example 5.

Taking work, charge and frequency as the fundamental quantities, find the dimensions of resistance.

Solution: Let dimensions of charge, work and frequency be q, W and ν, respectively.

$$\text{Resistance, } R = \frac{\text{Potential difference}}{\text{Current}}$$

$$= \frac{\dfrac{\text{Work}}{\text{Charge}}}{\dfrac{\text{Charge}}{\text{Time}}} = \frac{\text{Work} \times \text{Time}}{[\text{Charge}]^2}$$

$$= \frac{\text{Work}}{(\text{Charge})^2 \times \text{Frequency}}$$

$$\therefore \quad [R] = \mathbf{[W\nu^{-1}q^{-2}]}$$

Example 6.

Taking velocity of light, density and frequency as fundamental quantities, find the dimensions of surface tension.

Solution: $c = LT^{-1}$, $\rho = ML^{-3}$ and $\nu = T^{-1}$

Solving M, L and T terms of c, ρ, and ν, we get

$$T = \nu^{-1},\ L = c\nu^{-1} \text{ and } M = \rho c^3 \nu^{-3}$$

$$\text{Surface Tension} = \frac{\text{Force}}{\text{Length}} = \frac{[MLT^{-2}]}{[L]}$$

$$= MT^{-2} = [\rho c^3 \nu^{-3}][\nu^{+2}]$$

$$= \mathbf{[\rho c^3 \nu^{-1}]}$$

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10. DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

We have several physical quantities in physics and these quantities are interrelated with each other. There are some physical quantities which do not have any unit of measurement and such quantities are called **dimensionless quantities**, and for the rest, there is always some unit of measurement. Unit of measurement for a physical quantity may not be same as one of the units of fundamental quantities. But many times units of more than one fundamental quantities are required to represent the unit for a given physical quantity. Such types of units are called **derived units**. We know that any derived unit can be represented in terms of units corresponding to the fundamental quantities. When we write derived unit of a physical quantity in terms of units of fundamental quantities, then in symbolic form we can write the dimensional formula for the same physical quantity. Physical quantities having same unit of measurement will also have same dimensional formulae. We know that physical quantities having same units can be added or subtracted and hence we can say that physical quantities having same dimensions can be added or subtracted. Division and multiplication is possible between physical quantities having different units of measurement. In such a case of multiplication and division, units are treated like ordinary algebraic symbols. Identical units in numerator and denominator can be cancelled and similarly we can treat the dimensions of a physical quantity. **Dimensional analysis can be helpful in checking derivations, dimensional consistency or homogeneity of mathematical expressions representing relationships among physical quantities.**

10.1. Dimensional Consistency of Equations

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We know that physical quantities having same dimensions can only be added or subtracted from each other or we can say that only similar quantities can be added or subtracted from each other. For example, we cannot add or subtract length with area. And similarly we cannot add or subtract force with velocity or volume with temperature. This simple concept is called **principle of homogeneity of dimensions in an equation**. One can check the correctness of an equation with the help of principle of homogeneity of dimensions. If dimensions of all the terms in an equation are not same, the equation is certainly wrong. For example, suppose we have written force in terms of several other physical quantities. In such a case when individual dimensions on the other side of expression are simplified following algebraic rules then final remaining dimension must be that of the force. If some equation is describing speed in terms of some other physical quantities then dimensions on both the sides of the equation must be $[LT^{-1}]$. Let us understand this concept with the help of some example.

In circular motion we shall define centripetal force (F) in terms of mass (m), speed (v) and radius of the circle (r) as follows:

$$F = \frac{mv^2}{r}$$

Here for the dimensional consistency dimensions of right-hand side must be same as that of the force.

Dimensional formula of force can be written as:

$$[F] = [MLT^{-2}]$$

Dimensional formula for the right-hand side term can be determined as follows:

$$\left[\frac{mv^2}{r}\right] = \frac{[M][LT^{-1}]^2}{[L]} = \frac{[ML^2T^{-2}]}{[L]} = [MLT^{-2}]$$

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We can see that dimensions of right side of the above equation is same as that for the left side which is basically dimensions of force; hence above written equation for centripetal force is dimensionally correct.

If an equation is written as addition of subtraction of more than one terms then dimensions of each term must be same on both the sides of the equation.

Let us take an example of equation describing displacement (S) for a particle moving with constant acceleration (a), in terms of its initial velocity (u) and time (t). Equation is written as follows:

$$S = ut + \tfrac{1}{2}at^2$$

For dimensional consistency all the terms must have same dimensions on both the sides of the equation. Dimensions of each term in above equation can be written as follows:

$$[S] = [\mathrm{L}]$$
$$[ut] = [\mathrm{LT}^{-1}] \times [\mathrm{T}] = [\mathrm{L}]$$
$$\left[\frac{1}{2}at^2\right] = [\mathrm{LT}^{-2}] \times [\mathrm{T}^2] = [\mathrm{L}]$$

We can see that all the terms of above specified equation have same dimensions and hence the equation is dimensionally correct.

Checking dimensional consistency is just one preliminary test for the correctness of equation. If a given equation is dimensionally inconsistent then it is for sure that given equation is wrong, but we should be aware of the fact that if equation is dimensionally consistent then it is not a guarantee that equation is correct. We should be aware of the limitations behind this method.

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Let us take an example of above specified equation of displacement: $S = ut + \tfrac{1}{2}at^2$. If we look at the equation from dimensional consistency then $S = ut$ is also correct, $S = \tfrac{1}{2}at^2$ is also correct and $S = ut + \tfrac{1}{2}at^2$ is correct too. But we know that only relation $S = ut + \tfrac{1}{2}at^2$ is correct.

One more point here should be noted that constants multiplied in the equation are dimensionless and hence we cannot say anything about the constants (if any) used in the equation. For example, dimensionally $F = \dfrac{mv^2}{r}$ is correct but $F = \dfrac{7mv^2}{r}$ is also correct because nothing can be said about the presence of constants in the equation. So we should always remember that if a certain equation fails the test of dimensional consistency, then we can always say that given equation must be wrong but reverse is not true. If some equation passes the test of dimensional consistency then it is not a guarantee for the equation to be correct. ***Dimensionally wrong equation is certainly wrong, but dimensionally correct equation may not be the correct equation.***

FORMULAE USED

Principle of homogenity of dimensions is used to check the correctness of equations. This principle is obeyed if

Dimensions of M, L, T on L.H.S. = Dimensions of M, L, T on R.H.S.

SOLVED EXAMPLES

❑ **Example 1.**

Check the dimensional consistency of the relations:

(i) $S = ut + \dfrac{1}{2}at^2$

(ii) $\dfrac{1}{2}mv^2 = mgh$

Solution: (*i*) [S] = [L]

[ut] = [LT^{-1}] [T] = [L]

$\left[\dfrac{1}{2}at^2\right]$ = [LT^{-2}][T^2] = [L]

Dimensions of all the terms in the given equation are same. So, the given equation is dimensionally correct.

(*ii*) $\left[\frac{1}{2}mv^2\right] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$

$[mgh] = [M][LT^{-2}][L] = [ML^2T^{-2}]$

The equation is dimensionally correct.

Example 2.

The Poiseuille's equation:

$$V = \frac{\pi}{8}\frac{Pr^4}{\eta l}$$

gives the rate of flow (V) of a liquid flowing through a pipe of radius r and pressure gradient (P/l).

Calculate the dimensions of coefficient of viscosity η.

Solution: Here,

$$V = \text{Rate of flow of liquid} = \frac{\text{Volume}}{\text{Time}}$$

$$\therefore \quad [V] = \frac{[L^3]}{[T]} = [L^3T^{-1}]$$

$$\frac{P}{l} = \text{Pressure gradient} = \frac{\text{Pressure}}{\text{Length}}$$

$$\therefore \quad \left[\frac{P}{l}\right] = \frac{[ML^{-1}T^{-2}]}{[L]} = [ML^{-2}T^{-2}]$$

r = Radius

$$\therefore \quad [r] = [L]$$

Now, $$[\eta] = \left[\frac{\pi Pr^4}{8lV}\right]$$

$$= \frac{[ML^{-2}T^{-2}][L^4]}{[L^3T^{-1}]}$$

$$= \mathbf{[ML^{-1}T^{-1}]}$$

Example 3.

Check the dimensional correctness of the following equations:

(*i*) $T = K\sqrt{\frac{\rho r^3}{S}}$ *where* ρ *is the density, r is the radius and S is the surface tension and K is a dimensionless constant and T is the time period of oscillation.*

(*ii*) $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$, *when n is the frequency of vibration, l is the length of the string, T is the tension in the string and m is the mass per unit length.*

(*iii*) $d = \frac{mgl^3}{4bd^3Y}$, *where d is the depression produced in the bar, m is the mass of the bar, g is the acceleration due to gravity, l is the length of the bar, b is its breadth and d is its depth and Y is the Young's modulus of the material of the bar.*

Solution:

(*i*) [LHS] = [T] = [T]

$$[RHS] = \left[K\sqrt{\frac{\rho r^3}{S}}\right] = \left[\frac{[ML^{-3}][L^3]}{[MT^{-2}]}\right]^{\frac{1}{2}}$$

$$= [T^2]^{1/2} = [T] = [LHS]$$

The relation is dimensionally correct.

(*ii*) [LHS] = [n] = [T^{-1}]

$$[RHS] = \left[\frac{1}{2l}\sqrt{\frac{T}{m}}\right] = \frac{1}{[L]}\left[\frac{MLT^{-2}}{ML^{-1}}\right]^{\frac{1}{2}}$$

$$= \frac{1}{[L]}\cdot[LT^{-1}] = [T^{-1}] = [LHS]$$

The relation is dimensionally correct.

(*iii*) [LHS] = [d] = [L]

$$[RHS] = \left[\frac{mg\,l^3}{4bd^3Y}\right]$$

$$= \frac{[M][LT^{-2}][L]^3}{[L][L]^3[ML^{-1}T^{-2}]} = [L] = [LHS]$$

The equation is dimensionally correct.

Example 4.

The equation given below is the Vander Wall's equation for a gas

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where P is the pressure, V is the volume, R is the universal gas constant and T is the temperature. Find the dimensions of a/b.

Solution: $$[P] = \left(\frac{a}{V^2}\right)$$

(only quantities of similar dimension can be added or subtracted)

$$\therefore \quad [a] = [PV^2]$$

$$= [ML^{-1}T^{-2}]\ [L^3]^2$$

$$= [ML^5T^{-2}]$$

Similar, $$[b] = [V] = [L^3]$$

$$\therefore \quad \left[\frac{a}{b}\right] = \left[\frac{ML^5T^{-2}}{L^3}\right] = \mathbf{[ML^2T^{-2}]}$$

Example 5.

Rule out or accept the following formula for displacement y of particle undergoing periodic motion on the basis of dimensional arguments:

(*i*) *a* sin 2π*t*/*T*

(*ii*) *a* cos *vt*

(*iii*) *a* sin (ω*t* – *kx*)

where a = maximum displacement of the particle

T = time period of motion

t = time interval

v = speed of particle

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ω = *Angular speed of particle*

k = *displacement constant*

Solution: (*i*) $y = a \sin 2\pi t/T$

[LHS] = [y] = [L]

On [RHS], [a] = [L]

Also, for dimensional correctness, argument of sine function should be dimensionless.

$$\left[\frac{2\pi t}{T}\right] = \frac{[T]}{[T]} = [M^0L^0T^0]$$

$\therefore$ [RHS] = [a] = [L] = [LHS]

The formula is dimensionally acceptable for displacement.

(*ii*) $y = a \cos vt$

[LHS] = [y] = [L]

[RHS] = [a] = [L]

[vt] = [LT^{-1}] [T] = [L]

In this case, argument of the cosine function is not dimensionless and hence the formula is not acceptable.

(*iii*) $a \sin(\omega t - kx)$

[ωt] = [T^{-1}] [T] = [$M^0L^0T^0$]

[kx] = [L^{-1}] [L] = [$M^0L^0T^0$]

The argument of sine function is dimensionless. Also,

[RHS] = [a] = [L] = [LHS]

The formula is dimensionally acceptable.

Example 6.

Find the dimensions of $a \times b$ in the given equation:

$$P = \frac{a}{bx} - \frac{t^2}{b}$$

where P is the power, x is the distance and t is the time.

Solution: [P] = [$ML^2 T^{-3}$]

$$\left[\frac{t^2}{b}\right] = [P]$$

$$\Rightarrow \quad \frac{[T]^2}{[b]} = [ML^2 T^{-3}]$$

$$\Rightarrow \quad [b] = \frac{[T]^2}{[ML^2 T^{-3}]} = [M^{-1}L^{-2} T^5]$$

Also, $\left[\frac{a}{bx}\right] = [P]$

$$\Rightarrow \quad [a] = [P]\,[bx]$$

$$= [ML^2 T^{-3}]\,[M^{-1}L^{-2} T^5][L]$$

$$= [M^0 L^1T^2]$$

$$\therefore \quad [a \times b] = [M^0 L^1T^2]\,[M^{-1}L^{-2} T^{-5}]$$

$$= [M^{-1}L^{-1} T^{-3}]$$

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10.2. Deriving Relation Among Physical Quantities

In some situations we can take help of method of dimensions to formulate the relation among physical quantities. If we are aware that a particular physical quantity is dependent on some other physical quantities then we can follow the method of dimensional consistency to arrive at the relation. ***But number of physical quantities must not be greater than three.*** We need to take some example to understand this procedure.

In later chapters we shall learn that a net force named as centripetal force (F) is required to move a particle of mass m, with speed v, in a circle of radius r. We can easily guess that centripetal force required must be somehow dependent on mass, speed and radius of the circle. To start, let us assume the following equation:

$$\boldsymbol{F \propto m^a v^b r^c}$$

Here a, b and c are the exponents or powers raised on mass, velocity and radius, respectively to express the relation. Using the method of dimensional consistency we need to calculate a, b and c to finalise the relation. We can rewrite the above relation as follows:

$$F = k m^a v^b r^c$$

Here k is a dimensionless constant and limitation of this method is that we cannot say anything about k. Let us now write the above specified equation in terms of dimensions.

$$[M^1L^1T^{-2}] = [M^1]^a[L^1T^{-1}]^b[L^1]^c$$

$$\Rightarrow \quad [M^1L^1T^{-2}] = [M^aL^{b+c}T^{-b}]$$

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We can now equate dimensions on both the sides of the equation to get the following equations:

$$a = 1 \quad ...(1)$$

$$b + c = 1 \quad ...(2)$$

$$-b = -2 \Rightarrow b = 2 \quad ...(3)$$

Now using above equation (2) and (3), we get the following:

$$2 + c = 1 \Rightarrow c = -1$$

Hence we can write the relation of force as follows:

$$F = k\, m\, v^2\, r^{-1}$$

$$\Rightarrow \quad F = k\frac{mv^2}{r}$$

Here the value of k is 1, but proportionality constant like k, can never be determined using method of dimensions. After using the value of k we can write the above relation as follows:

$$\boxed{F = \frac{mv^2}{r}}$$

Let us take another simple example of time period of simple pendulum. Time period (T) of the simple pendulum is given to be dependent on mass (m) of the bob, length (l) of the thread and acceleration due to gravity (g).

Let us assume the following equation:

$$\boldsymbol{T = k\, m^a l^b g^c}$$

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Here k is proportionality constant, as used in the previous example and a, b and c are the powers raised on physical quantities on which it is dependent. Let us write this equation in terms of dimensions.

$$[M^0L^0T^1] = [M^1]^a\, [L^1]^b\, [L^1T^{-2}]^c$$

$$\Rightarrow \quad [M^0L^0T^1] = [M^aL^{b+c}\, T^{-2c}]$$

We can now equate the dimensions on both the sides of equation to get the following relations:

$$a = 0 \quad ...(1)$$

$$b + c = 0 \quad ...(2)$$

$$-2c = 1 \Rightarrow c = -1/2 \quad ...(3)$$

Substituting equation (3) in (2) we get

$$b + (-1/2) = 0 \Rightarrow b = 1/2.$$

Hence the relation of time period can be written as follows:

$$T = k\, l^{1/2} g^{-1/2}$$

$$\Rightarrow \quad \boxed{T = k\sqrt{\frac{l}{g}}}$$

Here also we cannot say anything about the value of k using method of dimensions. In this case value of k is 2π and after knowing its value we can write the relation of time period as follows:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Hence, method of dimensions can be helpful in formulating relationship among physical quantities but there are limitations and this method cannot be reliably used for this purpose. For example,

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dimensional method cannot distinguish between physical quantities having same dimensions. If we would have provided radius (r) of bob instead of length of thread of pendulum then we would have got r instead of l in the final result which would give an incorrect result. Apart from this we already know that this method cannot tell anything about the proportionality constant and moreover we cannot deal with more than three physical quantities in the perspective of mechanics. Because in mechanics all quantities can be expressed only in terms of M, L and T and hence we cannot formulate more than three equations. But still method of dimensions becomes useful in many situations.

SOLVED EXAMPLES

Example 1.

A particle of mass m is moving with a velocity v in a circle of radius r. Find an expression for the centripetal force F acting on it.

Take dimensionless constant K = 1

Solution: Let $F = km^a v^b r^c = m^a v^b r^c$ [k = 1]

Then

$$M^1L^1T^{-2} = [M]^a[LT^{-1}]^b[L]^c$$
$$= [M^aL^{b+c}T^{-b}]$$

Comparing the exponents of M, L and T on both sides, we get

$$a = 1, b + c = 1$$
$$b = 2$$
$$\Rightarrow \quad c = 1 - b = 1 - 2 = -1$$
$$\therefore \quad F = \frac{mv^2}{r}$$

This is the required expression.

Example 2.

Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using methods of dimensions. *[N.C.E.R.T. Solved Example]*

Solution: Given, $T \propto m^a l^b g^c$

or $$T = K\, m^a l^b g^c \qquad ...(i)$$

where K is a dimensionless constant = 2π (from experiments)

Taking dimensions on both sides of equation (i), we get

$$[M^0L^0T^1] = [M]^a[L]^b[LT^{-2}]^c$$
$$= M^aL^{b+c}T^{-2c}$$

Comparing the exponents of M, L and T on both sides, we get

$$a = 0, b + c = 0, -2c = 1 \text{ or } c = -1/2$$
$$b = -c = \frac{1}{2}$$
$$\therefore \quad T = Km^0 l^{1/2} g^{-1/2}$$
$$= K\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{g}} \qquad [K = 2\pi]$$

Example 3.

A planet revolves around the sun in a nearly circular orbit. Assuming that its time period of revolution 'T' depends upon radius 'r' of the orbit, mass 'M' of the sun and the gravitational constant 'G'. Show that square of 'T' varies with the cube of the radius r.

Solution: Let $T = Kr^aM^bG^c$ ...(i)

where K is a dimensionless constant

Taking dimensions on both sides of equation (i), we get

$$[M^0L^0T^1] = [L]^a[M]^b[M^{-1}L^3T^{-2}]^c$$

Comparing the exponents of M, L and T on both sides, we get

$$b - c = 0, a + 3c = 0, -2c = 1$$
$$\Rightarrow \quad c = \frac{-1}{2}$$
$$b = c = \frac{-1}{2}$$
$$a = -3c = -3 \times \left(\frac{-1}{2}\right) = \frac{3}{2}$$
$$\therefore \quad T = Kr^{3/2} M^{-1/2} G^{-1/2}$$
$$\text{or} \quad T^2 = \frac{K^2 r^3}{MG}$$
$$\Rightarrow \quad T^2 \propto r^3$$

Hence proved.

Example 4.

The surface tension 'S' of a liquid rising in a capillary tube is supposed to depend upon mass 'm' of the liquid, pressure 'p' of the liquid and radius 'r' of the capillary tube. Obtain an expression for 'S' using the method of dimensions.

Take dimensionless constant K = 1/2

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Solution: $S = Km^a p^b r^c$ ...(*i*)

Surface tension, $S = \dfrac{\text{Force}}{\text{Length}}$

$\therefore \quad [S] = \dfrac{[MLT^{-2}]}{[L]} = [MT^{-2}]$

Pressure, $p = \dfrac{\text{Force}}{\text{Area}}$

$\therefore \quad [p] = \dfrac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

$[r] = [L]$

Taking dimensions on both sides of equation (i), we get

$$[M^1L^0T^{-2}] = [M]^a[ML^{-1}T^{-2}]^b\,[L]^c$$
$$= [M]^{a+b}\,[L]^{-b+c}\,[T]^{-2b}$$

Comparing the exponents of M, L and T on both sides, we get

$$a + b = 1 \quad ...(ii)$$
$$-b + c = 0 \quad ...(iii)$$
$$-2b = -2 \quad ...(iv)$$

From (*iv*) $\quad b = 1$

From (*ii*) $\quad a = 1 - b = 1 - 1 = 0$

From (*iii*) $\quad b = c \Rightarrow c = 1$

$\therefore \quad S = Km^0p^1r^1$

$$= \frac{1}{2}pr$$

10.3. Conversion of Unit from One System to Other

While converting value of a certain physical quantity from one system to other, we must be aware of the fact that values in different system of units are representing same amount of given physical entity. Mathematically we can write it as follows:

$$Q = n_1u_1 = n_2u_2$$

Here u_1 and u_2 represent two different systems of units and n_1 and n_2 are corresponding numbers to represent given amount of physical quantity. One simple example is 1 m = 100 cm, or 1 kg = 1000 g.

Suppose n_1 is given for system of unit u_1 and we have to calculate n_2 for another system of unit u_2. We observe the following method:

Here, u_1 and u_2 are written in terms of dimensions according to the given physical entity. To differentiate between two different systems of units we can use M_1, L_1, and T_1 to represent fundamental units for system 1 and M_2, L_2, and T_2 for system 2. We can then proceed for the calculation of n_2 as follows:

$$n_2 = n_1\frac{u_1}{u_2}.$$

M, L and T used in dimensional formula represent fundamental units for mass, length and time respectively according to the system of unit. For example, in case of MKS system, M represents 1 kg, L represents 1 m and T represents 1 s. Similarly, for CGS system, M represents 1 g, L represents 1 cm and T represents 1 s.

Let us learn this concept of conversion of units by taking an example of energy. In previous classes you must have learned that joule is MKS unit of energy and ergs is CGS unit of energy. We want to convert 1 joule into equivalent value in ergs. We need dimensional formula of energy for this conversion and earlier we have already evaluated it as $[ML^2T^{-2}]$.

Let u_1 represent MKS system and u_2 represent CGS; hence we shall use subscript 1 to represent fundamental units of MKS system and 2 for the CGS system.

$$n_1u_1 = n_2u_2$$

We want to calculate value of 1 joule in ergs and hence $n_1 = 1$ and n_2 is to be calculated. We can write the above equation as follows:

$$\Rightarrow 1[M_1L_1^2T_1^{-2}] = n_2[M_2L_2^2T_2^{-2}]$$

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$$\Rightarrow \quad n_2 = \left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2}$$

$$\Rightarrow \quad n_2 = \left[\frac{1\,kg}{1\,g}\right]\left[\frac{1\,m}{1\,cm}\right]^2\left[\frac{1s}{1s}\right]^{-2}$$

$$\Rightarrow \quad n_2 = \left[\frac{1000\,g}{1\,g}\right]\left[\frac{100\,cm}{1\,cm}\right]^2\left[\frac{1s}{1s}\right]^{-2}$$

$$\Rightarrow \quad n_2 = (1000)\times(100)^2 = 10^7$$

Hence, **1 joule = 10^7 ergs**

Let us take another example of force. We know that in MKS system force is measured in Newton and in CGS system force is measured in dynes. We want to calculate how many dynes are there in 1 newton. We need dimensional formula of force for this conversion. Dimensional formula of force is $[MLT^{-2}]$. Let us again assume that u_1 represents MKS and u_2 represents CGS. We can proceed with the following equation:

$$n_1u_1 = n_2u_2$$

We have to calculate 1 Newton in Dynes hence, $n_1 = 1$ and n_2 is to be calculated. We can write the above equation as follows:

$$1[M_1L_1T_1^{-2}] = n_2[M_2L_2T_2^{-2}]$$

$$\Rightarrow \quad n_2 = \left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]\left[\frac{T_1}{T_2}\right]^{-2}$$

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$$\Rightarrow \quad n_2 = \left[\frac{1\,kg}{1\,g}\right]\left[\frac{1\,m}{1\,cm}\right]\left[\frac{1s}{1s}\right]^{-2}$$

$$\Rightarrow \quad n_2 = \left[\frac{1000\,g}{1\,g}\right]\left[\frac{100\,cm}{1\,cm}\right]\left[\frac{1s}{1s}\right]^{-2}$$

$$\Rightarrow \quad n_2 = (1000)\times(100) = 10^5$$

Hence, **1 newton = 10^5 dynes.**

For the conversion of units from one system to other we need dimensional formula of the corresponding physical quantity. Fundamental units of one system are related to the fundamental units of other system and we need to know the conversion factors. For example, 1 kg is same as 1000 g, or 1 m is same as 100 cm.

FORMULAE USED

For conversion of one system of units into another we use

$$n_2 = \frac{n_1u_1}{u_2}$$

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

Here u_1 and u_2 represent two different system of units.

n_1 is the number for system of unit u_1.

n_2 is the number for system of unit u_2.

SOLVED EXAMPLES

Example 1.

The value of universal gravitational constant is $6.6 \times 10^{-11}\ Nm^2\ kg^{-2}$ in SI units. Find its value in the CGS system.

Solution: We know, $F = G\dfrac{m_1m_2}{r^2}$

where F = gravitational force

m_1, m_2 = mass of two bodies exerting gravitational force on each other

r = distance between the bodies

$$\therefore \quad G = \frac{Fr^2}{m_1m_2}$$

$$\Rightarrow \quad [G] = \frac{[MLT^{-2}][L^2]}{[M]^2} = [M^{-1}L^3T^{-2}]$$

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Comparing with $[M^aL^bT^c]$, we have

$a = -1, b = 3$ and $c = -2$

SI units	CGS units
$n_1 = 6.6 \times 10^{-11}$	$n_2 = ?$
$M_1 = 1$ kg	$M_2 = 1$ g $= 10^{-3}$ kg
$L_1 = 1$ m	$L_2 = 1$ cm $= 10^{-2}$ m
$T_1 = 1$ s	$T_2 = 1$ s

$$\therefore \quad n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 6.6 \times 10^{-11}[10^3]^{-1}[10^2]^3[1]^{-2}$$

$$= 6.6 \times 10^{-11} \times 10^{-3} \times 10^6$$

$$= \mathbf{6.6 \times 10^{-8}\ dyne\ cm^2\ g^{-2}}$$

Example 2.

Express the value of following quantities in their SI units.

(i) Power of electric lamp = 2×10^9 erg s^{-1}

(ii) Surface tension of water = 72 dyne cm^{-1}

(iii) Stefan's constant = 5.67×10^{-5} erg s^{-1} cm^{-2} K^{-4}

Solution: *(i)* $\text{Power} = \dfrac{\text{Energy}}{\text{Time}}$

$$= \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$$

$\therefore \quad a = 1, b = 2, c = -3$

CGS units	SI units
$n_1 = 2 \times 10^9$	$n_2 = ?$
$M_1 = 1$ g	$M_2 = 1$ kg $= 1000$ g
$L_1 = 1$ cm	$L_2 = 1$ m $= 100$ cm
$T_1 = 1$ s	$T_2 = 1$ s

$$\therefore \quad n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 2 \times 10^9\left[\frac{1}{1000}\right]^1\left[\frac{1}{100}\right]^2\left[\frac{1}{1}\right]^{-3}$$

$$= 2 \times 10^9 \times 10^{-3} \times 10^{-4}$$

$$= 200$$

$\therefore$ Power in SI units = **200 W**

(ii) $\text{Surface Tension} = \dfrac{\text{Force}}{\text{Length}}$

$$= \frac{MLT^{-2}}{L} = MT^{-2}$$

$\therefore \quad a = 1, b = 0, c = -2$

CGS units	SI units
$n_1 = 72$	$n_2 = ?$
$M_1 = 1$ g	$M_2 = 1000$ g
$L_1 = 1$ cm	$L_2 = 100$ cm
$T_1 = 1$s	$T_2 = 1$s

$$\therefore n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 72\left[\frac{1}{1000}\right]^1\left[\frac{1}{100}\right]^0\left[\frac{1}{1}\right]^{-2}$$

$$= 72 \times 10^{-3} = 0.072$$

$\therefore$ Surface tension = **0.072 N m^{-1}**

(iii) Dimensional formula of Stefan's constant

$= ML^0T^{-3}K^{-4}$ $a = 1, b = 0, c = -3$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$\Rightarrow n_2 = 5.67 \times 10^{-5}\left[\frac{1\,g}{1\,kg}\right]^1\left[\frac{1\,cm}{100\,cm}\right]^0\left[\frac{1s}{1s}\right]^{-3} K^{-4}$$

$$\Rightarrow n_2 = 5.67 \times 10^{-5}\left[\frac{1\,g}{1000\,g}\right]^1[1][1]K^{-4}$$

$$\Rightarrow n_2 = 5.67 \times 10^{-8}$$

$$\Rightarrow \sigma = \mathbf{5.67 \times 10^{-8}\ Js^{-1}\ m^{-2}\ K^{-4}}$$

Example 3.

If the units of force, energy and time are 20 N, 100 J and 10 s, find the units of mass and length.

Solution: $[F] = [MLT^{-2}] = 20$ N ...(i)

$[E] = [ML^2T^{-2}] = 100$ J ...(ii)

$[T] = 10$ s ...(iii)

Dividing *(ii)* by *(i)*, $[L] = \dfrac{100}{20} = 5$ m

Also, $M = \dfrac{[F]}{[L][T^{-2}]} = \dfrac{20 \times (10)^2}{5}$

$= \mathbf{400\ kg}$

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PRACTICE PROBLEMS 3

1. How will you convert S.I. unit of energy (Joule) to C.G.S. unit of energy (erg)?
2. A pebble is dropped into water by a boy. He observes that ripples are formed in water. Velocity of ripples formed depends on the density of water (ρ), wavelength (λ) and surface tension $\left(T = \frac{F}{l}\right)$. Derive the formula for velocity of ripples.
3. Find the units of mass and length if the unit of force is Mega Newton, unit of power is watt and unit of time is 1 microsecond.
4. The depth to which a bullet can penetrate inside a human body d which depends on the coefficient of elasticity $\left(\eta = \frac{F}{A}\right)$ and kinetic energy (E). Derive the relation between d, η and E.
5. Calculate the value of 1 Joule/sec. in a system having 10 g, 10 cm and 1 min. as fundamental units.
6. Assume that the mass of a stone moved by a river depends on the velocity, density and acceleration due to gravity. Derive the dependence of mass on these factors.
7. Assume that the frequency of a oscillating drop depends on the factors: density (ρ), radius of drop (r) and surface tension (T) of liquid. Derive the relation between these quantities.
8. Check the accuracy of relation

$$p = \rho v \nu^{-3}$$

p is linear momentum, ρ is density, v is velocity, ν is frequency.

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9. Check the accuracy of the relation

$$\eta = \frac{\pi}{8}\frac{Pr^4}{lV}$$

Here, P is pressure, V = rate of flow of liquid through a pipe, η is coefficient of viscosity of liquid.

10. Find the dimensions of resistance in terms of mass, length, time and current.
11. Check whether the following formula is correct or incorrect.

$$v = \frac{1}{\lambda^3}\sqrt{\frac{K}{d^2}}$$

λ = wavelength, K is coefficient of volume elasticity, d is density of medium, v is the velocity of longitudinal wave.

12. The critical velocity (v) of flow of liquid through a pipe depends on the coefficient of viscosity of liquid (η), density (ρ) and radius of pipe (r).

Derive the relation between these physical quantities.

13. Find the dimensions of I, if the rotational kinetic energy is given by $\frac{1}{2} I\omega^2$, where ω is angular velocity.
14. If velocity (V), force (F) and time (T) are taken as fundamental quantities, find the dimensions of mass in terms of these quantities.
15. Check by the method of dimensions whether the given equation is correct or incorrect:

$$\nu = \frac{1}{2\pi}\sqrt{\frac{mgl}{I}}$$

where ν is the frequency, m is the mass, l is the length and I is the moment of inertia.

Answers to Practice Problems

1. $1\ J = 10^7$ ergs
2. $V = \sqrt{\frac{T}{\lambda\rho}}$
3. $M = 10^6$ Kg; $L = 10^{-12}$ m
4. $d \propto \eta^{-1/3}\ E^{\frac{1}{3}}$ or $\left(\frac{E}{\eta}\right)^{\frac{1}{3}}$
5. 216×10^6
6. $m \propto v^6 \rho g^{-3}$
7. Frequency $= \sqrt{\frac{T}{\rho r^3}}$
8. The given equation is incorrect.
9. The relation is correct.
10. $[ML^2T^{-3}I^{-2}]$
11. Incorrect
12. $v = \frac{\eta}{\rho r}$
13. $I = [ML^2]$
14. $[F\ T\ V^{-1}]$
15. The given relation is correct.

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16. The distance covered by a particle in time t is given by

$$x = at + bt^2 + ct^3 + dt^4$$

Calculate the dimensions of $\frac{a}{b}$ and $\frac{c}{d}$.

17. Find the value of p in the relation

$$\tau = \frac{YL^p}{\cos\theta}$$

where Y is the Young's modulus, τ is torque and L is the length.

18. Find the dimensions of V in the equation

$$y = A\sin\omega\left(\frac{x}{V} - k\right)$$

19. Calculate the dimensions of x in the relation

$$x = \frac{IFV^2}{Wl^3}$$

where W is work, l is length, F is force, V is velocity and I is moment of inertia.

20. Show that $\varepsilon_0 E^2$ has dimensions of pressure.

Answers to Practice Problems

16. [T], [T]
17. $p = 3$
18. $[V] = [LT^{-1}]$
19. $[MT^{-2}]$

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Chapter Summary

Key Terms & Laws

- **Unit:** An internationally accepted reference standard of a quantity, which is of the same nature as the physical quantity being measured is known as unit.
- **Fundamental Units:** Seven quantities, which are independent from each other, are required to express all the other physical quantities. These are known as fundamental or base quantities and they are defined and expressed in terms of a certain basic reference standard, known as fundamental or base units.
- **Derived units:** Quantities which can be expressed in terms of fundamental quantities are known as derived quantities and their units can be expressed as a combination of fundamental units. Such units obtained for the derived quantities are known as derived units.
- **System of Units:** A complete set of both the fundamental and derived units is known as system of units.
- **SI system of units:** An internationally accepted and universally used system of units, which is rational, coherent, consistent and the most convenient system for scientific, technical, industrial and commercial work, in all branches of science.
- **Accuracy:** Accuracy is the measure of degree or extent to which a measurement is close to the true value of the measured quantity.
- **Precision:** Precision is the measure of limit or resolution to which a physical quantity is being measured by a measuring device.
- **Error:** The uncertainty present in every measurement by any measuring instrument is known as error.
- **Least Count:** It is the value of smallest division on the scale of a measuring device that can be measured by that device.

FACTS

- Length, Mass, Time, Electric Current, Temperature, Amount of Substance and Luminous Intensity are fundamental quantities, having metre, kilogram, second, ampere, Kelvin, mole and candela as their respective fundamental units.
- SI system consists of seven base units, which are well defined on international standards. These are metre (m), kilogram (kg), second (s), Ampere (A), Kelvin (K), mole (mol) and Candela (Cd).
- Apart from base units, certain derived SI units are expressed by SI units with special names. E.g. pascal (unit of pressure), newton (unit of force), farad (unit of capacitance), watt (unit of power), etc.
- For very small and large quantities, physical measurements are usually expressed in scientific notation, having power of 10. Scientific notations and prefixes simplify measurement notation and numerical computation, which gives clear indication of the precision of the measured values.
- The size of objects we came across in universe vary over a wide range from 10^{-15} m (size of a neutron) to 10^{26} m (size of observable universe).
- Like sizes of objects, masses also vary over a wide range from 10^{-30} kg (mass of an electron) to 10^{55} kg (mass of known universe).
- Every measurement of physical quantities is approximate due to experimental uncertainties or errors in measurement. Basically, errors can be classified as **Systematic errors** (errors that tend to be in one direction) and **Random errors** (errors that occur irregularly).
- For proper recording and computation of data, accuracy of measurement and precision of measuring instrument, along with errors involved in measurements must be taken into account before expressing the final result for the measured value.
- To describe the nature of physical quantities, dimensions of base quantities and their combinations are used. With the help of dimensional analysis, one can check the dimensional consistency of equations, derive relations among the physical quantities etc.

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RELATIONS

- **Special units of length:**
  1. 1 fermi = 1 f = 1×10^{-15} m
  2. 1 Angstrom = 1 Å = 1×10^{-10} m
  3. 1 astronomical unit = 1 AU = 1.496×10^{11} m. It is the average distance of the sun from the earth.
  4. 1 light year = 1 ly = 9.46×10^{15} m. It is the distance travelled by light in one year, with a velocity of 3×10^8 m/s.
  5. 1 parsec = 3.08×10^{16} m. It is the distance at which average radius of earth's orbit subtends an angle of one arc second.
- 1 unified atomic mass unit = 1 u = (1/12) of the mass of an atom of carbon-12 isotope including the mass of electrons = 1.66×10^{-27} kg.
- Errors in Measurement: If a physical quantity is measured N times and measured values obtained are a_1, a_2, a_3, ……, a_N, then arithmetic mean of the measurement is

$$a_m = \sum_{i=1}^{N} a_i / N$$

1. Absolute error: For measurement a_1, absolute error is given by

$$\Delta a_1 = a_m - a_1$$

It can be calculated similarly for other measurements.

2. Mean absolute error: For absolute errors, a_1, a_2, a_3, ….a_N in N measurements, mean absolute error is given by

$$\Delta a_m = \frac{|\Delta a_1| + |\Delta a_2| + \ldots\ldots + |\Delta a_N|}{N} = \sum_{i=1}^{N} |\Delta a_i| / N$$

3. Relative error: It is given by:

$$\delta a = \frac{\Delta a_m}{a_m}$$

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4. Absolute error of a Sum or difference: For two measured quantities A and B with absolute errors ΔA and ΔB as their respective absolute errors, error in a sum or difference is

$$\pm\Delta X = \pm\Delta A \pm \Delta B$$

5. Relative error of a product or a quotient:

$$\frac{\Delta X}{X} = \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta B}{B}\right)$$

6. Relative error in case of a measured quantity raised to a power:

If $X = A^m B^n / C^r$, then

$$\frac{\Delta X}{X} = m\left(\frac{\Delta A}{A}\right) + n\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)$$

Conceptual Questions

Q.1. Length is one of the fundamental quantities. Justify this statement.

Ans. The length of an object is measured in metres. One metre is equal to the length of path travelled by light in vacuum in time interval 1/299,792,458 of a second. Since the value is always same, it is a fundamental quantity.

Q.2. Mention some ways to keep a track of time.

Ans. Time can be measured with the help of some periodic phenomenon or events like oscillations of a pendulum, motion of earth around the sun, motion of earth on its axis, etc.

Q.3. How is one second measured?

Ans. It is measured as the duration of 9,192,631,770 periods of radiation emitted during the transition in cesium 133 atom between two hyperfine levels. It is measured this way to assure accuracy and also the method is independent of the external conditions like, place, temperature and pressure.

Q.4. Standard international units are coherent system of units. Comment.

Ans. S.I. system of units consists of all the fundamental units which can derive all the other physical units by multiplication and division only. Thus it is coherent system of units.

Q.5. Mention some characteristics of standard international units.

Ans. (1) Standard international units are coherent systems of units.

(2) The system is also rational and only one unit is used in a particular type of physical quantity.

(3) Its multiple can be expressed in the power of 10.

Q.6. The distance between the sun and earth is one astronomical unit. Find the time taken by the sunlight to reach the earth.

Ans. One astronomical unit = 1.5×10^{11} m

$$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{1.5\times10^{11}\,\text{m}}{3\times10^{8}\,\text{m/s}} = 500\ \text{s} = 8.33\ \text{min}$$

Q.7. What is Chandrasekhar limit?

Ans. It is the largest possible value of mass of a white dwarf star and is equal to nearly 1.4 times the mass of the sun.

Q.8. How solar day is different from sidereal day?

Ans. Solar day is the time interval between successive passage of the sun crossing the meridian in the sky. However, the sidereal day is the time interval between successive passage of any star crossing the meridian or any other fixed point in the sky.

Q.9. Why angular diameter of the sun and the moon in the sky is same?

Ans. Angular diameter of any object in the sky can be measured as: $\alpha = \frac{d}{D}$

Here d is the diameter of object and D is the distance between object and the earth.

In case of the moon and the sun, by coincidence, the ratio d : D is same for both and hence their angular diameter is same in the sky.

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Q.10. How many radians are there in one second?

Ans. $1^\circ = 60' = 3600'' = \left(\frac{\pi}{180}\right)$

$\Rightarrow 1'' = \left(\frac{\pi}{3600 \times 180}\right) = 4.8 \times 10^{-6}$ rad

Q.11. What is one light year?

Ans. One light year is a unit of distance and is equal to the distance travelled by light in one year.

Q.12. Arrange the following units in increasing order of their value: One light year, one astronomical unit, one parsec.

Ans. one astronomical unit < one light year < one parsec.

Q.13. How can we measure distance of star which is nearly 50 light years away from us? Can we measure the distance from a star 150 light years away using the same method?

Ans. We can measure the distance of a star that is 50 light years away using parallax method. The star will be observed from two different locations on earth and by measuring the distance between both the locations and angle subtended by the star on both positions, the star distance can be calculated. This method cannot be used to measure a star which is more than 100 light years as the accuracy of the result is not achieved. Thus, we cannot measure the star 150 light years away from us using the same method.

Q.14. How are large time intervals measured?

Ans. Large time intervals are measured using carbon dating method. The time is measured by determining the ratio of decayed radioactive atoms to remaining undecayed atoms.

Q.15. What is the smallest mass in our knowledge till date?

Ans. Electron is the elementary particle with smallest mass observed till date, i.e. of the order 10^{-30} kg.

Q.16. Mention the uses of dimensional equations.

Ans. Dimensional equations are used to check the correctness of various relations or formulas.

These are also used to derive relations between physical quantities and converting one system of units to other.

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Q.17. In expression $A = \frac{BC^2}{2}$, the dimensions of A and C are $[ML^{-1}T^{-2}]$ and $[MLT^{-3}A^{-1}]$ respectively. Find the dimensions of B and identify it.

Ans. Here the dimensions of B can be calculated as:

$$A = \frac{BC^2}{2}$$

$$\Rightarrow B = \frac{2A}{C^2}$$

$$\Rightarrow [B] = \frac{[ML^{-1}T^{-2}]}{[MLT^{-3}A^{-1}]^2} = [M^{-1}L^{-3}T^4A^2]$$

We can see that the B is the electric permittivity of free space.

Q.18. All the constants are not dimensionless. Explain.

Ans. The given statement is correct, all the constants are not dimensionless. Constants like gravitational constant, Planck's constant, etc. have dimensions.

Q.19. What is the dimension of Reynold number?

Ans. It is a dimensionless quantity.

Q.20. How can we reduce least count error of any instrument?

Ans. It can be reduced by increasing the resolution and avoiding the errors during experiments.

Q.21. How can we reduce the random errors in an experiment?

Ans. Random errors can be decreased by increasing the number of times the reading is taken. Taking many readings and using their mean in the final calculation can reduce the random errors in a result.

Q.22. How accuracy and precision in an experiment are different?

Ans. Accuracy is the measure of the closeness of the measured value to the standard value. However, precision in any experiment depends on the least count of the instrument used and refers to the closeness of two or more measurements to each other.

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Q.23. In the given expression $X = \frac{abz^2}{h^3}$, which physical quantity should be measured more accurately to give the most accurate answer?

Ans. Since the power of h is highest, it should be measured most accurately to give accurate answer.

Q.24. How many significant digits are in 71450000?

Ans. There are four significant digits only.

Q.25. A physical quantity, U can be measured as $U = \frac{abz^2}{h^3}$. Write the expression for finding the relative error in U.

Ans. Relative error in U can be measured as: $\frac{\Delta U}{U} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + 2\frac{\Delta z}{z} + 3\frac{\Delta h}{h}$

Q.26. Which is more accurate 8.0 or 8.00?

Ans. Since 8.00 is correct to second place of decimal as compared to 8.0, which is correct only to first place of decimal, 8.00 is more accurate.

Q.27. How many significant digits are there in 0.008?

Ans. One

Tough & Tricky PROBLEMS

Problem 1. What is the value of G in CGS system if it is 6.67×10^{-11} in MKS system?

Solution Dimensional formula of $G = [M^{-1}L^3T^{-2}]$

Let n_1 and n_2 are the values of G in MKS and CGS system and let u_1 and u_2 represent MKS and CGS system, respectively.

$$n_1u_1 = n_2u_2$$

$$\Rightarrow \quad n_1[M_1^{-1}L_1^3T_1^{-2}] = n_2[M_2^{-1}L_2^3T_2^{-2}]$$

$$\Rightarrow \quad n_2 = n_1\left[\frac{M_2}{M_1}\right]^1\left[\frac{L_1}{L_2}\right]^3\left[\frac{T_2}{T_1}\right]^2$$

$$\Rightarrow \quad n_2 = (6.67\times10^{-11})\left[\frac{1\,g}{1000\,g}\right]^1\left[\frac{100\,cm}{1\,cm}\right]^3\left[\frac{1s}{1s}\right]^2$$

$$\Rightarrow \quad n_2 = (6.67\times10^{-11})\times10^3$$

$$\Rightarrow \quad n_2 = \mathbf{6.67\times10^{-8}}$$

Problem 2. Dimensions of length are expressed as $G^ac^bh^c$. Find the values of a, b and c.

Solution

$$[M^0LT^0] = [G]^a[c]^b[h]^c$$

$$\Rightarrow \quad [M^0LT^0] = [M^{-1}L^3T^{-2}]^a[LT^{-1}]^b[ML^2T^{-1}]^c$$

$$\Rightarrow \quad [M^0LT^0] = [M^{-a+c}L^{3a+b+2c}T^{-2a-b-c}]$$

On comparing dimensions on both sides.

$$-a + c = 0 \qquad \ldots(i)$$

$$3a + b + 2c = 1 \qquad \ldots(ii)$$

$$-2a - b - c = 0 \qquad \ldots(iii)$$

On solving the equations (i), (ii) and (iii) we get the following:

$$a = ½,\ b = -3/2 \text{ and } c = ½.$$

Problem 3. A Vernier caliper is used to measure the diameter of a cylindrical rod. Zero of the Vernier scale is found to lie between 4.15 cm and 4.20 cm of the main scale. Vernier scale has 50 divisions which are equivalent to 2.45 cm. The 36^{th} division of Vernier scale matches with one of the main scale divisions. What is the diameter of rod?

Solution It is given that zero of Vernier scale lies between 4.15 cm and 4.20 cm. Hence, length of one main scale division can be written as follows:

1 MSD = 4.20 – 4.15 = 0.05 cm

50 divisions of Vernier scale is equivalent to 2.45 cm. Hence, length of one Vernier scale division can be written as follows:

$$1\ VSD = \frac{2.45}{50} = 0.049\,cm$$

Least count is defined as difference in length of main scale and Vernier scale division.

LC = 1 MSD – 1 VSD = 0.05 – 0.049 = 0.001 cm.

Zero of the Vernier scale lies between 4.15 cm and 4.20 cm. Hence, main scale reading (MSD) is 4.15 cm. It is given that 36^{th} division of Vernier scale matches with some division of main scale. So, reading of the Vernier caliper can be written as follows:

$$d = MSR + VSR \times LC$$

$$\Rightarrow \quad d = 4.15 + 36\times0.001 = \mathbf{4.186\ cm}$$

Problem 4. The potential energy of a particle depends on its x-coordinates as $U = \frac{A\sqrt{x}}{x^2 + B}$, where A and B are dimensional constants. What will be the dimensional formula for A/B?

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Solution

$$U = \frac{A\sqrt{x}}{x^2 + B}$$

In denominator, B is added with x^2. So, dimensions of B must be same as that of x^2.

$$[B] = [x^2] = [L^2]$$

Further we can write the following:

$$[U] = \frac{[A\sqrt{x}]}{[x^2 + B]}$$

$$\Rightarrow \quad [ML^2T^{-2}] = \frac{[A][L^{1/2}]}{[L^2]}$$

$$\Rightarrow \quad [A] = [ML^{7/2}T^{-2}]$$

Hence, we can write dimensions of A/B as follows:

$$\left[\frac{A}{B}\right] = [ML^{3/2}T^{-2}]$$

Problem 5. Round off the following numbers up to 2 significant digits.

(a) 2462 (b) 76.14 (c) 3.75 (d) 16.5

Solution

(*a*) Digits to be dropped are 6 and 2. Hence, the preceding digit 4 should be increased by one and two zeroes are added after it which do not affect number of significant digits.

Hence, we can write the number as **2500**.

(*b*) We can simply drop last two digits after decimal point so that only two significant digits are retained.

Hence, we can write the number as **76**.

(*c*) We need to drop last digit which is 5 to retain two significant digits and the preceding digit is 7 which is an odd number hence it will be increased by one.

Hence, we can write the number as **3.8**.

(*d*) We need to drop last digit 5 to retain two significant digits and the preceding number is 6 which is an even number; so it will remain the same.

Hence, we can write the number as **16**.

Problem 6. *N* divisions on Vernier scale match with *N* – 1 divisions on mains scale. If one main scale division (MSD) is of *x* units, then find least count of the instrument.

Solution Let 1 MSD be x

One Vernier scale division (VSD) $= \left(\frac{N-1}{N}\right)x$

Least count (LC) = 1 MSD –1 VSD

$$\Rightarrow \quad \text{LC} = x - \left(\frac{N-1}{N}\right)x = \frac{x}{N}$$

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NCERT FILE Solved

NCERT Textbook Exercises

2.1. Fill in the blanks:

(*a*) The volume of a cube of side 1 cm is equal to ..... m^3.

(*b*) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to ..... $(mm)^2$.

(*c*) A vehicle moving with a speed of 18 km/h covers ..... m in 1s.

(*d*) The relative density of lead is 11.3. Its density is ..... g cm^{-3} or ..... kg m^{-3}.

Ans. (*a*) As length of the side = 1 cm

The volume of a cube of side 1 cm is equal to

$$= 1 \times 1 \times 1 \text{ cm}^3$$

$$= 1 \times \left(\frac{1}{100}\right)^3 \text{ m}^3 = \mathbf{10^{-6}\ m^3}.$$

(*b*) Total surface area of a solid cylinder,

$$A = 2\pi r^2 + 2\pi rh = 2\pi r\,(r + h)$$

r = 2 cm =20 mm, h = 10 cm = 100 mm

$$\therefore \quad A = 2 \times \frac{22}{7} \times 20\,(20 + 100)$$

$$= 15086 \text{ mm}^2$$

$$\approx \mathbf{1.5 \times 10^4\ mm^2}$$

(*c*) $18 \text{ km h}^{-1} = \frac{18 \times 1000}{60 \times 60} = 5 \text{ m s}^{-1}$

The distance covered by the vehicle in 1 second = **5 m**

(*d*) The relative density of lead = 11.3.

density =11.3 g cm^{-3} or 11300 kg m^{-3}.

1 g = 10^{-3} kg, 1 cm = 10^{-2} m

$\therefore$ 11.3 g cm^{-3} = 11.3 $(10^{-3}$ kg) $(10^{-2}$ m$)^{-3}$

$$= 11.3 \times 10^{-3} \times 10^6 \text{ kg m}^{-3}$$

$$= \mathbf{11.3 \times 10^3\ kg\ m^{-3} = 1.13 \times 10^4\ kg\ m^{-3}}$$

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2.2. Fill in the blanks by suitable conversion of units:

(a) 1 kg m^2 s^{-2} = ... g cm^2 s^{-2}

(b) 1 m = ... ly

(c) 3.0 ms^{-2} = ... kmh^{-2}

(d) $G = 6.67 \times 10^{-11}$ Nm^2 $(kg)^{-2}$ = ... $(cm)^3$ s^{-2} g^{-1}

Ans. (a) 1 kg m^2 s^{-2} = 1 $(10^3$ g$)$ $(10^2$ cm$)^2$ s^{-2}

$= \mathbf{10^7\ g\ cm^2\ s^{-2}}$.

(b) As 1 light year = 9.46×10^{15} m, therefore,

1 m = $1/9.46 \times 10^{15} = 1.057 \times 10^{-16}$ light year

$\simeq \mathbf{10^{-16}\ light\ year}$

(c) $3\ ms^{-2} = 3\left(\frac{1}{1000}\ km\right)\left(\frac{1}{(60\times60)}\ hr\right)^{-2}$

$= 3.888 \times 10^4\ km\ h^{-2}$

$= \mathbf{3.9 \times 10^4\ km\ h^{-2}}$

(d) $G = 6.67 \times 10^{-11}\ Nm^2\ (kg)^{-2}$

$G = 6.67 \times 10^{-11}\ (10^5\ dyne)\ (100\ cm)^2\ (1000\ g)^{-2}$

$= 6.67 \times 10^{-8}\ dyne\ cm^2\ g^{-2}$

$= 6.67 \times 10^{-8}\ g\ cm\ s^{-2}\ cm^2\ g^{-2}$

$= \mathbf{6.67 \times 10^{-8}\ cm^3\ s^{-2}\ g^{-1}}$.

2.3. A calorie is a unit of heat or energy and it equals about 4.2 J where 1 J = 1 kg m^2 s^{-2}. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude 4.2 α^{-1} β^{-2} γ^{+2} in terms of the new units.

Ans. Let n_1, n_2 be the numerical values when the fundamental units are (M_1, L_1, T_1) and (M_2, L_2, T_2) respectively, then

$n_1 u_1 = n_2 u_2$

$n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c]$

where *a, b, c* are the dimensions of *C* in terms of mass, length and time respectively.

As 1 cal = 4.2 J = 4.2 kg m^2 s^{-2}

$\therefore$ $a = 1, b = 2, c = -2$

$M_1 = 1$ kg, $L_1 = 1$ m, $T_1 = 1$ s

$M_2 = \alpha$ kg, $L_2 = \beta$ m, $T_2 = \gamma$ s

(Units in new system)

Here, $n_1 = 4.2$

$n_2 = ?$ $\quad n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$

$= 4.2\left(\frac{1\ kg}{\alpha\ kg}\right)^1\left(\frac{1\ m}{\beta\ m}\right)^2\left(\frac{1\ s}{\gamma\ s}\right)^{-2}$

$n_2 = \mathbf{4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^{+2}}$

Hence the result follows.

2.4. Explain this statement clearly:

"To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison". In view of this, reframe the following statements wherever necessary:

(a) atoms are very small objects

(b) a jet plane moves with great speed

(c) the mass of Jupiter is very large

(d) the air inside this room contains a large number of molecules

(e) a proton is more massive than an electron

(f) the speed of sound is much smaller than the speed of light.

Ans. The statement is true because a quantity can be small or large only in comparison to some standard.

(a) Atoms are very small objects of the order of 10^{-10} m and the size of atom is less than the sharp tip of a pin.

(b) A jet plane moves with great speed even faster than the superfast train.

(c) The mass of Jupiter is many times large as compared to the mass of earth.

(d) The air inside this room contains larger number of molecules than in one mole of air.

(e) The statement is already correct. A proton is 1840 times massive than that of an electron.

(f) The statement is already correct.

2.5. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

Ans. As speed of light in vacuum, $c = 1$ new unit of length $\times\ s^{-1}$

Time taken by light to cover the distance between the sun and the earth, $t = 8$ min and 20 s = 500 s

Hence the distance between the sun and the earth = $d = c \times t = 1$ new unit of length $\times\ s^{-1} \times 500$ s = 500 new unit of length

2.6. Which of the following is the most precise device for measuring length:

(a) a vernier callipers with 20 divisions on the sliding scale.

(b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale.

(c) an optical instrument that can measure length to within a wavelength of light?

Ans. The device having minimum least count is the most precise device for measuring length

(a) Least count of vernier callipers

= 1 M.S.D. – 1 V.S.D.

$= \left(1 - \frac{19}{20}\right)$ MSD

$= \mathbf{0.005\ cm}$

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(*b*) Least count of screw gauge

$$= \frac{\text{pitch}}{\text{total number of divisions on the circular scale}}$$

$$= \mathbf{0.001\ cm}$$

(*c*) Wavelength of light $\lambda = 10^{-6}$ cm = **0.000001 cm**

Hence, an optical instrument that can measure length to within a wavelength of light (which is approx. equal to 10^{-6} m) is the most precise device for measuring length.

2.7. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate of the thickness of hair?

Ans. Magnification = observed width/real width

observed width = 3.5 mm

Magnification = 100

$$\text{real width} = \frac{3.5\ \text{mm}}{100} = 0.035\ \text{mm}.$$

Hence, thickness of hair = 0.035 mm.

2.8. Answer the following:

(*a*) You are given a thread and a metre scale. How will you estimate the diameter of this thread?

(*b*) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale?

(*c*) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

Ans. (*a*) The diameter of the thread is very small and cannot be measured by a metre scale. Wind the thread in close turns on the metre scale. These turns are closely touching each other. Count the number of turns. Find the average length of the coiled thread on the metre scale. Divide this average length by the number of turns.

$$\text{Thus, diameter of the thread} = \frac{\text{Average length}}{\text{Number of turns}}.$$

(*b*) Yes, the accuracy of the gauge can be increased by increasing the number of divisions on the circular scale because when number of turns increases, the least count of the instrument becomes less.

Because, Least count

$$= \frac{\text{Pitch}}{\text{Number of division on the circular scale}}$$

(*c*) Yes, it is true that when we take a large number of observations say (100) and then take their mean value for the result, the random errors are greatly reduced as compared to the case when we take only five observations and then take their mean. Therefore, the result is more reliable when we take a set of 100 measurements than a set of 5 measurements only.

2.9. The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector-screen arrangement?

Ans. $$\text{Magnification of area} = \frac{\text{Area of image}}{\text{Area of object}}$$

$$= \frac{1.55 \times 100 \times 100}{1.75}$$

$$= 8857$$

$$\text{Linear magnification} = \sqrt{8857} = \mathbf{94.1}$$

2.10. State the number of significant figures in the following:

(*a*) 0.007 m^2

(*b*) 2.64×10^{24} kg

(*c*) 0.2370 g cm^{-3}

(*d*) 6.320 J

(*e*) 6.032 Nm^{-2}

(*f*) 0.0006032 m^2

Ans. (*a*) 1 (*b*) 3 (*c*) 4 (*d*) 4 (*e*) 4 (*f*) 4.

2.11. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Ans. Length (l) = 4.234 m, Breadth (b) = 1.005 m, Thickness (t) = 0.0201 m

$$\text{Area} = 2(lb + bt + tl)$$

$$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)$$

$$= 2\,[4.255 + 0.0211 + 0.0889]$$

As area can contain maximum of three significant figures

$\therefore$ Area = 8.73 m^2

Volume of the sheet = $l \times b \times t$

$$= 4.234 \times 1.005 \times 0.0201$$

$$= 0.0855\ m^3$$

As volume can also contain maximum of three significant figures

$\therefore$ Volume = **0.0855 m^3**

2.12. The mass of a box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the

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box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures?

Ans. (*a*) Total mass of box = 2.300 + 0.02015 + 0.02017

= 2.34032 kg = 2.3 kg

(correct up to one decimal place)

(*b*) Difference in the masses of pieces

= 20.17 – 20.15 = **0.02 g**

(correct upto one decimal place)

2.13. A physical quantity *P* is related to four observables *a*, *b*, *c* and *d* as follows:

$$P = \frac{a^3 b^2}{\sqrt{c}\, d}$$

The percentage errors of measurement in *a*, *b*, *c* and *d* are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity *P*? If the value of *P* calculated using the above relation turns out to be 3.763, to what value should you round off the result?

Ans. As $P = \frac{a^3 b^2}{c^{1/2} d}$

$$\frac{\Delta P}{P} = \frac{3\Delta a}{a} + \frac{2\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}$$

$$\therefore \frac{\Delta P}{P} \times 100 = \left(\frac{3\Delta a}{a} + \frac{2\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}\right) \times 100$$

$\frac{\Delta a}{a} = \frac{1}{100}$, $\frac{\Delta b}{b} = \frac{3}{100}$, $\frac{\Delta c}{c} = \frac{4}{100}$, $\frac{\Delta d}{d} = \frac{2}{100}$

So $\frac{\Delta P}{P} \times 100 = 3 + 2 \times 3 + \frac{1}{2} \times 4 + 2$

$\left(\because \frac{\Delta a}{a} \times 100 = 1 \text{ and so on}\right)$

= 3 + 6 + 2 + 2 = 13

$\therefore$ Percentage error in P = 13%

As the result (13%) have only 2 significant figures, therefore, if P = 3.763, then the result should be rounded off to 3.8.

2.14. A book with many printing errors contains four different formulas for the displacement *y* of a particle undergoing a certain periodic motion:

(*a*) $y = a \sin \frac{2\pi t}{T}$

(*b*) $y = a \sin vt$

(*c*) $y = (a/T) \sin t/a$

(*d*) $y = (a\sqrt{2})\ (\sin 2\pi t/T + \cos 2\pi t/T.)$

[*a* = maximum distance of the particle, *v* = speed of particle. *T* = Time period of motion]. Rule out the wrong formulas on dimensional grounds.

Ans. According to the principle of homogeneity,

The dimensions on left side of an equation = The dimensions on the right side of an equation. Also the argument of sine or cosine function must be dimensionless dimensionally.

(*a*) The relation is correct because $2\pi t/T = T/T = 1$ (dimensionless) and the dimensions of y and a are same.

(*b*) Dimensions of $vt = [L^1\ T^{-1} \times T^1] = [L^1]$. But vt should be dimensionless. Therefore, the relation is wrong.

(*c*) The relation is not correct because y and (a/T) have different dimensions and t/a is not dimensionless.

(*d*) Here the angle $\frac{2\pi t}{T}$ is dimensionless and the dimensions on both sides of the equation are equal . Therefore, it is dimensionally correct.

2.15. A famous relation in physics relates 'moving mass' *m* to the 'rest mass' m_0 of a particle in terms of its speed *v* and the speed of light, *c*. (This relation first arose as a consequence of special relativity theory due to Albert Einstein.) A boy recalls the relation almost correctly but forgets where to put the constant *c*. He writes:

$m = \frac{m_0}{(1 - v^2)^{1/2}}$. Guess where to put the missing *c*.

Ans. According to the principle of homogeneity

The dimensions on left side of an equation = The dimensions on the right side of an equation.

Hence, to make the dimensions on both the sides of equation equal, the denominator must be dimensionless, $\therefore$ instead of $(1 - v^2)^{1/2}$ factor at the denominator, there should be v^2/c^2.

$$\therefore \quad m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

2.16. The unit of length convenient on the atomic scale is known as an angstrom, and is denoted by Å : 1 Å = 10^{-10} m. The size of a hydrogen atom is about 0.5 Å.What is the total atomic volume in m^3 of a mole of hydrogen atoms?

Ans. Volume of 1 atom of hydrogen

= $4/3\ \pi r^3 = 4/3 \times 3.14 \times (0.5 \times 10^{-10})^3$

As one mole of hydrogen has 6.02×10^{23} atoms

$\therefore$ Total atomic volume = $6.02 \times 10^{23} \times 4/3 \times 3.14 \times 0.125 \times 10^{-30}$

$\therefore$ Atomic volume = $3.151 \times 10^{-7}\ m^3 \cong 3 \times 10^{-7}\ m^3$

2.17. One mole of an ideal gas at standard temperature and pressure occupies 22.4 litres (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large?

Ans. Molar volume = 22.4 litres = $22.4 \times 10^{-3} m^3$

(1 litre = $10^{-3}\ m^3$)

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Volume of 1 atom of hydrogen $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3$$

$$= \frac{4}{3} \times 3.14 \times 0.125 \times 10^{-30}\ \text{m}^3$$

Volume of 1 mole of hydrogen atoms

$$= 6.02 \times 10^{23} \times \frac{4}{3} \times 3.14 \times 0.125 \times 10^{-30}\ \text{m}^3$$

$$= 6.02 \times 0.125 \times \frac{4}{3} \times 3.14 \times 10^{-7}\ \text{m}^3$$

$$= 3.1 \times 10^{-7}\ \text{m}^3$$

$$\therefore \quad \frac{\text{Molar volume}}{\text{Atomic volume of a mole of hydrogen}}$$

$$= \frac{\mathbf{22.4 \times 10^{-3}\ m^3}}{\mathbf{3.1 \times 10^{-7}\ m^3}} \simeq \mathbf{7.2 \times 10^4}$$

This ratio is so large because inter-atomic separation in a gas is much larger than the size of a atom.

2.18. Explain this common observation clearly: If you look out of the window of a fast-moving train, the nearby trees, houses, etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars, etc.) seem to be stationary. In fact, since you are aware that you are moving, these distant objects seem to move with you.

Ans. Apparent distance moved by objects opposite to motion of train is proportional to the visual angle formed by two positions of object on our eyes. This visual angle is smaller for distant objects and thus objects very far off from observer seem stationary.

2.19. The principle of 'parallax' in Section 3.1 (Refer to Fig. 3.1) page 5 is used in the determination of distances of very distant stars. The role of distant object O there is now taken by nearby distant stars. The baseline AB is the line joining the earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the earth's orbit = 3×10^{11} m. However, even the nearest stars are so distant that with such a long baseline they show parallax only of the order 1″ (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1″ (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres?

Ans. l = Distance between the sun and earth = length of baseline AB= 1 AU = 1.5×10^{11} m

$$\theta = \text{Parallax angle} = 1'' = \frac{1^\circ}{3600} = \frac{\pi}{180} \times \frac{1}{3600}\ \text{radian}$$

$$d = 1\ \text{parsec} = ?$$

As $\quad l = 1.5 \times 10^{11}$ m

$$\theta = \frac{l}{d}$$

$$d = \frac{l}{\theta}$$

$$= \frac{1.5 \times 10^{11}}{\pi / 180 \times 3600}$$

$$d = \mathbf{3.09 \times 10^{16}\ m}$$

1 parsec is equal to 3.09×10^{16} m

2.20. The nearest star to our solar system is 4.29 light year away. How much is this distance in terms of parsec? How much parallax would this star (named Alpha Centauri) show when viewed from two locations on the Earth six months apart in its orbit around the sun?

Ans. As distance, d = 4.29 light years

$$= 4.29 \times 9.46 \times 10^{15}\ \text{m}$$

$$= \frac{4.29 \times 9.46 \times 10^{15}}{3.00 \times 10^{16}}\ \text{parsec}$$

$$= 1.317\ \text{parsec} \simeq 1.32\ \text{parsec}$$

A parsec distance subtends a parallax angle of 1″ for a base equal to radius of earth's orbit around the sun. In present case, base is the distance between two locations of earth six months apart, i.e. l = 2AU

Parallax angle subtended by 1 parsec distance = 2 second (by definition)

Parallax angle subtended by the star at given base, $\theta = 1.32 \times 2 = 2.64''$

2.21. Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.

Ans. (*a*) Precise measurements of length are made while finding the thickness of cornea for curing eye problems, another example is measurement of fringe system to Michelson-Morley experiment ($\sim 10^{-6}$ m).

(*b*) Precise measurements of time are done in cesium clock, and we can use these clocks to measure time taken by an electron to complete an orbit around the nucleus (10^{-15} s).

(*c*) Precise measurements of mass is done in measuring the masses of isotopes, mass of an electron, etc. Mass spectrometry is a technique used to measure mass of charged particles.

2.22. Just as precise measurements are necessary in science, it is equally important to be able

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to make rough estimates of quantities using rudimentary ideas and common observations.

Think of ways by which you can estimate the following: (where an estimate is difficult to obtain, try to get an upper bound on the quantity).

(*a*) the total mass of rain-bearing clouds over India during the Monsoon

(*b*) the mass of an elephant

(*c*) the wind speed during a storm

(*d*) the number of strands of hair on your head

(*e*) the number of air molecules in your classroom.

Ans. (*a*) Let the average rainfall recorded during monsoon = h = 100 cm = 1 m

Let Area of our country = A

Volume = $V = A \times h$

density of water = ρ

Here, A = 3.3 million sq km

$= 3.3 \times 10^6 \text{ km}^2 = 3.3 \times 10^{12} \text{ m}^2$

$\rho = 1000 \text{ kg/m}^3$

$\therefore$ Mass of rain-bearing clouds = $A\rho h$

$= 3.3 \times 10^{12} \times 1000 \times 1$

$= \mathbf{3.3 \times 10^{15} \text{ kg}}$

(*b*) To find the mass of an elephant, take a boat of area A.

Let depth of boat in water = d_1

$\therefore$ Volume of water displaced by boat $V_1 = Ad_1$

Let the elephant moves in the boat

Now, depth of boat inside water = d_2

Volume of water displaced by boat and elephant = $V_2 = Ad_2$

Volume of water displaced by elephant

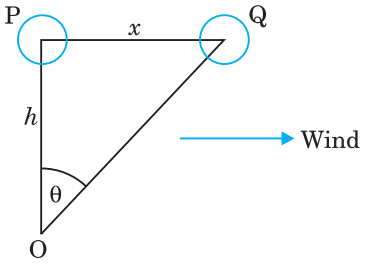
$= V = V_2 - V_1$

$= A(d_2 - d_1)$

density of water = ρ

Mass of elephant = $\boldsymbol{A\rho(d_2 - d_1)}$

(*c*) The wind speed during a storm can be calculated by using a gas filled ballon.



Let OP = Normal position of balloon when there is no wind.

Suppose the wind starts blowing towards right, the balloon shifts to position Q in one second.

Let θ be angle of shift.

h = height of balloon from ground

$\therefore$ Distance travelled by wind in 1 sec. = PQ = $x = \boldsymbol{h\theta}$

This is wind speed.

(*d*) Let area of our head = A

Let thickness of hair = $2t$ (diameter)

$\therefore$ area of cross section of hair = πt^2

No. of strands of hair, n

$$= \frac{\text{total area}}{\text{area of cross section of each hair}}$$

$$= \frac{\boldsymbol{A}}{\boldsymbol{\pi t^2}}$$

(*e*) Let volume of room = V

Let one mole of air occupy a volume of 22.4 litres = $22.4 \times 10^{-3} \text{ m}^3$

No. of molecules in $22.4 \times 10^{-3} \text{ m}^3 = 6.023 \times 10^{23}$

No. of molecules in $V \text{ m}^3 = \frac{6.023 \times 10^{23}}{22.4 \times 10^{-3}} \times V$

$= \mathbf{2.68 \times 10^{25}\ V}$

2.23. The Sun is a hot plasma (ionised matter) with its inner core at temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At these high temperatures no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be? In the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^8 m.

Ans. Radius of the sun = 7.0×10^8 m.

Volume = $4/3\ \pi r^3$

$= 4/3 \times 3.14 \times (7.0 \times 10^8)^3$

$= 4/3 \times 3.14 \times 49 \times 7 \times 10^{24}$

Mass of the sun = 2.0×10^{30} kg

$$\text{Density} = \frac{M}{V} = \frac{2.0 \times 10^{30} \times 3}{4 \times 3.14 \times 49 \times 7 \times 10^{24}}$$

$$= \frac{3000 \times 10^3}{2 \times 3.14 \times 49 \times 7}$$

$= 1.393 \times 10^3 \text{ kg m}^{-3}$

$= \mathbf{1.4 \times 10^3 \text{ kg m}^{-3}}$

Mass density of the sun should be 1.4×10^3 kg m^{-3}. The mass density of the sun is in the range of densities of solids and not gases. This high density of the hot plasma arises due to inward gravitational attraction on outer layers due to layers of the sun.

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2.24. When the planet Jupiter is at a distance of 824.7 million km from the earth, its angular diameter is measured to be 35.72 s of arc. Calculate the diameter of Jupiter.

Ans. Angular separation, $\alpha = 35.72$ s

$= 35.72 \times 4.85 \times 10^{-6}$ rad

$= 173.24 \times 10^{-6}$ rad.

Distance of Jupiter from earth

$= 824.7$ million km

$= 824.7 \times 10^6$ km

$\alpha = \frac{D}{d}$

$D = 173.2 \times 10^{-6} \times 824.7 \times 10^6$

$= \mathbf{1.428 \times 10^5\ km}$

NCERT Additional Exercises

2.25. A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v : $\tan\theta = v$ and checks that the relation has a correct limit, as $v \to 0$, $\theta \to 0$, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.

Ans. Because L.H.S. is dimensionless

whereas R.H.S. $= v = [M^0LT^{-1}]$

Dimension of L.H.S. is not equal to dimension of R.H.S

To make the relation correct

We should have $\tan\theta = \frac{v}{u}$

Here u is speed of rainfall.

2.26. It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s?

Ans. Total time = 100 years

$= 100 \times 365 \times 24 \times 60 \times 60$ s

Error in 100 years is 0.02 s

Error in 1 sec. is $\frac{0.02}{100 \times 365 \times 24 \times 60 \times 60}$

$= \mathbf{6.34 \times 10^{-12}}$

Therefore, the accuracy of cesium clock in 1 part is 10^{12}

2.27. Estimate the average mass density of a sodium atom assuming its size to be about 2.5 Å. (Use the known values of Avogadro's number and the atomic weight of sodium). Compare it with the density of sodium in its crystalline phase; 970 kg m^{-3}. Are the two densities of the same order of magnitude? If so, why?

Ans. Size of atom $= 2.5 \times 10^{-10}$ m

Volume of atom $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times (2.5 \times 10^{-10})^3$

$= \frac{4}{3} \times 3.14 \times 15.63 \times 10^{-30}$ m^3

Mass of 6.02×10^{23} atoms of sodium $= 23 \times 10^{-3}$ kg

Mass of 1 atom of sodium $= \frac{23 \times 10^{-3}}{6.02 \times 10^{23}}$ kg

Density $= \frac{M}{V}$

$= \frac{23 \times 10^{-3}}{6.02 \times 10^{23}} \times \frac{3}{4 \times 3.14 \times 15.63 \times 10^{-30}}$

$= \frac{69 \times 10^4}{6.02 \times 4 \times 3.14 \times 15.63}$ kg m^{-3}

$\approx \mathbf{0.6 \times 10^3\ kg\ m^{-3}}$

The two densities are not of the same order due to interatomic spacing in crystalline phase of sodium.

2.28. The unit of length convenient on the nuclear scale is a fermi, 1 f = 10^{-15} m. Nuclear sizes obey roughly the following empirical relation $r = r_0 A^{1/3}$ where r is the radius of the nucleus, A is its mass number and r_0 is a constant equal to about 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom in Q 2.27.

Ans. Radius of nucleus, $r = r_0 A^{1/3}$

Volume of the nucleus, $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_0^3 A$

According to Avogadro's hypothesis,

Mass of 6.02×10^{23} atoms of sodium $= A$ g

$= A \times 10^{-3}$ kg

Mass of 1 atom of sodium $= \frac{A \times 10^{-3}}{6.02 \times 10^{23}}$ kg

Density of nucleus $= \frac{\text{mass}}{\text{volume}}$

$= \frac{A \times 10^{-3}}{6.02 \times 10^{23} \times \frac{4}{3}\pi r_0^3 A}$ kg m^{-3}

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$$= \frac{10^{-3}}{6.02\times10^{23}\times\frac{4}{3}\pi\,(1.2\times10^{-15})^3}\ \text{kg m}^{-3}$$

$$= 0.3\times10^{18}\ \text{kg m}^{-3}.$$

This is independent of mass number, hence mass density is same for different nuclei.

Mass density of sodium nucleus = 0.3×10^{18} kg m^{-3}

mass density of sodium atom = 0.6×10^{3} kg m^{-3}

Mass density of sodium nucleus/mass density of sodium atom = $\frac{0.3\times10^{18}\ \text{kg m}^{-3}}{0.6\times10^{3}\ \text{kg m}^{-3}} \approx \mathbf{0.5\times10^{15}}$

2.29. A LASER is a source of very intense, monochromatic and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?

Ans. Velocity = 3×10^{8} ms^{-1}

Total time, $2t$ = 2.56 s

$$t = \frac{2.56}{2} = 1.28\ \text{s}$$

Distance travelled in 1.28 s = $3\times1.28\times10^{8}$ m

$$= 3.84\times10^{8}\ \text{m}$$

Radius of the lunar orbit around the earth

$$= \mathbf{3.84\times10^{8}\ m}.$$

2.30. A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m s^{-1}).

Ans. $v = 1450\ \text{m s}^{-1}$

$$d = \frac{vt}{2} = \frac{1450\ \text{m s}^{-1}\times77.0\ \text{s}}{2}$$

$$= 55810\ \text{m} = \mathbf{55.8\ km}$$

2.31. The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?

Ans. Speed of light, $c = 3\times10^{5}$ km s^{-1}

t = 3 billion years = $3\times10^{9}\times365\times24\times60\times60$ s

Distance in km,

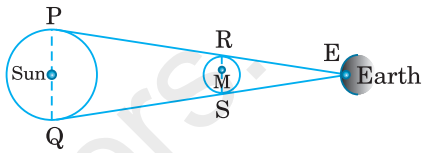
$d = 3\times10^{5}\times3\times10^{9}\times365\times24\times60\times60$

$$= 9\times365\times24\times36\times10^{16}$$

$$= \mathbf{2.8\times10^{22}\ km}$$

2.32. It is a well-known fact that during a total solar eclipse the disc of the moon almost covers the disc of the sun. Find the approximate diameter of the Moon.

Ans.



Distance (ME) of earth from the moon = 3.8×10^{8} m

Distance (SE) of earth from the sun = 1.496×10^{11} m

Diameter of the sun = PQ = 1.38×10^{9} m

During solar eclipse, the disc of the moon completely covers the disc of the sun.

Δ PQE and RSE are similar to each other.

$$\frac{PQ}{RS} = \frac{SE}{ME}$$

$$\frac{1.38\times10^{9}}{RS} = \frac{1.496\times10^{11}}{3.8\times10^{8}}$$

$$\Rightarrow \quad RS = \frac{1.38\times10^{9}\times3.8\times10^{8}}{1.496\times10^{11}}$$

$$= \mathbf{3.50\times10^{6}\ m}$$

2.33. A great physicist of this century (P.A.M. Dirac) loved playing with the numerical values of fundamental constant of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (c, e, mass of electron, mass of proton) and the gravitational constant G, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (~15 billion years). From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe is significant, what would this imply for the constancy of fundamental constants?

Ans. As the quantity $\frac{e^4}{16\pi^2\varepsilon_0^{\,2}m_p m_e^{\,2}c^3G}$ has the dimension of time. Therefore

$$t = \frac{(1.6\times10^{-19})^4\times(9\times10^{9})^2\times1}{1.67\times10^{-27}\times(9.1\times10^{-31})^2(3\times10^{8})^3\times6.67\times10^{-11}}$$

$$= \frac{530.84\times10^{-58}}{24905\times10^{-76}}$$

$$= 0.02\times10^{18}$$

$$= \mathbf{2\times10^{16}\ sec}$$

This time is of the order of age of universe.

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NCERT Exemplar Problems

Subjective Questions

Note: *Objective Questions (2.1 to 2.18) from Exemplar Problems are given in Competition File page 84.*

Very Short Answer Type Questions

2.19. Why do we have different units for the same physical quantity?

Ans: When we measure a physical quantity, its measured value may have a wide range in order of magnitude. For example, size of a nucleus is of the order of 1 f = 10^{-15} m, interatomic separations are of the order of 1 Å = 10^{-10} m and distance between the sun and earth is of the order of 1 AU = 10^{11} m.

So, it is convenient to have different units for the same physical quantity as it makes the expression of magnitude easier and more understandable.

2.20. The radius of atom is of the order of 1 Å and radius of nucleus is of the order of fermi. How many magnitudes higher is the volume of atom as compared to the volume of nucleus?

Ans: Given, Radius of atom, R_1 =1 Å = 10^{-10}m

Radius of nucleus, R_2 = 1 f = 10^{-15} m

Volume of atom, $V_1 = \frac{4}{3}\pi R_1^3$

Volume of nucleus, $V_2 = \frac{4}{3}\pi R_2^3$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{10^{-10}}{10^{-15}}\right)^3 = \mathbf{10^{15}}$$

2.21. Name the device used for measuring the mass of atoms and molecules.

Ans: Mass spectrograph is the device used for measuring the mass of atoms and molecules.

2.22. Express unified atomic mass unit in kg.

Ans: 1 u = $\frac{1}{12}$ × mass of an atom of carbon– 12 isotope $_6C^{12}$, including the mass of electrons.

$$= \frac{1}{12} \times \frac{12\,g}{6.023 \times 10^{23}}$$

$$= 1.66 \times 10^{-24}\ g$$

$$= \mathbf{1.66 \times 10^{-27}\ kg}$$

2.23. A function $f(\theta)$ is defined as:

$$f(\theta) = 1 - \theta + \frac{\theta^2}{2!} - \frac{\theta^3}{3!} + \frac{\theta^4}{4!} \ldots$$

Why is it necessary for θ to be a dimensionless quantity?

Ans: The function $f(\theta)$ is the sum of various powers of θ. So, θ has to be dimensionless as raising it to powers can change the dimensions of the quantity and quantities having different dimensions cannot be added or subtracted.

2.24. Why length, mass and time are chosen as base quantities in mechanics?

Ans: This is because all other quantities in mechanics can be expressed in terms of these quantities through simple relations.

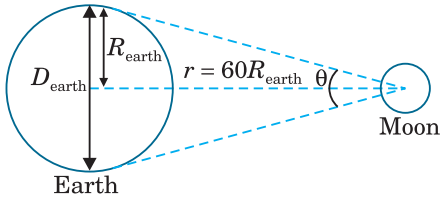
Short Answer Type Questions

2.25. (*a*) The earth-moon distance is about 60 earth radius. What will be the diameter of the earth (approximately in degrees) as seen from the moon?

(*b*) Moon is seen to be of (½)° diameter from the earth. What must be the relative size compared to the earth?

(*c*) From parallax measurement, the sun is found to be at a distance of about 400 times the earth-moon distance. Estimate the ratio of sun-earth diameters.

Ans: (*a*) Considering the given data, we have



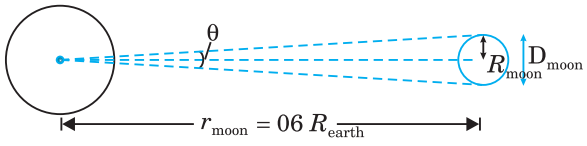
$$\therefore \quad \theta = \frac{\text{Diameter of earth, } D_{earth}}{\text{Distance between earth and moon, } r_{moon}}$$

$$= \frac{2R_{earth}}{60R_{earth}} = \frac{1}{30}\text{ rad}$$

$$= \frac{1}{30} \times \frac{180^\circ}{\pi} = 1.9^\circ = \mathbf{2^\circ}$$

Thus, the diameter of the earth as seen from the moon is 2°.

(*b*) Now, for moon, we have



Now, $\theta = \frac{D_{moon}}{r_{moon}} = \frac{2R_{moon}}{60R_{earth}} = \frac{R_{moon}}{30R_{earth}}$

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$$= \frac{1}{2} \times \frac{\pi}{180} \text{rad} = \frac{\pi}{360} \text{rad}$$

$$\therefore \frac{D_{moon}}{D_{earth}} = \frac{R_{moon}}{R_{earth}} = \frac{30\pi}{360} = \frac{\pi}{12} = 0.26 = \frac{1}{4}$$

$$\text{or } \frac{D_{earth}}{D_{moon}} = 4 \qquad ...(ii)$$

So, the earth is approximately four times the moon.

$$(c) \quad \frac{r_{sun}}{r_{moon}} = \frac{\text{Distance of sun from earth}}{\text{Distance of moon from earth}}$$

$$= \mathbf{400} \qquad ...(iii)$$

The sun and the moon both appear to be of the same angular diameter as seen from the earth.

$$\therefore \quad \theta = \frac{D_{sun}}{r_{sun}} = \frac{D_{moon}}{r_{moon}}$$

$$\text{or } \frac{D_{sun}}{D_{moon}} = \frac{r_{sun}}{r_{moon}} = 400 \qquad ...(iv)$$

But from equation (*ii*), we have

$$\frac{D_{earth}}{D_{moon}} = 4 \qquad ....(v)$$

Dividing equation (*iv*) by (*v*), we get

$$\frac{D_{sun}}{D_{earth}} = \frac{400}{4} = \mathbf{100}$$

2.26. Which of the following time-measuring devices is most precise?

(*a*) A wall clock.

(*b*) A stop watch.

(*c*) A digital watch.

(*d*) An atomic clock.

Give reason for your answer.

Ans: Atomic clock is the most precise with precision of 1 part in 10^{13}, i.e. the atomic oscillations of an atomic clock are repeated with a precision of 1 s in 10^{13}s.

2.27. The distance of a galaxy is of the order of 10^{25} m. Calculate the order of magnitude of time taken by light to reach us from the galaxy.

Ans: Given, distance of galaxy = 10^{25} m

Time taken by light to reach us from galaxy is

$$t = \frac{\text{Distance of galaxy}}{\text{Speed of light}}$$

$$= \frac{10^{25}\text{m}}{3 \times 10^{8}\text{m/s}} \approx \mathbf{3.3 \times 10^{16}\ s}$$

The order of the time taken is 10^{16} s.

2.28. The vernier scale of a travelling microscope has 50 divisions which coincide with 49 main scale divisions. If each main scale division is 0.5 mm, calculate the minimum inaccuracy in the measurement of distance.

Ans: Given,

$$50 \text{ VSD} = 49 \text{ MSD}$$

$$\Rightarrow \quad 1 \text{ VSD} = \left(\frac{49}{50}\right) \text{MSD}$$

Minimum inaccuracy in the measurement of distance = least count of the travelling microscope.

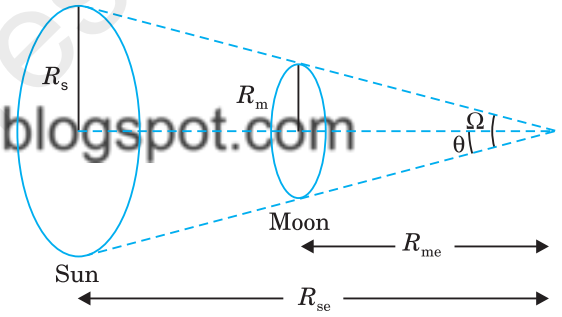
$$= 1 \text{ MSD} - 1 \text{ VSD}$$

$$= 1 \text{ MSD} - \left(\frac{49}{50}\right) \text{MSD} = \frac{1}{50} \text{ MSD} = \frac{1}{50} \times 0.5 \text{ mm}$$

$$= \mathbf{0.01\ mm}$$

2.29. During a total solar eclipse the moon almost entirely covers the sphere of the sun. Write the relation between the distances and sizes of the sun and the moon.

Ans: From the figure, we find that



Radius of the sun = R_s

Diameter of the sun = $2R_s = D_s$

Distance of the sun from earth = R_{se}

Radius of the moon = R_m

Diameter of the moon = $2R_m = D_m$

Distance of the moon from earth = R_{me}

Using property of similar triangles we can write the following:

$$\therefore \quad \frac{D_s}{D_m} = \frac{R_{se}}{R_{me}}$$

$$\text{or} \quad \frac{R_s}{R_m} = \frac{\boldsymbol{R_{se}}}{\boldsymbol{R_{me}}}$$

2.30. If the unit of force is 100 N, unit of length is 10 m and unit of time is 100 s, what is the unit of mass in this system of units?

Ans: $[F] = [MLT^{-2}] = 100$ N

$[L] = 10$ m

$[T] = 100$ s

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$$\therefore \quad M = \frac{[F]}{[L][T]} = \frac{[MLT^{-2}]}{[L][T]^{-2}}$$

$$= \frac{100}{10 \times (100)^{-2}}$$

$$= \frac{100 \times (100)^2}{10}$$

$$= \mathbf{10^5\ kg}$$

2.31. Give an example of

(*a*) a physical quantity which has a unit but no dimensions.

(*b*) a physical quantity which has neither unit nor dimensions.

(*c*) a constant which has a unit.

(*d*) a constant which has no unit.

Ans: (*a*) Solid angle/Planar angle

(*b*) Relative density/Efficiency

(*c*) Universal gravitational constant/Planck's constant

(*d*) Reynold's number

2.32. Calculate the length of the arc of a circle of radius 31.0 cm which subtends an angle of $\frac{\pi}{6}$ at the centre.

Ans: Radius, $r = 31.0$ cm

Angle, $\theta = \frac{\pi}{6}$ rad

Since, $\theta = \frac{\text{Length of arc}, L}{\text{Radius}, r}$

$\therefore \quad L = \theta r$

$$= \frac{\pi}{6} r = \frac{\pi}{6} \times 31.0 \text{ cm}$$

$$= \mathbf{16.2\ cm}$$

2.33. Calculate the solid angle subtended by the periphery of an area of 1cm^2 at a point situated symmetrically at a distance of 5 cm from the area.

Ans: Area, $A = 1\ \text{cm}^2$

Radius, $r = 5$ cm

$$\text{Solid angle} = \frac{A}{r^2}$$

$$= \frac{1\ \text{cm}^2}{(5\,\text{cm})^2} = \frac{1}{25}\ \text{sr}$$

$$= \mathbf{4 \times 10^{-2}\ sr}$$

2.34. The displacement of a progressive wave is represented by $y = A \sin(\omega t - kx)$, where x is distance and t is time. Write the dimensional formula of (*i*) ω and (*ii*) k.

Ans: Given, $y = A \sin(\omega t - kx)$

Argument of any trigonometric function is dimensionless

Here, both ωt and kx are dimensionless

$\therefore \quad [\omega t] = 1$

$$\Rightarrow \quad [\omega] = \frac{1}{[t]} = [T]^{-1}$$

$$= [M^0 L^0 T^{-1}]$$

Also, $[kx] = 1$

$$[k] = \frac{1}{[x]} = [L^{-1}]$$

$$= \mathbf{[M^0 L^{-1} T^0]}$$

2.35. Time for 20 oscillations of a pendulum is measured as $t_1 = 39.6$ s; $t_2 = 39.9$ s; $t_3 = 39.5$ s. What is the precision in the measurements?

What is the accuracy of the measurement?

Ans: Given, no of oscillations, $N = 20$

Time taken for 20 oscillations;

$t_1 = 39.6$ s, $t_2 = 39.9$ s and $t_3 = 39.5$ s

The precision in the time measurement is equal to the least count of measuring instrument which is 0.1 s.

Mean value of time per 20 oscillations is

$$t \text{ mean} = \frac{t_1 + t_2 + t_3}{3}$$

$$= \frac{39.6 + 39.9 + 39.5\,\text{s}}{3}$$

$$= 39.7 \text{ s}$$

Absolute error in each measurement is

$$\Delta t_1 = t_{\text{mean}} - t_1$$
$$= 39.7 - 39.6 = 0.1 \text{ s}$$
$$\Delta t_2 = t_{\text{mean}} - t_2$$
$$= 39.7 - 39.9 = -0.2 \text{ s}$$
$$\Delta t_3 = t_{\text{mean}} - t_3$$
$$= 39.7 - 39.5 = 0.2 \text{ s}$$

Mean absolute error is

$$\Delta t_{\text{mean}} = \frac{|\Delta t_1| + |\Delta t_2| + |\Delta t_3|}{3}$$

$$= \frac{0.1 + 0.2 + 0.2}{3}$$

$$= 0.17 \text{ s}$$

$$= 0.2 \text{ s (1 significant figure)}$$

$\therefore$ Accuracy of measurement = $\pm$ **0.2 s**

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HOTS Higher Order Thinking Skills & Advanced Level

QUESTIONS WITH ANSWERS

Q.1. If velocity (V), acceleration (A) and force (F) are assumed as fundamental quantities in place of mass (M), length (L) and time (T) respectively, what will be the dimensional formula of Young's modulus?

Ans. Let Young's modulus is expressed as follows:

$$Y \propto V^a A^b F^c$$

$$[ML^{-1}T^{-2}] = [LT^{-1}]^a [LT^{-2}]^b [MLT^{-2}]^c$$

$$[ML^{-1}T^{-2}] = [M^c L^{a+b+c} T^{-a-2b-2c}]$$

Comparing dimensions on both the sides we get the following:

$$c = 1 \quad \ldots(i)$$

$$a + b + c = -1$$

$$\Rightarrow a + b + 1 = -1$$

$$\Rightarrow a + b = -2 \quad \ldots(ii)$$

$$-a - 2b - 2c = -2$$

$$\Rightarrow -a - 2b - 2 = -2$$

$$\Rightarrow a + 2b = 0 \quad \ldots(iii)$$

Subtracting equation (ii) from equation (iii) we get

$$b = 2 \quad \ldots(iv)$$

Substituting equation (iv) in equation (ii) we get

$$a = -4 \quad \ldots(v)$$

Using values of a, b and c we can write the dimensional formula for Young's modulus as follows:

$$[Y] = [V^{-4}A^2F^1]$$

Q.2. An object kept inside the water starts oscillating with time period T when an explosion occurs inside water. It is found that $T \propto P^a \rho^b E^c$. What are the values of a, b and c? P is pressure, ρ is density of water and E is energy of explosion.

Ans. $$T \propto P^a \rho^b E^c$$

$$\Rightarrow [T] = [P]^a [\rho]^b [E]^c$$

$$\Rightarrow [M^0L^0T] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [ML^2T^{-2}]^c$$

$$\Rightarrow [M^0L^0T] = [M^{a+b+c}L^{-a-3b+2c}T^{-2a-2c}]$$

On comparing individual dimensions we can write the following equations:

$$a + b + c = 0 \quad \ldots(i)$$

$$-a - 3b + 2c = 0 \quad \ldots(ii)$$

$$-2a - 2c = 1 \quad \ldots(iii)$$

On solving above equations we get the following:

$$a = -5/6,\ b = \tfrac{1}{2},\ \text{and}\ c = 1/3.$$

Q.3. If energy E, velocity V and time T are taken as fundamental units, then what will be the dimensional formula of surface tension? Note that surface tension is defined as force per unit length.

Ans. Let surface tension (S) is written as follows:

$$S \propto E^x V^y T^z$$

$$\Rightarrow [S] \propto [E]^x [V]^y [T]^z$$

$$\Rightarrow [ML^0T^{-2}] = [ML^2T^{-2}]^x [LT^{-1}]^y [T]^z$$

$$\Rightarrow [ML^0T^{-2}] = [M^x L^{2x+y} T^{-2x-y+z}]$$

On comparing dimensions on both the sides we get the following equations:

$$x = 1$$

$$2x + y = 0 \Rightarrow y = -2x \Rightarrow y = -2$$

$$-2x - y + z = -2 \Rightarrow -2 + 2 + z = -2 \Rightarrow z = -2$$

Hence, the dimensional formula for surface tension is:

$$[S] = [EV^{-2}T^{-2}]$$

Q.4. In a new system of units, value of speed of light ($c = 3 \times 10^8$ m/s), Universal gravitational constant ($G = 6.67 \times 10^{-11}$) and acceleration due to gravity ($g = 9.8$ m/s^2) are found to be unity. What are the units of mass, length and time in this new system of units?

Ans. Let M_1, L_1 and T_1 represent fundamental units in MKS system and M_2, L_2 and T_2 represent the same in new system of units. Let us assume that $M_2 = x\ M_1$, $L_2 = y\ L_1$ and $T_2 = z\ T_1$.

Dimensional formula for speed of light is LT^{-1}. Let n_1 and n_2 be the values for speed of light in MKS and new system respectively.

$$n_1 u_1 = n_2 u_2$$

$$\Rightarrow (n_1)[L_1T_1^{-1}] = (n_2)[L_2T_2^{-1}]$$

$$\Rightarrow (n_1) = (n_2)\left[\frac{L_2}{L_1}\right]\left[\frac{T_2^{-1}}{T_1^{-1}}\right]$$

$$\Rightarrow (n_1) = (n_2)\left[\frac{L_2}{L_1}\right]\left[\frac{T_1}{T_2}\right]$$

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$$\Rightarrow \quad (3\times10^{8}) = (1)(y)\left(\frac{1}{z}\right)$$

$$\Rightarrow \quad \frac{y}{z} = 3\times10^{8} \qquad \ldots(i)$$

Dimensional formula for Gravitational constant is $M^{-1}L^{3}T^{-2}$. Let n_1 and n_2 be the values for gravitational constant in MKS and new system respectively.

$$n_1u_1 = n_2u_2$$

$$\Rightarrow (n_1)[M_1^{-1}L_1^{3}T_1^{-2}] = (n_2)[M_2^{-1}L_2^{3}T_2^{-2}]$$

$$\Rightarrow \quad (n_1) = (n_2)\left[\frac{M_2^{-1}}{M_1^{-1}}\frac{L_2^{3}}{L_1^{3}}\frac{T_2^{-2}}{T_1^{-2}}\right]$$

$$\Rightarrow \quad (n_1) = (n_2)\left[\frac{M_1}{M_2}\right]\left[\frac{L_2}{L_1}\right]^{3}\left[\frac{T_1}{T_2}\right]^{2}$$

$$\Rightarrow \quad (6.67\times10^{-11}) = (1)\left(\frac{1}{x}\right)(y^{3})\left(\frac{1}{z^{2}}\right)$$

$$\Rightarrow \quad \frac{y^{3}}{xz^{2}} = 6.67\times10^{-11} \qquad \ldots(ii)$$

Dimensional formula for acceleration due to gravity LT^{-2}. Let n_1 and n_2 be the values for acceleration due to gravity in MKS and new system respectively.

$$n_1u_1 = n_2u_2$$

$$\Rightarrow \quad (n_1)[L_1T_1^{-2}] = (n_2)[L_2T_2^{-2}]$$

$$\Rightarrow \quad (n_1) = (n_2)\left[\frac{L_2}{L_1}\frac{T_2^{-2}}{T_1^{-2}}\right]$$

$$\Rightarrow \quad (n_1) = (n_2)\left[\frac{L_2}{L_1}\right]\left[\frac{T_1}{T_2}\right]^{2}$$

$$\Rightarrow \quad (9.8) = (1)(y)\left(\frac{1}{z^{2}}\right)$$

$$\Rightarrow \quad \frac{y}{z^{2}} = 9.8 \qquad \ldots(iii)$$

On dividing equation (iii) from equation (i) we get the following:

$$z = \frac{3\times10^{8}}{9.8}$$

$$\Rightarrow \quad z = 3.06\times10^{7} \qquad \ldots(iv)$$

Substituting the value of z from equation (iv) in equation (i) we get the following:

$$\frac{y}{3.06\times10^{7}} = 3\times10^{8}$$

$$\Rightarrow \quad y = 9.18\times10^{15} \qquad \ldots(v)$$

Now substituting the values of y and z from equation (iv) and (v) in equation (ii) we get the following:

$$x = \frac{(9.18\times10^{15})^{3}}{(6.67\times10^{-11})(3.06\times10^{7})^{2}}$$

$$\Rightarrow \quad = \frac{773.62\times10^{45}}{(6.67\times10^{-11})(9.364\times10^{14})}$$

$$\Rightarrow \quad x = 12.4\times10^{42} \qquad \ldots(vi)$$

Now from results in equations (iv), (v) and (vi) we can describe the new system of units as follows:

Unit of mass = 12.4×10^{42} kg.

Unit of length = 9.18×10^{15} m.

Unit of time = 3.06×10^{7} s.

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Revision Exercises

Very Short Answer Questions *Carrying 1 mark*

1. What are physical quantities?
2. Define unit.
3. What are fundamental units and derived units?
4. Is it possible for a body to have zero mass?
5. What is length? What is its SI unit?
6. Define SI unit of luminous intensity.
7. Define SI unit of thermodynamic temperature.
8. What is one Ampere?
9. Define Chandrasekhar limit.
10. How is one second measured?
11. What are different systems of units used for measuring all kinds of physical quantities?
12. Why are units required?
13. What is the smallest measured mass?
14. Why are SI units required?
15. Define one light year.
16. What is one astronomical unit? Why is it required?
17. Define one parsec.
18. How many astronomical units are equal to one light year?
19. How many radians are there in one minute?
20. What is angular diameter of the sun and the moon as seen from earth?

21. A rectangle has an area equal to 7.654 cm^2. If its breadth is 2.3 cm, then calculate the length in accordance with the rules of significant figures.
22. What is an electron microscope?
23. How many unified atomic mass units are there in one kg?
24. What is radioactive dating?
25. What is an atomic clock? How does it work?
26. What is the accuracy of a cesium clock?
27. Express giga in terms of femto.
28. Define error in a measurement of any physical quantity.
29. Define random errors.
30. What are systematic errors?
31. What are instrumental errors? How can they be reduced?
32. How can we reduce systematic errors?
33. What is least count error? How can we reduce it?
34. Round off 1.00876 to four significant digits.
35. Define gross errors. Give example.
36. How can we define fractional and percentage error?
37. How is absolute error different from mean absolute error?
38. What do we mean by accuracy?
39. Define precision.
40. Which of the following physical quantity should be measured more accurately in the expression $X = \frac{h^2 y^3 z}{b}$ for most accurate results?
41. How many significant digits are there in 0.007?
42. How many significant digits are there in 82450000?
43. Which of the two readings is more accurate –7.0 or 7.00?
44. How can we reduce least count of any instrument?
45. Can we reduce the permissible error in a result using same instrument?
46. What are dimensions of physical quantities? How do we represent them?
47. How is dimensional formula related to dimensional equation?
48. What is the dimension for Reynold number?
49. Is it possible for a constant to have dimensions?
50. What is the dimensional formula of electric permittivity of free space?

Additional Questions *Carrying 1 mark*

51. The number of significant digits in 0.002 is
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4
52. In expression $m = 3zx^3$, the significant figures in factor 3 are
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) ∞
53. Which of the following has largest value
(*a*) One astronomical unit (*b*) One parsec
(*c*) One light year (*d*) One light minute
54. Dimensional formula for Joule's mechanical equivalent of heat is
(*a*) $[M^{-1}L^{-3}T^4A^2]$ (*b*) $[M^0L^0T^0A^0]$
(*c*) $[M^{-1}LTA^{-1}]$ (*d*) $[M^{-1}L^{-1}T^4]$

Fill in the Blanks *Carrying 1 mark*

1. Planck's constant is a quantitiy which has dimensions …… .
2. The coefficient of elasticity has dimensions of …… .
3. Momentum per unit volume has same dimensions as the …… .
4. The percentage error in the determination of $g = 4\pi^2 l/T^2$ is …… % when l and T are measured with 2% and 3% errors respectively.
5. On adding 2.375 to 1.23, the correct result with regard to significant figures is …… .
6. …… errors are those errors which occur irregularly and by …… .
7. 8.0505 grams of a substance occupies a volume of 2.3 cm^3. The density of the substance with regard to significant figures is …… .
8. Device used for measuring the mass of atoms and molecules is …… .
9. Accuracy of screw gauge can be increased by …… the number of divisions on circular scale.
10. The number of significant figures in 0.080 is ……

Short Answer Questions *Carrying 2 or 3 marks*

1. What are some basic characteristics of standard units?
2. Mention some advantages of SI system of units.
3. Mention drawback of C.G.S. system of units?
4. Derive a relation between one parsec and one light year.
5. Calculate the number of light minutes in one metre?
6. Why is length a fundamental unit?
7. How many light minutes are there in one AU?
8. Mention some important conventions to be followed while writing a unit.
9. What is parallax method? How is it used to measure large distances?
10. Why angular diameter of the sun and the moon in the sky is same?
11. Convert 1° and 1′ to radians.
12. Length, mass and time are fundamental units. Explain.
13. How many microseconds are there in milliseconds?
14. How many femto metres are there in one A.U.?
15. Give one example of the following:
(*a*) Physical quantity that is dimensionless but has a unit
(*b*) A constant having dimensions

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16. Explain three types of errors observed in measurement.
17. Write mathematical expression for absolute error and mean absolute error.
18. Two identical resistors of resistance $(2 \pm 0.1)\ \Omega$ are connected in series. Calculate the effective resistance along with the percentage error.
19. The diameters of two cylinders are $d_1 = (2.02 \pm 0.01)$ cm and $d_2 = (3.21 \pm 0.03)$. Find the difference between their diameters along with the error limits.
20. Differentiate between precision and accuracy with an example.
21. Solve the following with regard to significant figures.
    (*a*) $781 + 0.5321$ (*b*) $9.0 - 0.521$
22. Compute the following with regard to significant figures.
    (*a*) 5.2×0.135 (*b*) $\dfrac{0.8851 \times 1.23}{1.985}$
23. For physical quantity z, given by expression $z = \dfrac{gh^{\frac{1}{2}}I}{mx^3}$, write the expression for relative error in it.
24. How can we minimise the random errors in an experiment?
25. Define dimensional equations and give their uses.
26. Express Newton in terms of dyne.
27. Check the dimensional consistency of the equation $t = 2\pi\sqrt{\dfrac{l}{g}}$, where t, l, g are time period, length and acceleration respectively.
28. Find the dimensions of Z in the given expression $Z = \dfrac{2C}{A^2}$. Given dimension for C and A are $[ML^{-1}T^{-2}]$ and $[MLT^{-3}A^{-1}]$.

Long Answer Questions *Carrying 5 marks*

1. Write a short note on standard international units. Mention their importance.
2. How do we measure the standard unit for mass, length and time? Explain why they are also called fundamental units.
3. Describe some methods to measure time of different ranges.
4. What are some limitations of dimensional analysis?
5. Write a short note on applications and uses of dimensional analysis.
6. Define error in measurement. What are different types of possible errors which occur in experiments?
7. Briefly explain the causes of systematic errors.

Numerical Problems

1. Express the given angles into radian.
   (*a*) 1° (*b*) 12 sec
2. A distant star is observed from two diametrically opposite points on earth. If both the points are at distance 1.27×10^7 m and angle subtended by both the points at star is 0.80″, calculate the distance between earth and the star.
3. The diameter of the moon is 3474 km and is 384,400 km away from the earth. Calculate the angular diameter of the moon in arc minutes.
4. In an experiment, the time period measurements for an oscillating simple pendulum are 2.10 s, 2.12 sec, 2.23, 2.61 sec and 2.91 sec. Find the relative error and absolute error in the experiment.
5. In Ohm's law experimental verification, the reading of voltage and resistance are (120 ± 5) V and (2 ± 0.1) Ω respectively. What is the percentage error in the value of current obtained?
6. Two identical resistors of resistance 100 ± 5 ohm are connected parallel. Calculate the equivalent resistance of the combination.
7. The diameter of two cylinders are found to be $d_1 = (2 \pm 0.01)$ cm and $d_2 = (3.1 \pm 0.03)$ cm. Find the difference between both the diameters along with the error limits.
8. Gravitational force between two objects with mass m_1 and m_2 kept r distance apart is given as $F_G = \dfrac{Gm_1m_2}{r^2}$, where G is gravitational constant. Express the relative error in F_G.
9. Calculate the volume of 2.101 cm cylinder with radius 1.123 cm in appropriate significant figures.
10. Check the dimensional consistency of the equation $\dfrac{mV^2}{r} = \dfrac{Gm_1m_2}{r^2}$.
11. An oil drop of radius 0.10 mm spreads on a circular film of diameter 20 cm. Calculate the molecular size of the oil drop on the film.
12. Calculate the percentage error in the volume of a cube if error in the measurement of its each side is 12%.
13. State the number of significant figures in the following measurements:
    (*a*) 2.80×10^8 (*b*) 8.034
    (*c*) 0.0009
14. Round off the given numbers up to 3 significant digits.
    (*a*) 18.65 (*b*) 429253
    (*c*) 13.653
15. Deduce the expression for centripetal force F acting on an object of mass m moving with velocity v in a circle of radius r.
16. For a given expression of power, $P = \dfrac{a}{bx} - \dfrac{t^2}{b}$. Determine the dimensions of a and b. Here, x and t are distance and time respectively.
17. For a planet of mass M, moving around the sun in an orbit of radius r, time period T depends on its radius (r), mass M and universal gravitational constant G and can be written as: $T^2 = \dfrac{K^x r^y}{MG}$. Find the value of y.

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Hints & Answers for Revision Exercises

Very Short Answer Questions

1. A physical quantity is a quantity or a physical property in physics that can be quantified or measured. For example, mass, volume, force, electric current, etc.

2. A basic and internationally accepted reference standard for a physical quantity is known as unit. Measurement of a quantity is expressed by a numerical value accompanied by a unit. For example: 5 kg of mass, where 5 is the numerical value and kg is the unit of mass.

3. Very few physical quantities (independent of each other) are used to express all the other physical quantities. These are known as fundamental or base quantities. The units for the fundamental quantities are known as fundamental units. For example: Mass, length and time with kg, m and s as their units.

The units of all the other physical quantities (derived quantities) can be expressed in terms of fundamental quantities. These units are known as derived units. For example: Velocity is a derived quantity with unit m/s.

4. No

5. Length is generally a measured dimension of an object or distance between two points. Its SI unit is metre (m).

6. The SI unit of luminous intensity is candela (cd). It is defined as the luminous intensity, in a given direction of a source that emits monochromatic radiation of frequency 540×10^{12} Hz and has a radiant intensity in that direction of 1/683 watt per steradian.

7. The SI unit of thermodynamic temperature is kelvin (K). It is defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

8. One ampere is defined as that constant current which, if maintained in two straight parallel conductors of infinite length, negligible circular cross section and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{7} Newton per metre of length.

9. It is the largest possible value of mass of a white dwarf star which is equal to nearly 1.4 times the mass of the sun.

10. One second is measured as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

11. SI system of units, CGS system of units, MKS system of units and FPS system of units.

12. Measurement of a quantity is significant only if there is some reference with which you can compare the measured value. Without units, all the numerical statements for a physical quantity are insignificant. A unit is thus required to act as a reference standard for a quantity to be measured.

13. Smallest measured mass is the mass of an electron ($\approx 10^{-31}$ kg).

14. SI units are required in order to maintain a consistency in international usage of units for scientific, technical, industrial and commercial work. Also, the system is quite simple and convenient and allows hassle-free conversions.

15. The distance travelled by the light in 1 year, with a velocity of 3×10^{8} m/s, is known as one light year. It is equal to 9.46×10^{15} m.

16. It is equal to the average distance of the sun from the earth and is equal to 1.496×10^{11} m. It is required to express interstellar distances conveniently.

17. One parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second. It is equal to 3.08×10^{16} m.

18. 1 light year has approximately 6.3×10^{4} AU.

19. 180° degrees = π radians or 1° = π/180 rad. 1 minute (1') = 1/60 degrees = 1/60 × π/180 rad = π/10,800 rad

20. Angular diameter of the sun from earth = 0.5° = 30 arc minute. Angular diameter of the moon from earth is similar to that for the sun = 0.5°.

21. 3.3

22. An electron microscope is a microscope with high resolving power and magnification and uses a beam of electrons to create an image of a specimen.

23. 1 kg = 6.02×10^{26} u

24. It is a technique which is used to compare the abundance ratio of a radioactive isotope of an element to a reference isotope in order to determine the age of a rock or material.

25. An atomic clock is the clock which uses atomic frequencies of atoms (particularly caesium –133) as its resonator. They are the most precise time-measuring instruments so far.

26. It has an accuracy of about 1 second in 1.4 million years.

27. 1 giga = 10^{9} a and 1 femto = 10^{-15} a, where a is a reference standard. So 1 giga = $10^{9}/10^{-15} = 10^{24}$ femto.

28. The uncertainty in measurement of any physical quantity by any instrument is known as error.

29. Random errors are those errors which occur irregularly and hence are random with respect to sign and size.

30. Systematic errors are the unidirectional errors which are either positive or negative. They include errors due to instrument, personal errors while measurement, least count error, etc.

31. Instrumental errors are associated with the instrument due to its design or calibration. It can be reduced by replacing the instrument by more accurate instrument or by using zero correction.

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32. Systematic errors can be reduced by using instruments with less error, adapting efficient or errorless techniques and by minimising possible personal errors.

33. Least count error is associated with the resolution of the instrument used in measurement. It can be reduced by replacing the instrument with a higher resolution instrument and adapting better experimental techniques.

34. 1.009

35. Gross errors are those which occur due to carelessness of the observer. These errors include false methods or set ups, wrong readings, ignoring other sources of errors, etc.

36. Fractional error is ratio of mean absolute error of a quantity to mean value of the quantity. It can be further expressed into percentage and is then known as percentage error.

37. Absolute error is the magnitude of the difference between the measured value and the original value. However, the mean absolute error is the mean of magnitudes of the absolute errors in all the measurements throughout the experiment.

38. Accuracy is the measure of the closeness of the measured value to the original value.

39. Precision in any experiment depends on the least count of the instrument used and refers to the closeness of two or more measurements to each other.

40. y has the highest power, thus it should be measured accurately for most accurate results.

41. one

42. four

43. 7.00 is measured to two decimal places and hence is more accurate than 7.0

44. To reduce least count, we can increase the resolution of scale.

45. No, they can be only decreased by changing instrument with less error.

46. Dimensions are the powers to which the unit of mass, length and time are raised to represent the physical quantity.

47. Dimensional formula of a physical quantity is representation of mass, length and time of the quantity. Dimensional equation contains equated dimensional formula and physical quantity.

48. it is a dimensionless quantity.

49. yes, gravitational constant has dimensions.

50. $[M^{-1}L^{-3}T^4A^2]$

Additional Questions

51. (a) **52.** (d) **53.** (b) **54.** (b)

Fill in the Blanks

1. $[M^1L^2T^{-1}]$ **2.** stress or force/Area **3.** Pressure
4. 8 **5.** 3.61 s **6.** Random, chance
7. $3.5 g/cm^3$ **8.** Mass spectrometer **9.** increasing
10. 2

Short Answer Questions

4. 1 parsec $= 3.08 \times 10^{16}$ m

1 light year $= 9.46 \times 10^{15}$ m

$$\therefore 1 \text{ Parsec} = \frac{9.46 \times 10^{15}}{3.08 \times 10^{16}} = 3.26 \text{ light year}$$

5. Speed of light $= 3 \times 10^8$ ms^{-1}

time = 1 minute = 60 sec.

$\therefore$ One light minute $= 3 \times 10^8 \times 60$ m

$= 1.8 \times 10^{10}$ m

7. 1 A.U. $= 1.5 \times 10^{11}$ m

1 light minute $= 1.8 \times 10^{10}$ m

$\therefore$ 1 A.U. = 8.33 light minutes

15. (a) Angle, it has no dimensions but it has unit, radian

(b) Velocity of light in vacuum, $c = 3 \times 10^8$ m/s, dimensional formula: $[LT^{-1}]$

18. Given $R_1 = (2 \pm 0.1)\ \Omega$

$R_2 = (2 \pm 0.1)\ \Omega$

$R' = R_1 + R_2$ (series connection)

$\therefore$ $R' = 2 + 2 = 4\ \Omega$

$\Delta R' = \pm(\Delta R_1 + \Delta R_2)$

$\Rightarrow$ $\Delta R' = \pm(0.1 + 0.1) = \pm 0.2$

$$\text{Percentage error} = \pm\frac{0.2}{4} \times 100$$

$= \pm 5\%$

$\therefore$ $R' = (4 \pm 0.2)\Omega = (4\Omega \pm 5\%)$

19. Given, $d_1 = (2.02 + 0.01)$ cm

$d_2 = (3.21 \pm 0.03)$ cm

$d = d_2 - d_1 = 3.21 - 2.02 = 1.19$ cm

$\Delta d = \pm(\Delta d_1 + \Delta d_2) = \pm(0.01 + 0.03)$

$= \pm 0.04$ cm

$$\% \text{ age error} = \pm\frac{0.04}{1.19} \times 100 = \pm 3.36\%$$

$\therefore$ Difference in diameters $= (1.19 \pm 0.04)$ cm

or $(1.19 \text{ cm} \pm 3.36\%)$

Numerical Problems

1. (a) We know, (2π) rad $= 360°$

$$\therefore \quad 1° = \left(\frac{\pi}{180}\right)\text{rad}$$

$$\Rightarrow \quad 1° = 1.74 \times 10^{-2} \text{ rad}$$

(b) $1° = 60' = 3600''$

$$\Rightarrow \quad 1'' = \frac{1°}{3600}$$

$$\Rightarrow \quad 12'' = 12 \times \left(\frac{1}{3600}\right)^\circ$$

$$\Rightarrow \quad 12'' = 12 \times \frac{1}{3600} \times 1.74 \times 10^{-2} \text{ rad}$$

$$\cong \mathbf{5.8 \times 10^{-5}\ rad}$$

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2. Given, $\theta = 0.80'' = 0.80 \times 4.85 \times 10^{-6}$ rad

(using $1'' = 4.85 \times 10^{-6}$ rad)

$\theta = 3.88 \times 10^{-6}$ rad

Distance between both points, $d = 1.27 \times 10^7$ m

$\therefore$ Distance between earth and star can be calculated as:

$$D = \frac{d}{\theta} = \frac{1.27 \times 10^7}{3.88 \times 10^{-6}}$$

$$\Rightarrow \quad D = 0.327 \times 10^{13}$$

$$= 0.33 \times 10^{13} \text{ m}$$

$$D = \mathbf{3.3 \times 10^{12}\ m}$$

3. Given diameter of the moon, $d = 3474$ km

distance between earth and the moon, $D = 384,400$ km

angular diameter of the moon, $\alpha = \frac{d}{D} = \frac{3474}{384400}$

$$\Rightarrow \quad \alpha = 9.03 \times 10^{-3} \text{ rad}$$

$$1 \text{ rad} = \left(\frac{1}{2.91 \times 10^{-4}}\right)'$$

$$\therefore \quad \alpha = \left(\frac{9.03 \times 10^{-3}}{2.91 \times 10^{-4}}\right)'$$

$$\therefore \quad \alpha = 3.103 \times 10'$$

$$= \mathbf{31.03'}$$

4. Given readings are:

$T_1 = 2.10$ s

$T_2 = 2.12$ s

$T_3 = 2.23$ s

$T_4 = 2.61$ s

$T_5 = 2.91$ s

Mean time period of $T = \frac{(T_1 + T_2 + T_3 + T_4 + T_5)}{5}$

$$\Rightarrow \quad T = \frac{(2.10 + 2.12 + 2.23 + 2.61 + 2.91)}{5}$$

$$\Rightarrow \quad T = \frac{11.97}{5} = 2.394 \text{ s} = 2.39 \text{ s}$$

The errors in each time period will be

$\Delta T_1 = T_1 - T = 2.10 - 2.39 = -0.29$

$\Delta T_2 = T_2 - T = 2.12 - 2.39 = -0.27$

$\Delta T_3 = T_3 - T = 2.23 - 2.39 = -0.16$

$\Delta T_4 = T_4 - T = 2.61 - 2.39 = 0.22$

$\Delta T_5 = T_5 - T = 2.91 - 2.39 = 0.52$

Mean of absolute errors can be calculated as:

$$\Delta T_{mean} = \frac{(0.29 + 0.27 + 0.16 + 0.22 + 0.52)}{5}$$

$$\Delta T_{mean} = 0.29 \text{ s}$$

Thus, $T = 2.3 \pm 0.2$

Relative error can be calculated as:

$$\delta_T = \frac{0.2}{2.3} \times 100\% = \mathbf{8.69\%}$$

5. $$I = \frac{V}{R} \Rightarrow \frac{\Delta I}{I} = \frac{\Delta V}{V} + \frac{\Delta R}{R}$$

Percentage error in $V = \frac{5}{120} \times 100\% = 4.17\%$

Percentage error in $R = \frac{0.1}{2} \times 100\% = 5\%$

Percentage error in value of current will be

$4.17\% + 5\% = \mathbf{9.17\%}$

6. Given, $R = (100 \pm 5)\ \Omega$

In parallel combination

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

or $$\frac{\Delta R'}{(R')^2} = \frac{2\Delta R}{R^2}$$

$$\therefore \quad \Delta R' = \frac{(R')^2 \times 2 \times \Delta R}{(R)^2}$$

$$\Rightarrow \quad \Delta R' = \frac{\left(\frac{R}{2}\right)^2 \times 2\Delta R}{(R)^2}$$

$$\Rightarrow \quad \Delta R' = \frac{\left(\frac{100}{2}\right)^2 \times 2 \times 5}{(100)^2}$$

$$\Rightarrow \quad \Delta R' = 2.5$$

$$\therefore \quad R' = \frac{100}{2} \pm 2.5$$

$$\Rightarrow \quad R' = \mathbf{(50 \pm 2.5)\ ohm}$$

7. Given, $d_1 = (2 \pm 0.01)$ cm

$d_2 = (3.1 \pm 0.03)$ cm

difference, $d' = d_2 - d_1 = 3.1 - 2 = 1.1$ cm

$\Delta d_1 = \pm(\Delta d_2 - \Delta d_1) = \pm(0.01 + 0.03)$

$\Delta d_1 = \pm 0.04$

$\therefore$ Percentage error $= \frac{\pm 0.04}{1.1} \times 100 = 3.636\%$

Thus difference between diameter $= (1.1 \pm 0.04)$ cm

$= \mathbf{1.1\ cm \pm 3.64\%}$

8. Here, $$F_G = \frac{Gm_1m_2}{r^2}$$

$$\Rightarrow \quad \frac{\Delta F}{F} = \frac{\Delta m_1}{m_1} + \frac{\Delta m_2}{m_2} + 2\left(\frac{\Delta r}{r}\right)$$

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9. Volume of cylinder, $V = \pi r^2 h$

here $r = 1.123$ cm

$h = 2.101$ cm

$V = 3.14 \times (1.123)^2 \times 2.101$

$= 8.31984$

$\Rightarrow$ $V = \mathbf{8.31984\ cm^3 = 8.320\ cm^3}$

(four significant digits)

10. Dimensions of L.H.S. are

$[M]\ [LT^{-1}]^2\ [L]^{-1} = [MLT^{-2}]$

Dimensions of R.H.S. are

$[L^3M^{-1}T^{-2}]\ [M]\ [M]\ [L^{-2}]$

$= LMT^{-2} = [MLT^{-2}]$

Since the L.H.S. dimensions are equal to the R.H.S. dimensions, the given equation is dimensionally consistent.

11. Given, radius of oil drop, $r = 0.10$ mm $= 0.010$ cm

Radius of film, $R = 10$ cm

$$\text{Thickness of oil film} = \frac{\text{Volume of oil drop}}{\text{Area of the film}}$$

$$= \frac{\frac{4}{3}\pi \times (0.010)^3\ \text{cm}^3}{\pi \times (10)^2\ \text{cm}^2}$$

$$= \frac{4}{3} \times (10)^3 \times 10^{-11}\ \text{cm}$$

$$= \mathbf{1.33 \times 10^{-8}\ cm}$$

Considering the film with one molecular thickness, the molecular size of the drop will be 1.33×10^{-8} cm.

12. Given, percentage error in its each side = 12%

or $\frac{\Delta a}{a} \times 100 = 12\%$

Volume of cube, $V = a^3$

$\therefore$ Percentage error in volume of the cube is

$$\frac{\Delta V}{V} \times 100 = 3\left(\frac{\Delta a}{a} \times 100\right) = 3 \times 12\% = \mathbf{36\%}$$

13. (*a*) Number of significant digits in 2.80×10^8 is three, i.e. 2, 8 and 0.

(*b*) Number of significant digits in 8.034 is four, i.e. 8, 0, 3 and 4.

(*c*) Number of significant digits in 0.0009 is one, i.e. 9.

14. (*a*) 18.6 (*b*) 429000 (*c*) 13.6

15. Consider centripetal force $F = km^a v^b r^c$, here k is proportionality constant and is dimensionless.

Equating for dimensions on both the sides, we get

$M^1L^1T^{-2} = [M]^a[LT^{-1}]^b[L]^c$

$= M^aL^{b+c}T^{-b}$

Comparing the exponents of M, L and T on both sides, we get

$a = 1,\ b + c = 1$ and $b = 2$

$\Rightarrow$ $c = 1 - 2 = -1$

$\therefore$ $F = \dfrac{\boldsymbol{mv^2}}{\boldsymbol{r}}$

This is the required expression for centripetal force.

16. As we know, dimensions of power can be given as:

$[P] = [ML^2\ T^{-3}]$

Also, $\left[\frac{t^2}{b}\right] = [P]$ and $\left[\frac{a}{bx}\right] = [P]$

$\Rightarrow$ $\frac{[T]^2}{[b]} = [ML^2\ T^{-3}]$

$\Rightarrow$ $[b] = \frac{[T]^2}{[ML^2\ T^{-3}]} = [M^{-1}L^{-2}\ T^5]$

And, $\left[\frac{a}{bx}\right] = [P]$

$\Rightarrow$ $[a] = [P]\ [bx]$

$= [ML^2T^{-3}]\ [M^{-1}L^{-2}\ T^5]\ [L]$

$= [M^0L^1T^2]$

$\therefore$ $[a] = [M^0L^1T^2]$ and $[b] = \mathbf{[M^{-1}L^{-2}T^5]}$

17. Consider time period, $T = Kr^aM^bG^c$. Here, K is dimensionless constant. Equating dimensions on both the sides, we get

$[M^0L^0T^1] = [L]^a[M]^b[M^{-1}L^3T^{-2}]^c$

Comparing the exponents of M, L and T on both sides, we get

$\Rightarrow$ $b - c = 0,$

$\Rightarrow$ $a + 3c = 0,$

and $-2c = 1$

$\Rightarrow$ $c = \frac{-1}{2}$

$b = c = \frac{-1}{2}$

and $a = -3c = -3 \times \left(\frac{-1}{2}\right) = \frac{3}{2}$

$\therefore$ $T = Kr^{3/2}\ M^{-1/2}\ G^{-1/2}$

or $T^2 = \dfrac{\boldsymbol{K^2r^3}}{\boldsymbol{MG}}$

Thus value of y is 3.

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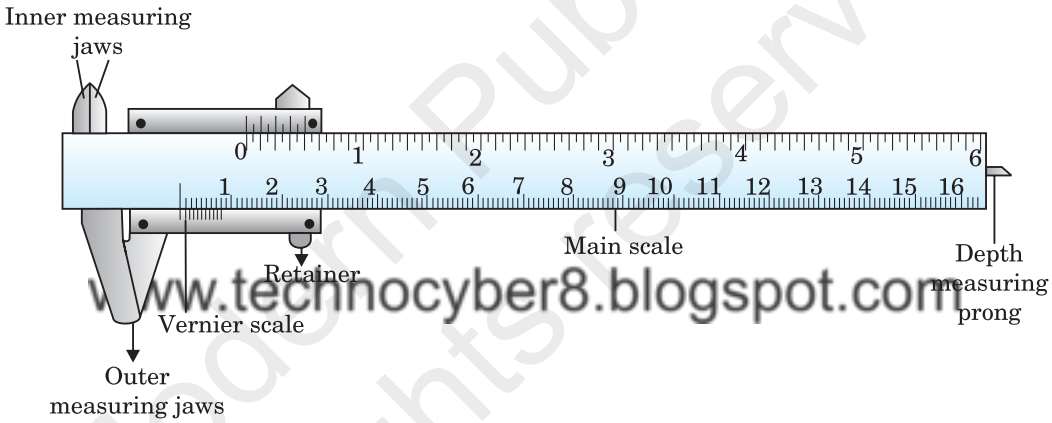
Competition File

Additional Useful Information and Objective Questions

ADDITIONAL USEFUL INFORMATION

VERNIER CALIPERS

Vernier caliper is a device to measure the distance between two points. It is designed to help measure dimensions of objects. We can measure length of a certain object, diameter of a cylinder, depth of hole, etc. There is one main scale which is like some ordinary scale and in addition to that there is Vernier scale to add more precision to the measurement. Two jaws are there to hold the object in between. One jaw is fixed with the main scale and the other is fixed with the Vernier scale, which in turn slides on the main scale. When both the jaws are in contact then zero of the main scale matches with zero of Vernier scale. When both the jaws are in contact and zero of the main scale does not match with zero of Vernier scale then we say that there is zero error in the device. When there is zero error in the instrument then it affects final reading accordingly. At present we are assuming that there is no zero error in the instrument.


Inner measuring jaws
0 1 2 3 4 5 6
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Main scale
Depth measuring prong
Retainer
Vernier scale
Outer measuring jaws

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Markings on main scale are made at regular separation and in most cases it is 1 mm. Hence each division of main scale corresponds to a particular value and we are assuming it to be 1 mm. On Vernier scale ten divisions are made in such a manner that these 10 divisions of Vernier scale match with 9 divisions of main scale. So we can understand that each division of Vernier scale is shorter than each division on main scale, by an amount equivalent to one tenth of main scale division. If main scale division is 1 mm then division on Vernier scale is shorter by 0.1 mm than division on main scale. Difference in length of main scale division and Vernier scale division is called least count of Vernier calipers. Least count of instrument in above explained case is 0.1 mm. ***Least count (LC) is the minimum value which can be measured accurately by the instrument***.

Least count = Length of one main scale division
– Length of one vernier scale division

LC = 1 MSD – 1 VSD

Now assume that zero of Vernier scale matches with zero of main scale. We know that each division in Vernier scale is smaller by an amount 0.1 mm (LC) than that of main scale. When Vernier is moved by an amount 0.1 mm then first mark of Vernier scale matches with the first mark of main scale. Similarly when Vernier scale is moved by 0.2 mm then 2nd mark of Vernier scale matches with second mark on main scale and so on. Hence by finding which mark on Vernier scale is matching with a mark on main scale, we can measure the length with a precision of 0.1 mm.

When we measure a length by holding corresponding object between the jaws then reading is taken in two steps.

Main scale reading (MSR): Value corresponding to the mark on main scale, which is nearest to the zero of Vernier scale on left side is called ***main scale reading***.

Vernier scale reading (VSR): If n^{th} division of Vernier scale coincides with some division on main scale, then Vernier scale reading is $n \times$ LC.

Total reading = MSR + VSR

$\Rightarrow$ Total reading = MSR + $n \times$ Least Count

Competition File

Inner measuring jaws are provided to measure inner dimensions of an object and depth measuring prong is provided to measure depth of an object. Retainer is used to restrict the movement of jaws after holding the object appropriately.

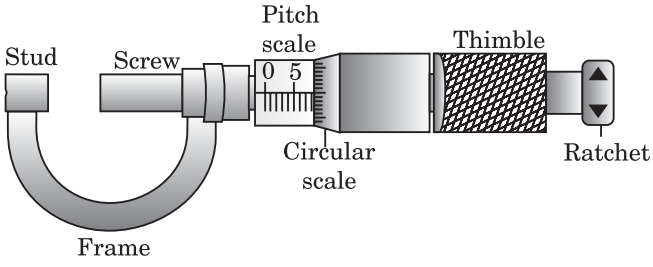
When two jaws of the Vernier are brought in contact then zero of the Vernier scale should match with zero of main scale. If it is so then there is no zero error in the instrument or else we need to take care of zero error also.

When the two jaws are in contact and zero of the Vernier scale lies to the right of zero of main scale then it is called positive zero error. When jaws are in contact and n^{th} division of Vernier scale matches with some division of main scale then positive zero error is equal to $n \times$ LC. In case of positive error, reading taken by the instrument is more than the actual and thus zero correction is negative. Hence positive zero error is subtracted from the final reading.

When the two jaws are in contact and zero of the Vernier scale lies to the left of zero of main scale then error is called negative error. Zero correction is positive. If N is total number of divisions on Vernier scale and n^{th} division of Vernier scale matches with some division of main scale then negative zero error can be calculated as $(N - n) \times$ LC. In case of negative error, reading taken by the instrument is less than the actual and thus zero error is added to the final reading.

SCREW GAUGE

Screw gauge is also used to measure distance between two points with more precision than Vernier calipers. It can be used to measure the diameter of a thin wire or thickness of thin metal sheet, etc.



Screw gauge consists of a U-shaped frame holding a screwed rod which is attached to a thimble at one end. This screwed rod can be moved along the axis of thimble by rotating it. A stud with plane end is fitted with U-shaped frame and when thimble is rotated then end of the screwed rod comes in contact with the stud. In fact, this state corresponds to zero reading. A scale is fixed with the U-frame parallel to the axis of thimble and this scale is called Pitch scale or linear scale. A reference line is made parallel to the axis of thimble and divisions are made. Let us assume that each division of linear scale corresponds to 1 mm. A circular scale is attached to the thimble using a sleeve, which can move on the pitch scale or linear scale along with the screwed rod, when thimble is turned. Circular scale is divided in 50 or 100 equal parts.

Pitch of the screw gauge is defined as the distance moved by screwed rod along its length when thimble is given one full rotation. To measure the pitch we note down the distance travelled by circular scale relative to linear scale when certain number of full rotations are given to the thimble.

Pitch of the screw

$$= \frac{\text{Distance moved by screwed rod}}{\text{Number of rotations}}$$

Least count (LC) of the screw gauge is defined as distance moved by screwed rod when it is turned through 1 division on circular scale. Hence we can write the least count as follows:

Least Count

$$= \frac{\text{Pitch}}{\text{Total number of divisions on circular scale}}$$

If pitch of the screw is 1 mm and total number of divisions on circular scale are 100 then least count becomes 0.01 mm.

When some object is there between stud and end of the screwed rod then reading is taken in two steps. One is linear scale reading (LSR) and the other is circular scale reading (CSR). When screw gauge is completely closed then linear scale is completely covered by circular scale. When thimble is rotated to take the reading then linear scale is exposed. Mark on the linear scale which is just outside the circular scale corresponds to linear scale reading and the corresponding value is noted as linear scale reading. Now we note which division on circular scale is in line with reference line. Let n^{th} division of circular scale is aligned with reference line then circular scale reading is n multiplied with least count (CSR $= n \times$ LC).

Linear scale reading and circular scale reading are added to get the total reading.

Total reading = LSR + CSR

$\Rightarrow$ Total Reading = LSR + $n \times$ LC.

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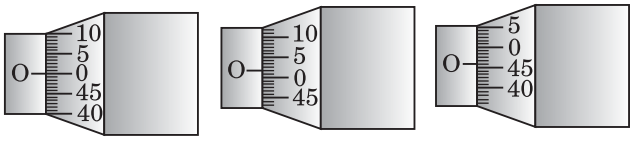
It may be the case that when screwed rod completely touches the stud then zero of the circular scale don't get aligned with the reference line and in this case we say that zero error exists in the instrument.



Zero error nil Zero error positive Zero error negative

When zero of the circular scale lies below the reference line hence zero mark on circular scale crosses the reference line when screw gauge is completely closed then error is called positive error. Zero correction is negative. We can note the number of division of circular scale in line with reference line and then multiply it with least count to get the zero error. This positive zero verror is subtracted in the final result as zero correction is negative. In case of positive error, reading taken by the instrument is more than the actual; hence zero error is subtracted from the final reading.

When zero of the circular scale lies above the reference line then error is called negative. Corresponding zero correction is positive. If N is total number of divisions on circular scale and n^{th} division lies in line with the reference line then zero error is $(N - n) \times$ LC. As we know that zero correction is positive and hence zero error is added in the final result. In case of negative error, reading taken by the instrument is less than actual; hence zero error is added to the final result.

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions *M. C. Q.*

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A MULTIPLE CHOICE QUESTIONS *with only one correct answer*

Select the Correct Answer:

A1. What will be the value of 3.725 after rounding off to three significant digits?

(*a*) 3.73 (*b*) 3.72

(*c*) 3.71 (*d*) None of these

A2. What will be the value of 3.735 after rounding off to three significant digits?

(*a*) 3.73 (*b*) 3.74

(*c*) 3.72 (*d*) None of these

A3. A sample of material having mass 8.89 g occupies a volume of 1.9 cm^3. What will be the density of material with correct significant digits?

(*a*) 4.678 g/cc (*b*) 4.6 g/cc

(*c*) 4.7 g/cc (*d*) 4.68 g/cc

A4. In a Vernier caliper, 9 divisions of main scale match with $9 + n$ divisions of Vernier scale. What should be the value of n so that least count of the instrument remains at minimum possible value?

(*a*) 1 (*b*) 9

(*c*) 2 (*d*) None of these

A5. Result of measurement of a certain physical entity is expressed as nu, where n is numerical value and u is its unit of measurement. If the same result is written in various units then,

(*a*) $n \propto 1/u$ (*b*) $n \propto u$

(*c*) $n \propto u^2$ (*d*) $n \propto u^3$

A6. How the unit of energy will change if the unit of length and force are increased four times?

(*a*) 4 times (*b*) 8 times

(*c*) 16 times (*d*) None of these

A7. What are the dimensions of a and b in Vander Wall's equation?

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

(*a*) ML^5T^{-2}, L^3 (*b*) $ML^{-1}T^{-1}$, L^3

(*c*) MLT, L^2 (*d*) None of these

A8. Velocity of a particle at time t is expressed as follows:

$$v = \alpha t + \frac{\beta}{t + \gamma}$$

Dimensions of α, β and γ are respectively.

(*a*) L^3, T, LT^{-1} (*b*) LT^{-2}, L, T

(*c*) L, T, LT^{-1} (*d*) None of these

A9. If pressure (P) is expressed as, $P = \frac{a}{b} e^{-\frac{ax}{k\theta}}$, then what will be dimensions of b. Here x is distance, k is Boltzmann constant and θ is temperature.

(*a*) $M^1L^2T^2$ (*b*) $M^0L^2T^0$

(*c*) $M^2L^0T^1$ (*d*) None of these

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A10. A physical entity given by $z = \varepsilon_0 l \frac{\Delta V}{\Delta t}$. Here ε_0 is electrical permittivity of free space, l is length, ΔV is potential difference and Δt is time interval. Write the dimensional formula for z.

(a) $M^0L^0T^0A$ (b) $M^1L^0T^0A$

(c) $M^0L^1T^0A$ (d) None of these

A11. If E is electric field intensity and ε_0 is electrical permittivity of free space, then dimension of $u = \frac{1}{2}\varepsilon_0 E^2$ is:

(a) $ML^{-1}T^{-2}$ (b) $ML^{-1}T^{-1}$

(c) MLT^{-2} (d) None of these

A12. Number of particles crossing per unit area, per unit time perpendicular to Z-axis is $N = -D\frac{n_2 - n_1}{z_2 - z_1}$ where n_1 and n_2 are the number of particles per unit volume for z_1 and z_2 respectively. Here D is called diffusion constant. What are the dimensions of D?

(a) $[M^0L^2T^{-1}]$ (b) $[ML^2T^{-1}]$

(c) $[MLT^{-1}]$ (d) None of these

A13. Volume flow rate through a capillary tube is given by $V = \frac{K(r_2 - r_1)^4}{(x_2 - x_1)}$, where r_1 and r_2 are the inner and outer radii and x_1 and x_2 are the positions crossed by the liquid. What will be the dimensional formula for K?

(a) $[MLT^{-1}]$ (b) $[M^0L^0T^{-2}]$

(c) $[M^0L^0T^{-1}]$ (d) None of these

A14. Marks on main scale of Vernier caliper are made at 1 mm separation. There are 25 equal divisions on Vernier scale and the same matches with 20 divisions on main scale. What is the least count of given Vernier caliper?

(a) 0.1 mm (b) 0.25 mm

(c) 0.15 mm (d) 0.2 mm

A15. Mass (m), length (l) and radius (r) of a wire are as follows:

$m = 0.3 \pm 0.003$ g

$l = 6 \pm 0.06$ cm

$r = 0.5 \pm 0.005$ mm

What will be the maximum percentage error in the measurement of its density?

(a) 1 (b) 2

(c) 3 (d) 4

A16. Edge length of a cube is 1.7×10^{-2} m. What will be the volume of cube with correct significant digits?

(a) 4.913×10^{-6} m^3 (b) 4.91×10^{-6} m^3

(c) 4.9×10^{-6} m^3 (d) 4.92×10^{-6} m^3

A17. Percentage error in mass and momentum are 3% and 2% respectively. Maximum possible percentage error in the kinetic energy is

(a) 7% (b) 2%

(c) 4% (d) 3%

A18. Least count of a stop watch is 0.2 sec. Time of 20 oscillations is found to be 25 seconds. Maximum possible percentage error in the measurement of time period is

(a) 8% (b) 0.8%

(c) 1.8% (d) 2%

A19. Acceleration due to gravity is to be measured using simple pendulum. If a and b are percentage errors in the measurement of length and time period respectively, then percentage error in the measurement of acceleration due to gravity is

(a) $a - 2b$ (b) $2a - b$

(c) $a + 2b$ (d) $b + 2a$

A20. Radius of a circle is 2.23 m. Area of the circle is

(a) 15.6 m^2 (b) 15.6149 m^2

(c) 15.615 m^2 (d) 15.62 m^2

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Answers

A1. (b)	A2. (b)	A3. (c)	A4. (a)	A5. (a)	A6. (c)	A7. (a)	A8. (b)	A9. (b)	A10. (a)
A11. (a)	A12. (a)	A13. (c)	A14. (d)	A15. (d)	A16. (c)	A17. (a)	A18. (b)	A19. (c)	A20. (a)

B MULTIPLE CHOICE QUESTIONS *from Competitive Examinations*

AIPMT/NEET & Other State Boards for Medical Entrance

BA1. Which of the following pairs does not have similar dimensions?

(a) Stress and pressure

(b) Angle and strain

(c) Tension and surface tension

(d) Planck's constant and angular momentum

(A.I.I.M.S. 2001)

BA2. The length and breadth of a metal sheet are 3·124 m and 3·002 m respectively. The area of the sheet up to four correct significant digits is:

(a) 9·37 m^2 (b) 9·378 m^2

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(c) 9·3782 m^2 (d) 9·378248 m^2

(*A.I.I.M.S. 2001*)

BA3. The dimensions of the electric susceptibility are:

(a) $[M^0L^0T^0A^0]$ (b) $[M^0L^{-2}T^0A^2]$

(c) $[ML\ T^{-2}A]$ (d) $[M^0L^{-1}T^0A]$

(*J.I.P.M.E.R. 2001*)

BA4. The dimensional formula for coefficient of restitutions:

(a) MLT^{-2} (b) $ML1T^{-2}$

(c) $ML2T^{-1}$ (d) $M^0L^0T^0$

(*J.I.P.M.E.R. 2001*)

BA5. The quantity that has dimensions of time is:

(a) $\frac{R}{L}$ (b) $\frac{1}{RC}$

(c) $\sqrt{LC}$ (d) light year

(*B.H.U. 2002*)

BA6. If the energy (E) velocity (v) and force (F) be taken as fundamental quantity, then the dimensional formula of mass will be:

(a) $F\,v^{-2}$ (b) $F\,v^{-1}$

(c) $E\,v^{-2}$ (d) $E\,v^2$

(*B.H.U. 2002*)

BA7. Pick out the wrong pair:

(a) charge-coulomb

(b) temperature-thermometer

(c) pressure-barometer

(d) sp. gravity-hydrometer

(*A.F.M.C. 2003*)

BA8. The instrument to measure time is:

(a) barometer (b) chronometer

(c) radiometer (d) None of these

(*A.F.M.C. 2003*)

BA9. The main scale of a vernier callipers has n divisions/cm. n divisions of the vernier scale coincide with $(n - 1)$ divisions of main scale. The least count of the vernier callipers is,

(a) $\frac{1}{n(n+1)}$ cm (b) $\frac{1}{(n+1)(n-1)}$ cm

(c) $\frac{1}{n}$ cm (d) $\frac{1}{n^2}$ cm (*N.E.E.T. 2019*)

BA10. A cube has a side of length 1.2×10^{-2} m. Calculate its volume:

(a) $1.7 \times 10^{-6}\ m^3$ (b) $1.73 \times 10^{-6}\ m^3$

(c) $1.70 \times 10^{-6}\ m^3$ (d) $1.732 \times 10^{-6}\ m^3$

BA11. A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and zero of circular scale division coincides with 25 divisions above the reference level. If screw gauge has a zero error of –0.004 cm, the correct diameter of the ball is

(a) 0.053 cm (b) 0.525 cm

(c) 0.521 cm (d) 0.529 cm

(*N.E.E.T. 2018*)

BA12. Pressure gradient has the same dimensions as that of:

(a) velocity gradient

(b) potential gradient

(c) energy gradient

(d) none of these. (*A.F.M.C. 2004*)

BA13. The unit of permittivity of free space, ε_0, is:

(a) newton metre^2/coulomb^2

(b) coulomb^2/newton metre^2

(c) coulomb^2/(newton metre)^2

(d) coulomb/newton metre (*C.B.S.E. 2004*)

BA14. The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$ where a, b, c are constants. The dimensions of a, b, and c are respectively:

(a) L, L T and T^2 (b) LT^{-2}, L and T

(c) L^2, T and LT^2 (d) LT^2, LT and L

(*A.I.P.M.T. 2006*)

BA15. The physical quantity having the dimensions $[M^{-1}L^{-3}.\ T^3\ A^2]$ is:

(a) electromotive force

(b) electrical conductivity

(c) electrical resistivity

(d) electrical resistance (*P.M.T. 2006*)

BA16. If the error in the measurement of radius of a sphere is 2%, then the error in the determination of volume of the sphere will be

(a) 6% (b) 8%

(c) 12% (d) 4%

(*C.B.S.E. 2008*)

BA17. Which two of the following five physical parameters have the same dimensions?

(A) Energy density

(B) Refractive index

(C) Dielectric constant

(D) Young's modulus

(E) Magnetic field

(a) (C) and (E) (b) (A) and (D)

(c) (A) and (E) (d) (B) and (D)

(*C.B.S.E. 2008*)

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BA18. Dimensions of resistance in an electrical circuit, in terms of dimension of mass M, of length L, of time T and of current I, would be:
(*a*) ML^2T^{-2} (*b*) $ML^2T^{-1}I^{-1}$
(*c*) $ML^2T^{-3}I^{-2}$ (*d*) $ML^2T^{-3}I^{-1}$
(*C.B.S.E. 2007*)

BA19. A force '*F*' is given by
$F = at + bt^2$
where *t* is time. What are the dimensions of '*a*' and '*b*'
(*a*) MLT^{-1} and MLT^0
(*b*) MLT^{-3} and ML^2T^4
(*c*) MLT^{-4} and MLT^1
(*d*) MLT^{-3} and MLT^{-4} (*C.B.S.E. 2006*)

BA20. If the dimensions of a physical quantity are given by $M^aL^bT^c$, then the physical quantity will be:
(*a*) Velocity if $a = 1, b = 0, c = -1$
(*b*) Acceleration if $a = 1, b = 1, c = -2$
(*c*) Force if $a = 0, b = -1, c = -2$
(*d*) Pressure if $a = 1, b = -1, c = -2$
(*C.B.S.E. 2009*)

BA21. The dimensional formula of $\frac{1}{2}\epsilon_0 E^2$, where ϵ_0 is the permittivity of free space and E is electric field, is:
(*a*) $M L^2 T^{-1}$ (*b*) MLT^{-1}
(*c*) ML^2T^{-2} (*d*) $ML^{-1}T^{-2}$
(*C.B.S.E. 2010*)

BA22. The position *x* of a particle at time *t* is given by:
$x = \frac{v_0}{a}(1 - e^{-at})$
where v_0 is a constant and $a > 0$. The dimensional formula of v_0 and *a* are:
(*a*) $[M^0LT^{-1}]$ and $[T^{-1}]$
(*b*) $[M^0LT^0]$ and $[T^{-1}]$
(*c*) $[M^0LT^{-1}]$ and $[LT^{-2}]$
(*d*) $[M^0LT^{-1}]$ and [T] (*B.H.U. 2002*)

BA23. An object is moving through the liquid. The viscous damping force acting on it is proportional to the velocity. Then dimensional formula of constant of proportionality is:
(*a*) $ML^{-1}T^{-1}$ (*b*) MLT^{-1}
(*c*) $M^0L^{-1}T^{-1}$ (*d*) ML^0T^{-1}
(*J.I.P.M.E.R. 2002*)

BA24. If L, C and R denote the inductance, capacitance and resistance respectively, then the dimensional formula for C^2 LR is:
(*a*) $ML^2T^{-1}I^0$ (*b*) $M^0L^0T^3I^0$
(*c*) $M^{-1}L^{-2}T^6I^2$ (*d*) $M^0L^0T^2I^0$
(*J.I.P.M.E.R. 2002*)

BA25. If F, T and V denote Force, Time and Velocity, then the dimensional formula for mass 'M' is:
(*a*) $F\,TV^{-1}$ (*b*) $F^{-1}\,TV$
(*c*) $F^{-1}T^{-1}V$ (*d*) $FT^{-1}V^{-1}$
(*A.I.P.M.T. 2014*)

BA26. Length cannot be measured by:
(*a*) fermi (*b*) debye
(*c*) micron (*d*) light year
(*A.I.I.M.S. 2002*)

BA27. Using mass (M), length (L), time (T) and current (A) as fundamental quantities, the dimensional formula of permeability is:
(*a*) $M^{-1}LT^{-2}A$ (*b*) $ML^2T^{-2}A^{-1}$
(*c*) $MLT^{-2}A^{-2}$ (*d*) $MLT^{-1}A^{-1}$
(*A.I.I.M.S. 2003*)

BA28. The dimensional formula of $\frac{a}{b}$ in the equation, where $P = \frac{a - t^2}{bx}$ is pressure, *x* is distance and *t* is time, is:
(*a*) LT^{-3} (*b*) ML^3T^{-1}
(*c*) M^2LT^{-3} (*d*) MT^{-2}
(*J.I.P.M.E.R. 2003*)

BA29. The length, breadth and thickness of a block are given by $l = 12$ cm, $b = 6$ cm and $t = 2.45$ cm. The volume of the block according to the idea of significant figures should be:
(*a*) 1×10^2 cm^3 (*b*) 2×10^2 cm^3
(*c*) 1.763×10^2 cm^3 (*d*) none of these
(*B.H.U. 2003*)

BA30. The dimensional formula of light year is:
(*a*) $[L^{-1}]$ (*b*) $[T^{-1}]$
(*c*) [L] (*d*) [T]
(*Manipal 2003*)

BA31. Using mass (M), length (L), time (T) and electric current (A) as fundamental quantities, the dimensions of permittivity will be:
(*a*) $[MLT^{-1}A^{-1}]$ (*b*) $[MLT^{-2}A^{-2}]$
(*c*) $[M^{-1}L^{-3}T^4A^2]$ (*d*) $[M^2L^{-2}T^{-2}A]$
(*A.I.I.M.S. 2004*)

BA32. The magnetic moment has dimensions of:
(*a*) [LA] (*b*) $[L^2A]$
(*c*) $[LT^{-1}A]$ (*d*) $[L^2T^{-1}A]$
(*A.I.I.M.S. 2006*)

BA33. The dimensions of $[\mu_0 \epsilon_0]^{\frac{1}{2}}$ are:
(*a*) $[L^{\frac{1}{2}}T^{-\frac{1}{2}}]$ (*b*) $[L^{-1}T]$
(*c*) $[LT^{-1}]$ (*d*) $[L^{-\frac{1}{2}}T^{-\frac{1}{2}}]$
(*C.B.S.E. 2011*)

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BA34. In an experiment, the percentage of error occurred in the measurement of physical quantities A, B, C and D are 1%, 2%, 3% and 4% respectively. Then the maximum percentage of error in the measurement X, where $X = \frac{A^2B^{1/2}}{C^{1/3}D^3}$ will be:

(*a*) $\left(\frac{3}{13}\right)\%$ (*b*) 16%

(*c*) –10% (*d*) 10% (*N.E.E.T. 2019*)

BA35. In an experiment four quantities *a*, *b*, *c* and *d* are measured with percentage error 1%, 2%, 3% and 4% respectively. Quantity *P* is calculated as follows: $P = \frac{a^3b^2}{cd}$, % error in P is:

(*a*) 10% (*b*) 7%

(*c*) 4% (*d*) 14%

(*N.E.E.T. 2013*)

BA36. If energy (*E*), velocity (*V*) and time (*T*) are chosen as the fundamental quantities, the dimensional formula of surface tension will be:

(*a*) $[EV^{-2}T^{-1}]$ (*b*) $[EV^{-1}T^{-2}]$

(*c*) $[EV^{-2}T^{-2}]$ (*d*) $[E^{-2}V^{-1}T^{-3}]$

(*A.I.P.M.T. 2015*)

BA37. A physical quantity of the dimensions of the length can be formed out of *c*, *G* and $\frac{e^2}{4\pi\varepsilon_0}$ is [*c* is velocity of light, *G* is universal constant of gravitational and *e* is charge]:

(*a*) $\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{\frac{1}{2}}$ (*b*) $c^2\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{\frac{1}{2}}$

(*c*) $\frac{1}{c^2}\left[\frac{e^2}{G4\pi\varepsilon_0}\right]^{\frac{1}{2}}$ (*d*) $\frac{1}{c}G\frac{e^2}{4\pi\varepsilon_0}$

(*N.E.E.T. 2017*)

JEE (Main) & Other State Boards for Engineering Entrance

BB1. The relative error in the determination of the surface area of a sphere is α. Then the relative error in the determination of its volume is:

(*a*) $\frac{3}{2}\alpha$ (*b*) $\frac{2}{3}\alpha$

(*c*) α (*d*) $\frac{5}{2}\alpha$

(*J.E.E. Main 2018*)

BB2. In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is:

(*a*) 0.4300 cm (*b*) 0.3150 cm

(*c*) 0.0430 cm (*d*) 0.2150 cm

(*J.E.E. Main 2018*)

BB3. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is:

(*a*) 4.5% (*b*) 6%

(*c*) 2.5% (*d*) 3.5%

(*J.E.E. Main 2018*)

BB4. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further it is found that the screw gauge has a zero error of –0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale division in line with the main scale as 35. The diameter of the wire is

(*a*) 3.32 mm (*b*) 3.73 mm

(*c*) 3.67 mm (*d*) 3.38 mm

(*A.I.E.E.E. 2008*)

BB5. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line. The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is:

(*a*) 5.740 mm (*b*) 5.950 mm

(*c*) 5.725 mm (*d*) 5.755 mm

(*J.E.E. Main 2019*)

BB6. The percentage errors in quantities *P*, *Q*, *R* and *S* are 0.5%, 1%, 3% and 1.5% respectively in the measurement of a physical quantity $A = \frac{P^3Q^2}{\sqrt{R}S}$. The maximum percentage error in the value of *A* will be:

(*a*) 6.5% (*b*) 7.5%

(*c*) 6.0% (*d*) 8.5%

(*J.E.E. Main 2018*)

BB7. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°) then the least count of the instrument is

(*a*) 1 minute (*b*) half minute

(*c*) one degree (*d*) half degree

(*A.I.E.E.E. 2009*)

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BB8. In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is:

(*a*) 0.4300 cm (*b*) 0.3150 cm

(*c*) 0.0430 cm (*d*) 0.2150 cm

(*J.E.E. Main 2011*)

BB9. A spectrometer gives the following reading when used to measure the angle of a prism.

Main scale reading: 58.5 degree

Vernier scale reading: 09 divisions

Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data:

(*a*) 59 degree (*b*) 58.59 degree

(*c*) 58.77 degree (*d*) 58.65 degree

(*A.I.E.E.E. 2012*)

BB10. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is:

(*a*) 3% (*b*) 6%

(*c*) zero (*d*) 1%

(*A.I.E.E.E. 2012*)

BB11. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then:

(*a*) $[\varepsilon_0] = [M^{-1} L^{-3} T^4 A^2]$

(*b*) $[\varepsilon_0] = [M^{-1} L^2 T^{-1} A^{-2}]$

(*c*) $[\varepsilon_0] = [M^{-1} L^2 T^{-1} A]$

(*d*) $[\varepsilon_0] = [M^{-1} L^{-3} T^2 A]$ (*J.E.E. Main 2013*)

BB12. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to:

(*a*) $\sqrt{\frac{hc^5}{G}}$ (*b*) $\sqrt{\frac{Gh}{c^3}}$

(*c*) $\sqrt{\frac{c^3}{Gh}}$ (*d*) $\sqrt{\frac{Gh}{c^5}}$

(*J.E.E. Main 2019*)

BB13. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. The accuracy in the determination of g is:

(*a*) 3% (*b*) 1%

(*c*) 5% (*d*) 2%

(*J.E.E. Main 2015*)

BB14. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be:

(*a*) 92 ± 2 s (*b*) 92 ± 5.0 s

(*c*) 92 ± 1.8 s (*d*) 92 ± 3 s

(*J.E.E. Main 2016*)

BB15. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line?

(*a*) 0.75 mm (*b*) 0.80 mm

(*c*) 0.70 mm (*d*) 0.50 mm

(*J.E.E. Main 2016*)

BB16. From the following combinations of physical constants (expressed through their usual symbols) the only combination that would have the same value in different systems of units is:

(*a*) $\frac{ch}{2\pi\varepsilon_0^2}$ (*b*) $\frac{e^2}{2\pi\varepsilon_0 G m_e^2}$

(*c*) $\frac{m_0\varepsilon_0}{c^2}\frac{G}{he^2}$ (*d*) $\frac{2\pi\sqrt{\mu_0\varepsilon_0}}{ce^2}\frac{h}{G}$

(*J.E.E. Main 2014*)

BB17. In terms of resistance R and time T, the dimension of ratio $\frac{\mu}{\varepsilon}$ of the permeability μ and permittivity ε is:

(*a*) RT^{-2} (*b*) R^2T^{-2}

(*c*) R^2 (*d*) R^2T^2

(*J.E.E. Main 2014*)

BB18. An experiment is performed to obtain the value of acceleration due to gravity by using a simple pendulum of length L. In this experiment time for 100 oscillations is measured by using a watch of 1 second least count and the value is 90.0 seconds. The length L is measured by using a metre scale of least count 1 mm and the value is 20.0 cm. The error in the determination of g would be:

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(a) 1.7% (b) 2.7%
(c) 4.4% (d) 2.27%
(*J.E.E. Main 2014*)

BB19. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. The accuracy in the determination of g is:
(a) 1% (b) 5%
(c) 2% (d) 3%
(*J.E.E. Main 2015*)

BB20. If electronic charge e, electron's mass m, speed of light in vacuum c and Planck's constant h are taken as fundamental quantities, the permeability of vacuum μ_0 can be expressed in units of:
(a) $\left(\frac{hc}{me^2}\right)$ (b) $\left(\frac{h}{me^2}\right)$
(c) $\left(\frac{h}{ce^2}\right)$ (d) $\left(\frac{mc^2}{he^2}\right)$
(*J.E.E. Main 2015*)

BB21. In the following 'I' refers to current and other symbols have their usual meaning. Choose the option that corresponds to the dimensions of electrical conductivity:
(a) $ML^{-3}T^{-3}I^2$ (b) $M^{-1}L^3T^3I$
(c) $M^{-1}L^{-3}T^3I^2$ (d) $M^{-1}L^{-3}T^3I$
(*J.E.E. Main 2016*)

BB22. *A, B, C* and *D* are four different physical quantities having different dimensions. None of them is dimensionless. But we know that the equation $AD = C\ln(BD)$ holds true. Then which of the combination is not a meaningful quantity?
(a) $A^2 - B^2C^2$ (b) $\frac{(A-C)}{D}$
(c) $\frac{A}{B} - C$ (d) $\frac{C}{BD} - \frac{AD^2}{C}$
(*J.E.E. Main 2016*)

BB23. Time (T), velocity (C) and angular momentum (h) are chosen as fundamental quantities instead of mass, length and time. In terms of these, the dimensions of mass would be:
(a) $[M] = [T^{-1} C^{-2} h]$ (b) $[M] = [T^{-1} C^2 h]$
(c) $[M] = [T^{-1} C^{-2} h^{-1}]$ (d) $[M] = [T C^{-2} h]$
(*J.E.E. Main 2017*)

BB24. A physical quantity P is described by the relation $P = a^{\frac{1}{2}} b^2c^3d^{-4}$ If the relative errors in the measurement of a, b, c and d, respectively, are 2%, 1%, 3% and 5%, then the relative error in P will be:
(a) 8% (b) 12%
(c) 32% (d) 25%
(*J.E.E. Main 2017*)

BB25. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of
(a) 9 (b) $\frac{1}{9}$
(c) 81 (d) $\frac{1}{81}$
(*J.E.E. Main 2017*)

BB26. The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is:
(a) 37.0 cm^2 (b) 37 cm^2
(c) 37.03 cm^2 (d) 37.030 cm^2
(*J.E.E. Main 2019*)

JEE (Advanced) for IIT Entrance

BC1. A vernier caliper has 1 mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main scale divisions. For the vernier calipers, the least count is:
(a) 0.02 mm (b) 0.05 mm
(c) 0.1 mm (d) 0.2 mm
(*I.I.T. 2010*)

BC2. Which of the following set have different dimensions?
(a) Pressure, Young's modulus, stress
(b) emf, potential difference, electric potential
(c) Heat, work done, energy
(d) Dipole moment, electric flux, electric field
(*I.I.T. 2005*)

BC3. A quantity 'x' is given by $\varepsilon_0 L\frac{\Delta V}{\Delta t}$ where ε_0 is permittivity of free space, L is length, ΔV is a potential difference and Δt is the time interval. The dimensional formula for 'x' is the same as that of:
(a) resistance (b) charge
(c) voltage (d) current
(*I.I.T. 2001*)

BC4. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond 11 main

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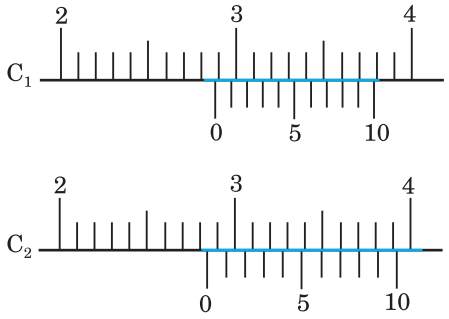
scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2, respectively, are



(*a*) 2.87 and 2.86 (*b*) 2.85 and 2.82

(*c*) 2.87 and 2.87 (*d*) 2.87 and 2.83

(*J.E.E. Advanced 2016*)

BC5. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is (are) true?

(*a*) The error in the measurement of r is 10%.

(*b*) The error in the measurement of T is 3.57%.

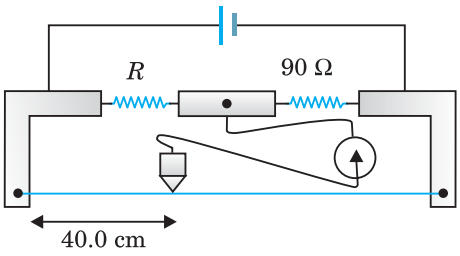
(*c*) The error in the measurement of T is 2%.

(*d*) The error in the determined value of g is 11%.

(*J.E.E. Advanced 2016*)

BC6. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance is

(*J.E.E. Advanced 2014*)



(*a*) $60 \pm 0.15\ \Omega$ (*b*) $135 \pm 0.56\ \Omega$

(*c*) $60 \pm 0.25\ \Omega$ (*d*) $135 \pm 0.23\ \Omega$

BC7. The diameter of a cylinder is measured using a Vernier caliper with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24^{th} division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

(*a*) 5.112 cm (*b*) 5.124 cm

(*c*) 5.136 cm (*d*) 5.148 cm

(*J.E.E. Advanced 2013*)

BC8. Using the expression $2d \sin\theta = \lambda$, one calculate the values of d by measuring the corresponding angles θ in the range 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0° to 90°.

(*a*) the absolute error in d remains constant.

(*b*) the absolute error in d increases.

(*c*) the fractional error in d remains constant.

(*d*) the fractional error in d decreases.

(*J.E.E. Advanced 2013*)

BC9. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be $L = 20$ metres. Take the acceleration due to gravity $g = 10\ ms^{-2}$ and the velocity of sound is $300\ ms^{-1}$. Then the fractional error in the measurement $\frac{\delta L}{L}$ is closest to:

(*a*) 0.2% (*b*) 3%

(*c*) 5% (*d*) 1%

(*J.E.E. Advanced 2017*)

BC10. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ±0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ±0.01 mm. Take $g = 9.8\ m/s^2$ (exact). The Young's modulus obtained from the reading is

(*a*) $(2.0 \pm 0.3) \times 10^{11}\ N/m^2$

(*b*) $(2.0 \pm 0.2) \times 10^{11}\ N/m^2$

(*c*) $(2.0 \pm 0.1) \times 10^{11}\ N/m^2$

(*d*) $(2.0 \pm 0.05) \times 10^{11}\ N/m^2$

(*I.I.T. 2007*)

BC11. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa ($\Delta a/a << 1$), then what is the error Δr in determining r?

(*a*) $\frac{\Delta a}{(1+a)^2}$ (*b*) $\frac{2\Delta a}{(1+a)^2}$

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(c) $\frac{2\Delta a}{(1-a^2)}$ (d) $\frac{2a\Delta a}{(1-a^2)}$

(*J.E.E. Advanced 2018*)

BC12. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(a) 0.9% (b) 2.4%

(c) 3.1% (d) 4.2%

(*I.I.T. 2011*)

BC13. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length L = 2 m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension l = 0.25 mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of division on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

(a) due to the errors in the measurements of d and l are the same.

(b) due to the error in the measurement of d is twice that due to the error in the measurement of l.

(c) due to the error in the measurement of l is twice that due to the error in the measurement of d.

(d) due to the error in the measurement of d is four times that due to the error in the measurement of l.

(*I.I.T. 2012*)

Answers

BA1. (c)	**BA2.** (b)	**BA3.** (a)	**BA4.** (d)	**BA5.** (c)	**BA6.** (c)	**BA7.** (a)	**BA8.** (b)	**BA9.** (d)	**BA10.** (a)
BA11. (d)	**BA12.** (d)	**BA13.** (b)	**BA14.** (b)	**BA15.** (b)	**BA16.** (a)	**BA17.** (b)	**BA18.** (c)	**BA19.** (d)	**BA20.** (d)
BA21. (d)	**BA22.** (a)	**BA23.** (d)	**BA24.** (b)	**BA25.** (a)	**BA26.** (b)	**BA27.** (c)	**BA28.** (d)	**BA29.** (b)	**BA30.** (c)
BA31. (b)	**BA32.** (b)	**BA33.** (b)	**BA34.** (b)	**BA35.** (d)	**BA36.** (c)	**BA37.** (a)	**BB1.** (a)	**BB2.** (d)	**BB3.** (a)
BB4. (d)	**BB5.** (d)	**BB6.** (a)	**BB7.** (a)	**BB8.** (d)	**BB9.** (d)	**BB10.** (b)	**BB11.** (a)	**BB12.** (d)	**BB13.** (a)
BB14. (c)	**BB15.** (b)	**BB16.** (b)	**BB17.** (c)	**BB18.** (b)	**BB19.** (d)	**BB20.** (c)	**BB21.** (c)	**BB22.** (b, d)	
BB23. (a)	**BB24.** (c)	**BB25.** (a)	**BB26.** (c)	**BC1.** (d)	**BC2.** (d)	**BC3.** (d)	**BC4.** (d)	**BC5.** (a)	**BC6.** (c)
BC7. (b)	**BC8.** (d)	**BC9.** (d)	**BC10.** (b)	**BC11.** (b)	**BC12.** (c)	**BC13.** (a)			

C MULTIPLE CHOICE QUESTIONS *with more than one correct answer*

C1. Pairs of physical quantities having same dimensions are

(a) Reynolds number and strain

(b) Torque and force

(c) Curie and frequency

(d) Gravitational potential and latent heat

C2. A Vernier caliper has 50 divisions which are equivalent to 2.45 cm. Main scale reading of vernier caliper is found to be 4.15 cm. Markings on main scale are at 0.05 cm. The 24th division of Vernier scale matches with one of the main scale divisions.

(a) Least count = 0.001 cm

(b) Least count = 0.001 mm

(c) Final reading = 4.174 cm

(d) Final reading = 4.157 cm

C3. Two physical quantities P and Q have different dimensions. Which of the following mathematical operation is/are possible?

(a) $\frac{P}{Q}$ (b) PQ

(c) $P+Q$ (d) $P-Q$

C4. 10 Vernier scale divisions (VSD) match with 9 main scale divisions (MSD). Least count of the given Vernier caliper is

(a) 0.1 mm if 1 MSD = 1 mm

(b) 0.05 cm if 1 MSD = 0.05 cm

(c) 0.01 mm if 1 MSD = 1 mm

(d) 0.005 cm if 1 MSD = 0.05 cm

C5. Select the correct option(s)

(a) Dimensionally correct equation must be correct.

(b) Dimensionally correct equation may be incorrect.

(c) Dimensionally correct equation may be correct.

(d) Dimensionally incorrect equation must be incorrect.

C6. Accuracy of measurement is determined by

(a) Absolute error (b) Relative error

(c) Percentage error (d) None of these

C7. A dimensionless quantity

(a) Must not have a unit

(b) May have a unit

(c) May not have a unit

(d) Must have a unit

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C8. Unitless physical quantity
(*a*) Can have non-zero dimensions
(*b*) Cannot have non-zero dimensions
(*c*) Must have zero dimensions
(*d*) Does not exist

C9. The dimensional formula $ML^{-1}T^{-2}$ may correspond to
(*a*) Impulse of a force
(*b*) Pressure
(*c*) Energy
(*d*) Energy per unit volume

C10. Which of the following sets of physical quantities can be used as fundamental quantities in a system of units?
(*a*) Length, velocity and acceleration
(*b*) Length, mass and acceleration
(*c*) Length, mass and time
(*d*) mass, time and acceleration

C11. A length-scale (l) depends on the permittivity (ε) of a dielectric materials, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles and the charge (q) carried by each of the particles. Which of the following expression(s) for l is(are) dimensionally correct?

(*a*) $l = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (*b*) $l = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$

(*c*) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (*d*) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

(*J.E.E. Advanced 2016*)

C12. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stopwatch with the least count of 1 sec. for this and records 40 seconds for 20 oscillations. For this observation, which of the following statements(s) is (are) true?
(*a*) Error ΔT in measuring T, the time period, is 0.05 second
(*b*) Error ΔT in measuring T, the time period, is 1 second
(*c*) Percentage error in the determination of g is 5%
(*d*) Percentage error in the determination of g is 2.5% (*I.I.T. 2010*)

C13. A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.5 metre. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40) (66. 33), (78, 39). The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is(are)
(*a*) (42, 56) (*b*) (48, 48)
(*c*) (66, 33) (*d*) (78, 39)
(*I.I.T. 2009*)

C14. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement (s) is/are correct ?
(*a*) The dimension of force is L^{-3}
(*b*) The dimension of energy is L^{-2}
(*c*) The dimension of power is L^{-5}
(*d*) The dimension of linear momentum is L^{-1}
(*J.E.E. Advanced 2019*)

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Answers

C1. (*a, c, d*)	**C2.** (*a, c*)	**C3.** (*a, b*)	**C4.** (*a, d*)	**C5.** (*b, c, d*)	**C6.** (*b, c*)	**C7.** (*b, c*)
C8. (*b, c*)	**C9.** (*b, d*)	**C10.** (*b, c, d*)	**C11.** (*b, d*)	**C12.** (*a, c*)	**C13.** (*c, d*)	**C14.** (*a,b,d*)

D MULTIPLE CHOICE QUESTIONS
based on a given passage / comprehension

Passage 1

There is one Vernier caliper in which 1 main scale division (MSD) is 1 mm. It is given that 10 divisions of Vernier scale match with 9 divisions of main scale. Given Vernier caliper is used to measure the edge of a cube. Main scale reading (MSR) was found to be 10 and first division of the Vernier scale was found to be coinciding with some division of the main scale. Mass of the cube is 2.736 g.

Answer the following questions:

D1. What is least count of the given Vernier caliper?
(*a*) 0.1 mm (*b*) 0.1 cm
(*c*) 0.01 mm (*d*) 1 μm

D2. What is the edge length of the cube?
(*a*) 1.01 mm (*b*) 1.01 cm
(*c*) 10.1 cm (*d*) 1.001 cm

D3. What is the number of significant digits in the edge length of cube, measured with the given Vernier caliper?
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

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D4. What is the volume of cube with correct number of significant digits?

(*a*) 1.03 cm^3 (*b*) 1.0303 cm^3

(*c*) 1.03 mm^3 (*d*) 1.0303 mm^3

D5. What is the density of material with correct number of significant digits?

(*a*) 2.65 g/cm^3 (*b*) 2.66 g/cm^3

(*c*) 2.656 g/cm^3 (*d*) 2.650 g/cm^3

Answers

Passage 1 **D1.** (*a*) **D2.** (*b*) **D3.** (*c*) **D4.** (*a*) **D5.** (*b*)

ASSERTION REASON TYPE QUESTIONS

The questions given below consist of an assertion and the reason. Use the following key to choose appropriate answer:

(*a*) If both assertion and reason are *correct* and reason is a *correct explanation* of the assertion.

(*b*) If both assertion and reason are *correct* but reason is *not the correct explanation* of assertion.

(*c*) If assertion is *correct* but reason is *incorrect*.

(*d*) If assertion is *incorrect* but reason is *correct*.

(*e*) If both assertion and reason are *incorrect*.

1. **Assertion:** All equations which are physically correct must also be dimensionally correct.
   Reason: All equations which are dimensionally correct must also be physically correct.
2. **Assertion:** Out of the two readings 15.25 and 15.256, second one is more precise.
   Reason: Measurements with more decimal places are said to be more accurate.
3. **Assertion:** When unit of measurement for a physical quantity is changed, its numerical value also changes.
   Reason: Product of numerical value and unit of measurement for a physical quantity remains constant.
4. **Assertion:** A physical quantity which has zero dimension may have unit of measurement.
   Reason: Two physical entities having same dimensional formula may have different units of measurement.
5. **Assertion:** Maximum possible error in a reading is taken as the least count of the measuring instrument.
   Reason: Error in the measurement can never be greater than the least count of the instrument.
6. **Assertion:** All physical quantities can be represented dimensionally in terms of the base quantities.
   Reason: A base quantity cannot be represented dimensionally in terms of remaining base quantities.

Answers

1. (*c*) **2.** (*a*) **3.** (*a*) **4.** (*b*) **5.** (*c*) **6.** (*b*)

MATCHING TYPE QUESTIONS

Each question has a matching list. The codes for the lists have choices (*a*), (*b*), (*c*) and (*d*) out of which only one is correct.

1. Match the two lists.

List-I		List-II	
P	Gravitational constant	1	$[ML^0T^{-2}]$
Q	Surface Tension	2	$[ML^2T^{-2}K^{-1}]$
R	Planck's constant	3	$[ML^2T^{-1}]$
S	Boltzmann's constant	4	$[M^{-1}L^3T^{-2}]$

	P	Q	R	S
(*a*)	1	4	3	2
(*b*)	2	3	1	4
(*c*)	4	1	3	2
(*d*)	3	4	2	1

2. Match List I with List II and select the correct answer using the codes given below the lists:

List-I		List-II	
P	Boltzmann constant	1	$[ML^2T^{-1}]$
Q	Coefficient of viscosity	2	$[ML^{-1}T^{-1}]$
R	Planck constant	3	$[MLT^{-3}K^{-1}]$
S	Thermal conductivity	4	$[ML^2T^{-2}K^{-1}]$

(*J.E.E. Advanced 2013*)

	P	Q	R	S
(*a*)	3	1	2	4
(*b*)	3	2	1	4
(*c*)	4	2	1	3
(*d*)	4	1	2	3

Answers

1. (*c*) **2.** (*c*)

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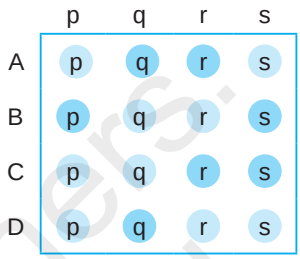
MATRIX MATCH TYPE QUESTIONS

Each question contains statements given in two columns, which have to be matched. Statements in column-I are labelled as A, B, C and D whereas statements in column-II are labelled as p, q, r and s. Match the entries of column-I with appropriate entries of column-II. Each entry in column-I may have one or more than one correct option from column-II. The answers to these questions have to be appropriately bubbled as illustrated in the given example. If the correct matches are A→(q, r), B→(p, s), C→(r, s) and D→(q), then correctly bubbled matrix will look like the following:



1. Match the two columns.

Column I		Column II	
(A)	Stress	(p)	Joule
(B)	Coefficient of elasticity	(q)	ML^2T^{-2}
(C)	Work	(r)	N/m^2
(D)	Energy	(s)	$ML^{-1}T^{-2}$

Answers

1. A → (r, s) B → (r, s) C → (p, q) D → (p, q)

INTEGER TYPE QUESTIONS

The answer to each of the following questions is a single-digit integer ranging from 0 to 9. Darken the correct digit.

1. What is the dimension of length in moment of inertia?

0	1	2	3	4	5	6	7	8	9

2. Consider the following equation $F = a\sqrt{x} + bt^2$. If the dimensions of a/b are $M^pL^qT^r$ then calculate value of: $-qr$

0	1	2	3	4	5	6	7	8	9

3. Mass (M) of the largest stone that can be moved by flowing river depends on the velocity of flow of river (v), density of water (ρ) and acceleration due to gravity (g). If M depends on n^{th} power of velocity then calculate n.

0	1	2	3	4	5	6	7	8	9

4. The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2\ s^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of $E(t)$ at $t = 5$ s is

0	1	2	3	4	5	6	7	8	9

5. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross sectional area is $8 \times 10^{-7}\ m^2$. The least count of Vernier scale is 1.0×10^{-5} m. the maximum percentage error in the Young's modulus of the wire is

0	1	2	3	4	5	6	7	8	9

(*J.E.E. Advanced 2014*)

6. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is

0	1	2	3	4	5	6	7	8	9

(*J.E.E. Advanced 2014*)

Answers

1. [2] **2.** [1] **3.** [6] **4.** [4] **5.** [8] **6.** [3]

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NCERT Exemplar Problems

Objective Questions

Multiple Choice Questions (Type-I)

2.1. The number of significant figures in 0.06900 is
(*a*) 5 (*b*) 4
(*c*) 2 (*d*) 3

2.2. The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is
(*a*) 663.821 (*b*) 664
(*c*) 663.8 (*d*) 663.82

2.3. The mass and volume of a body are 4.237 g and 2.5 cm^3, respectively. The density of the material of the body in correct significant figures is
(*a*) 1.6048 g cm^{-3} (*b*) 1.69 g cm^{-3}
(*c*) 1.7 g cm^{-3} (*d*) 1.695 g cm^{-3}

2.4. The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give
(*a*) 2.75 and 2.74 (*b*) 2.74 and 2.73
(*c*) 2.75 and 2.73 (*d*) 2.74 and 2.74

2.5. The length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm, respectively. The area of the sheet in appropriate significant figures and error is
(*a*) 164 ± 3 cm^2
(*b*) 163.62 ± 2.6 cm^2
(*c*) 163.6 ± 2.6 cm^2
(*d*) 163.62 ± 3 cm^2

2.6. Which of the following pairs of physical quantities does not have same dimensional formula?
(*a*) Work and torque.
(*b*) Angular momentum and Planck's constant.
(*c*) Tension and surface tension.
(*d*) Impulse and linear momentum.

2.7. Measure of two quantities along with the precision of respective measuring instrument is
A = 2.5 m $s^{-1} \pm 0.5$ m s^{-1}
B = 0.10 s ± 0.01 s
The value of *A B* will be
(*a*) (0.25 ± 0.08) m
(*b*) (0.25 ± 0.5) m
(*c*) (0.25 ± 0.05) m
(*d*) (0.25 ± 0.135) m

2.8. You measure two quantities as *A* = 1.0 m ± 0.2 m, *B* = 2.0 m ± 0.2 m. We should report correct value for $\sqrt{AB}$ as:
(*a*) 1.4 m ± 0.4 m (*b*) 1.41 m ± 0.15 m
(*c*) 1.4 m ± 0.3 m (*d*) 1.4 m ± 0.2 m

2.9. Which of the following measurements is most precise?
(*a*) 5.00 mm (*b*) 5.00 cm
(*c*) 5.00 m (*d*) 5.00 km.

2.10. The mean length of an object is 5 cm. Which of the following measurements is most accurate?
(*a*) 4.9 cm (*b*) 4.805 cm
(*c*) 5.25 cm (*d*) 5.4 cm

2.11. Young's modulus of steel is 1.9×10^{11} N/m^2. When expressed in CGS units of dynes/cm^2, it will be equal to (1N = 10^5 dyne, 1 m^2 = 10^4 cm^2)
(*a*) 1.9×10^{10} (*b*) 1.9×10^{11}
(*c*) 1.9×10^{12} (*d*) 1.9×10^{13}

2.12. If momentum (*P*), area (*A*) and time (*T*) are taken to be fundamental quantities, then energy has the dimensional formula
(*a*) (P^1 A^{-1} T^1) (*b*) (P^2 A^1 T^1)
(*c*) (P^1 $A^{-1/2}$ T^1) (*d*) (P^1 $A^{1/2}$ T^{-1})

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Multiple Choice Questions (Type-II)

2.13. On the basis of dimensions, decide which of the following relations for the displacement of a particle undergoing simple harmonic motion is *not* correct:
(*a*) $y = a \sin 2\pi t/T$
(*b*) $y = a \sin \nu t$.
(*c*) $y = \frac{a}{T} \sin\left(\frac{t}{a}\right)$
(*d*) $y = a\sqrt{2}\left(\sin\frac{2\pi t}{T} - \cos\frac{2\pi t}{T}\right)$

2.14. If *P, Q, R* are physical quantities, having different dimensions, which of the following combinations can never be a meaningful quantity?
(*a*) $(P - Q)/R$ (*b*) $PQ - R$
(*c*) PQ/R (*d*) $(PR - Q^2)/R$
(*e*) $(R + Q)/P$

2.15. Photon is quantum of radiation with energy $E = h\nu$ where ν is frequency and *h* is Planck's constant. The dimensions of *h* are the same as that of
(*a*) Linear impulse
(*b*) Angular impulse
(*c*) Linear momentum
(*d*) Angular momentum Planck

2.16. If Planck's constant (*h*) and speed of light in vacuum (*c*) are taken as two fundamental quantities, which

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one of the following can, in addition, be taken to express length, mass and time in terms of the three chosen fundamental quantities?

(*a*) Mass of electron (m_e)

(*b*) Universal gravitational constant (G)

(*c*) Charge of electron (e)

(*d*) Mass of proton (m_p)

2.17. Which of the following ratios express pressure?

(*a*) Force/Area

(*b*) Energy/Volume

(*c*) Energy/Area

(*d*) Force/Volume

2.18. Which of the following is not a unit of time?

(*a*) Second (*b*) Parsec

(*c*) Year (*d*) Light year

Answers

2.1. (*b*)	**2.2.** (*b*)	**2.3.** (*c*)	**2.4.** (*d*)	**2.5.** (*a*)	**2.6.** (*c*)	**2.7.** (*a*)	**2.8.** (*d*)	**2.9.** (*a*)
2.10. (*a*)	**2.11.** (*c*)	**2.12.** (*d*)	**2.13.** (*b*, *c*)	**2.14.** (*a*, *e*)	**2.15.** (*b*, *d*)	**2.16.** (*a*, *b*, *d*)	**2.17.** (*a*, *b*)	**2.18.** (*b*, *d*)

Hints & Solutions for *Difficult Objective Type Questions*

A. *MCQs with only one correct answer*

A1. (*b*): Last digit to be dropped is 5 and preceding digit is even hence it will remain unchanged. Hence option (*b*) is correct.

A2. (*b*): Last digit to be dropped is 5 and preceding digit is odd hence it will be increased by 1. Hence option (*b*) is correct.

A3. (*c*): Density of the material $= \frac{\text{mass}}{\text{volume}} = \frac{8.89}{1.9}$

$= \mathbf{4.6789\ g/cc}$

According to rule, number of significant digits in the answer must be 2 because volume has minimum significant digits equal to 2. Hence final answer rounded off to correct number of significant digits is 4.7 g/cc. Hence option (*c*) is correct.

A4. (*a*): Least count is equal to the difference in length of one main scale division and one Vernier scale division. Hence for minimum possible difference we need n=1. Hence, option (*a*) is correct.

A5. (*a*): When result is expressed in various units then nu remains constant.

$n_1u_1 = n_2u_2$

Hence option (*a*) is correct.

A6. (*c*): Energy can be expressed as force multiplied with length; hence when both are increased to four times then unit of energy becomes 16 times. Hence option (*c*) is correct.

A7. (*a*): a/V^2, must have same dimensions as that of pressure, P.

$\frac{a}{L^6} = \mathrm{ML^{-1}T^{-2}} \Rightarrow [a] = \mathrm{ML^5T^{-2}}$

Dimensions of b must be same as that of V.

$[b] = L^3$.

Hence option (*a*) is correct.

A8. (*b*): Dimensions of αt must be same as that of velocity.

$\alpha(\mathrm{T}) = \mathrm{LT^{-1}} \Rightarrow [\alpha] = \mathrm{LT^{-2}}$

Dimensions of b must be same as that of length. $[\beta] = \mathrm{L}$

Since γ is added to time hence its dimension must be same as that of time.

$[\gamma] = \mathrm{T}$

Hence option (*b*) is correct.

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A9. (*b*): Here $ax/k\theta$ is power raised to exponent hence it must be dimensionless.

$[a][x]=[k][\theta]$

$\Rightarrow [a] = \frac{\mathrm{ML^2T^{-2}}K^{-1} \times K}{\mathrm{L}}$

$\Rightarrow [a] = \mathrm{MLT^{-2}}$

Now we can understand that a/b must have same dimensions as that of pressure.

$$\frac{[a]}{[b]} = [P]$$

$$\Rightarrow \quad \frac{\mathrm{MLT^{-2}}}{[b]} = \mathrm{ML^{-1}T^{-2}}$$

$$\Rightarrow \quad [b] = \mathbf{M^0L^2T^0}$$

Hence option (*b*) is correct.

A10. (*a*): $[z[= [\varepsilon_0][l]\frac{[\Delta V]}{[\Delta t]}$

$\Rightarrow \quad [z] = [\mathrm{M^{-1}L^{-3}T^4}A^2][\mathrm{L}]\frac{[\mathrm{ML^2T^{-3}}A^{-1}]}{[\mathrm{T}]}$

$\Rightarrow \quad [z] = \mathbf{[M^0L^0T^0}A\mathbf{]}$

Hence option (*a*) is correct.

A11. (*a*): $[u] = \left[\frac{1}{2}\varepsilon_0 E^2\right]$

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$\Rightarrow \quad [u] = [M^{-1}L^{-3}A^2T^4][MLT^{-3}A^{-1}]^2$

$\Rightarrow \quad [u] = [ML^{-1}T^{-2}]$

Hence option (*a*) is correct. In class XII we shall study that given expression stands for energy per unit volume and so we can evaluate dimensions on that basis also.

A12. (*a*): $N = -D\dfrac{n_2 - n_1}{z_2 - z_1}$

$\Rightarrow \quad [N] = [D]\dfrac{[n_2 - n_1]}{[z_2 - z_1]}$

$\Rightarrow \quad [L^{-2}T^{-1}] = [D]\dfrac{[L^{-3}]}{[L]}$

$\Rightarrow \quad [D] = [L^2T^{-1}]$

$\Rightarrow \quad [D] = \mathbf{[M^0L^2T^{-1}]}$

Hence option (*a*) is correct.

A13. (*c*): $V = \dfrac{K(r_2 - r_1)^4}{(x_2 - x_1)}$

$\Rightarrow \quad [V] = \dfrac{[K][(r_2 - r_1)]^4}{[(x_2 - x_1)]}$

$\Rightarrow \quad [L^3T^{-1}] = \dfrac{[K][L]^4}{[L]}$

$\Rightarrow \quad [K] = [T^{-1}] = \mathbf{[M^0L^0T^{-1}]}$

Hence option (*c*) is correct.

A14. (*d*): Main scale divisions are made at 1 mm separation

$\Rightarrow$ 1 MSD = 1 mm

25 Vernier scale divisions match with 20 divisions on main scale hence

1VSD = 20/25 = 0.8 mm

Least count is difference in length of MSD and VSD.

LC = 1MSD – 1VSD = 1 - 0.8 = **0.2 mm**

Hence option (*d*) is correct.

A15. (*d*): Density of the material of wire is given by

$$\rho = \frac{m}{\pi r^2 l}$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta l}{l}$$

$$\Rightarrow \quad \frac{\Delta\rho}{\rho} = \frac{0.003}{0.3} + 2\frac{0.005}{0.5} + \frac{0.06}{6} = 0.01 + 0.02 + 0.01$$

$$= 0.04$$

$$\Rightarrow \quad \frac{\Delta\rho}{\rho} \times 100 = \mathbf{4}$$

Hence, option (*d*) is correct.

A16. (*c*): Volume of cube can be written as: $V = a^3 = (1.7 \times 10^{-2})^3 = 4.913 \times 10^{-6}$ m^3.

Edge of the cube is provided with 2 significant digits; hence volume must also contain 2 significant digits only. Hence volume of the cube should be written as follows:

$V = \mathbf{4.9 \times 10^{-6}\ m^3}$.

Hence option (*c*) is correct.

A17. (*a*): Kinetic energy can be written in terms of linear momentum as follows:

$$K = \frac{p^2}{2m}$$

$$\Rightarrow \quad \frac{\Delta K}{K} = 2\frac{\Delta p}{p} + \frac{\Delta m}{m}$$

$$\Rightarrow \quad \frac{\Delta K}{K} \times 100 = 2\frac{\Delta p}{p} \times 100 + \frac{\Delta m}{m} \times 100$$

$$\Rightarrow \quad \frac{\Delta K}{K} \times 100 = 2 \times 2 + 3 = \mathbf{7\%}$$

Hence option (*a*) is correct.

A18. (*b*): Time period of oscillation can be written as follows:

$$T = \frac{25}{20} = 1.25s$$

Error in the measurement of time of 20 oscillations is 0.2 sec; hence corresponding error in the measurement of time period can be written as follows:

$$\Delta T = \frac{0.2}{20} = 0.01s$$

Percentage error in the measurement of time period can be written as follows:

$$\frac{\Delta T}{T} \times 100 = \frac{0.01}{1.25} \times 100 = \mathbf{0.8\%}$$

Hence, option (*b*) is correct.

A19. (*c*): $T = 2\pi\sqrt{\dfrac{l}{g}}$

$$\Rightarrow \quad g = \frac{4\pi^2 l}{T^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

$$\Rightarrow \quad \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2\frac{\Delta T}{T} \times 100$$

$$\Rightarrow \quad \frac{\Delta g}{g} \times 100 = \boldsymbol{a} + \mathbf{2}\boldsymbol{b}$$

Hence, option (*c*) is correct.

A20. (*a*): Area of the circle is: $A = \pi r^2 = 3.14 \times (2.23)^2 = 15.6149$ m^2.

Number of significant digits in radius is 3; hence final answer should also contain only 3 significant digits.

Hence $A = \mathbf{15.6\ m^2}$.

Hence, option (*a*) is correct.

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B. *MCQs from Competitive Examinations*

AIPMT/NEET & Other State Boards for Medical Entrance

BA1. (c): Tension is force but surface tension is force per unit length.

BA2. (b): $l = 3.124$ m (four significant figures), $b = 3.002$ m (four significant figures)

$$A = lb = 3.124 \times 3.002$$
$$= 9.378248$$

From given data, least significant number is four, so the result of this multiplication must have four significant figure

$$A = \mathbf{9.378\ m^2}$$

BA3. (a): Electric susceptibility is dimensionless.

BA4. (d): Coefficient of restitution $e = \dfrac{(v_2 - v_1)}{(u_2 - u_1)}$

e has no dimensions

BA5. (c): $\omega = \dfrac{1}{\sqrt{LC}} \Rightarrow T = 2\pi\sqrt{LC}$

BA6. (c): $M = k[E]^a[v]^b[F]^c$

$$= [M^aL^{2a}T^{-2a}][M^0L^bT^{-b}][M^cL^cT^{-2c}]$$
$$= [M^{a+c}L^{2a+b+c}T^{-2a-b-2c}]$$

$a + c = 1, \quad 2a + b + c = 0, \quad 2a + b + 2c = 0,$

$a = 1, \quad b = -2, \quad c = 0$

$M = \boldsymbol{k[E]^1[v]^{-2}}$

BA7. (a): Rest all pairs are measuring devices and respective quantities to be measured.

BA8. (b): Chronometer

BA9. (d): It is given that

n VSD = $(n - 1)$ MSD

$$1 \text{ VSD} = \frac{(n-1)}{n} \text{ MSD}$$

Least count is difference between main scale division and Vernier scale division.

$$\text{L.C.} = 1 \text{ MSD} - 1 \text{ VSD} = 1 \text{ MSD} - \frac{(n-1)}{n} \text{ MSD}$$
$$= \frac{1}{n} \text{ MSD}$$

There are n divisions per cm; hence each main scale division is equivalent to $\dfrac{1}{n}$ cm.

$$\text{L.C.} = \frac{1}{n} \times \frac{1}{n} \text{ cm} = \boldsymbol{\frac{1}{n^2}\ \text{cm}}$$

Hence option (d) is correct.

BA10. (a): $a = 1.2 \times 10^{-2}$ (two significant digits)

$$\text{Volume} = a^3 = 1.728 \times 10^{-6}\ \text{m}^3$$

Result of volume must have two significant figure.

$$\therefore \quad a^3 = \mathbf{1.7 \times 10^{-6}\ m^3}$$

BA11. (d): Diameter of the ball

= M.S.R + C.S.R × (Least count) – Zero error

$= 0.5 \text{ cm} + 25 \times 0.001 - (-0.004)$

$= 0.5 + 0.025 + 0.004 = \mathbf{0.529\ cm}$

Hence option (d) is correct.

BA12. (d)

BA13. (b): $\varepsilon_0 = \left(\dfrac{1}{4\pi}\right)\left(\dfrac{q_1q_2}{Fr^2}\right) = \dfrac{C^2}{Nm^2} = \mathbf{C^2/Nm^2}$

BA14. (b): Same dimensions can be added

This given formula is dimensionally valid

$$v = at + \frac{b}{t+c}$$

$at \rightarrow v$ (velocity)

$a[T] \rightarrow [LT^{-1}]$

$\Rightarrow \quad a \rightarrow [LT^{-2}]$

Again $\dfrac{b}{t+c} \rightarrow v$

But, $t + c$ = time = [T] $\Rightarrow c$ = [T]

$$\frac{b}{[T]} = [LT^{-1}]$$

$b = \mathbf{[L]}$

BA15. (b): $D = [M^{-1}L^{-3}T^3A^2]$

$$\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{Il}{VA} = \frac{I^2l}{(VI)A} = \frac{[A^2][L]}{[ML^2T^{-3}][L^2]}$$
$$= \mathbf{[M^{-1}L^{-3}T^3I^2]}$$

BA16. (a): Given error percentage of radius is $r = 2\%$

$$\frac{\Delta V}{V} = \frac{3\Delta r}{r} = 3 \times 2\% = \mathbf{6\%}$$

BA17. (b): Energy density = $\dfrac{\text{Energy}}{\text{Volume}} = \dfrac{Fl}{l^3}$

$$= \frac{F}{l^2} = \mathbf{[ML^{-1}T^{-2}]}$$

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = \mathbf{[ML^{-1}T^{-2}]}$$

BA18. (c): $R = \dfrac{V}{I} = \dfrac{W}{qI} = \dfrac{W}{I^2t} = \dfrac{[ML^2\ T^{-2}]}{[A^2T]}$

$= [ML^2T^{-3}A^{-2}]$ or $\mathbf{[ML^2T^{-3}I^{-2}]}$

BA19. (d): $F = at + bt^2$

$at \rightarrow$ Force = $[MLT^{-2}]$

$a[T] = [MLT^{-2}]$

$\Rightarrow \quad a = \mathbf{[MLT^{-3}]}$

Now $bt^2 \rightarrow$ Force = $[MLT^{-2}]$

$b[T^2] \rightarrow [MLT^{-2}]$

$\Rightarrow \quad b = \mathbf{[MLT^{-4}]}$

BA20. (d): $P = \dfrac{F}{A} = [ML^{-1}T^{-2}] \Rightarrow [M]^a\ [L]^b\ [T]^c$

$a = \mathbf{1}, b = \mathbf{-1}, c = \mathbf{-2}$

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BA21. (*d*): This formula is energy density for capacitor,

$$\frac{1}{2}\varepsilon_0 E^2 = \frac{\text{Energy}}{\text{Volume}} = \frac{W}{V} = \frac{Fd}{V} = \frac{ML^2T^{-2}}{L^3}$$

$$= \mathbf{[ML^{-1}T^{-2}]}$$

BA22. (*a*): $x = \frac{v_0}{a}(1 - e^{-at})$

$at \rightarrow$ constant

$\Rightarrow \quad a[T] \rightarrow k$

$\Rightarrow \quad a = [T^{-1}]$

$\therefore x$ is the position

$$[L] = \frac{v_0}{[T^{-1}]}$$

$$v_0 = \mathbf{[M^0LT^{-1}]}$$

BA23. (*d*): $F \propto v \Rightarrow F = kv$

$[MLT^{-2}] = k[LT^{-1}] \Rightarrow K = \mathbf{[ML^0T^{-1}]}$

BA24. (*b*): $C^2LR = [M^{-1}L^{-2}T^4I^2]^2[ML^2T^{-2}I^{-2}][ML^2T^{-3}I^{-2}]$

$= \mathbf{[M^0L^0T^3I^0]}$

BA25. (*a*): F = force T = time v = velocity

$F = [MLT^{-2}]$, $T = [T]$, $v = [LT^{-1}]$

$$\text{Mass} = M = \frac{F}{a} \quad (a \text{ is acceleration})$$

$$= \frac{Ft}{v} = \frac{[FT]}{[V]}$$

$$M = \mathbf{[FTV^{-1}]}$$

BA26. (*b*): debye

BA27. (*c*): $\mu \rightarrow$ permeability, $F = qvB$, $B = \frac{F}{qv}$

$$B = \frac{\mu}{4\pi} \times \left(\frac{Idl}{r^2}\right)$$

$$\frac{F}{qv} = \frac{\mu}{4\pi}\left(\frac{A\,L}{L^2}\right)$$

$$\left[\frac{MLT^{-2}}{AT \cdot LT^{-1}}\right] = \mu\left[\frac{A}{L}\right]$$

$$\mu = \frac{ML^2T^{-2}}{A^2T^0L}$$

$$= \mathbf{[MLT^{-2}\,A^{-2}]}$$

BA28. (*d*): Pressure $= P = \frac{a - t^2}{bx} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$

$t \rightarrow$ time, A $\rightarrow$ area

$$a = [T^2]$$

$$bx = \left(\frac{AT^2}{F}\right)$$

$$b[L] = \frac{L^2T^2}{MLT^{-2}}$$

$$b = M^{-1}L^0T^4$$

$$\frac{a}{b} = \frac{T^2}{M^{-1}L^0T^4} = \mathbf{ML^0T^{-2}}$$

BA29. (*b*): $l = 12$ cm, $b = 6$ cm, $t = 2.45$ cm

Volume $= l\,b\,t$

$= 12 \times 6 \times 2.45$

$= 176.4 \Rightarrow$ it must be converted into least significant figures in l, b and t

$= 1.764 \times 10^2$

$= \mathbf{2 \times 10^2\ cm^3}$

BA30. (*c*): 1 Light year $= 9.4607 \times 10^{12}$ Km

It is distance travelled by light in one year so,

Dimensions of light year, 1 ly = **[L]**

BA31. (*b*)

BA32. (*b*): Magnetic moment $= I.A$

$$M = \mathbf{[AL^2]}$$

BA33. (*b*): $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$[\mu_0\varepsilon_0]^{1/2} = \frac{1}{c} = \frac{1}{LT^{-1}} = \mathbf{[L^{-1}\,T]}$$

BA34. (*b*): $x = \frac{A^2B^{1/2}}{C^{1/3}D^3}$

$$\frac{\Delta x}{x} = \frac{2\Delta A}{A} + \frac{1}{2}\frac{\Delta B}{B} + \frac{1}{3}\frac{\Delta C}{C} \times 3\frac{\Delta D}{D}$$

$$\frac{\Delta x}{x} \times 100 = 2(1\%) + \frac{1}{2}(2\%) + \frac{1}{3}(3\%) + 3(4\%) = \mathbf{16\%}$$

Hence option (*b*) is correct.

BA35. (*d*):

$$\frac{\Delta P}{P} \times 100 = \pm\left[3\left(\frac{\Delta a}{a} \times 100\right) + 2\left(\frac{\Delta b}{b} \times 100\right) + \left(\frac{\Delta c}{c} \times 100\right) + \left(\frac{\Delta d}{d} \times 100\right)\right]$$

$$= \pm[3 \times 1\% + 2 \times 2\% + 3\% + 4\%]$$

% error in P = **14%**

BA36. (*c*): Surface Tension $= \frac{F}{l} = \frac{F \cdot l}{l^2} = \frac{E}{l^2}\frac{t^{-2}}{t^{-2}}$

$$= \frac{E\,T^{-2}}{L^2T^{-2}} = \frac{E\,T^{-2}}{[V]^2} = \mathbf{[EV^{-2}T^{-2}]}$$

BA37. (*a*): The dimensions of the quantities c, G and $\frac{e^2}{4\pi\varepsilon_0}$ are as follows:

$$[c] = [LT^{-1}]$$

$$[G] = [M^{-1}L^3T^{-2}]$$

$$\left[\frac{e^2}{4\pi\varepsilon_0}\right] = \left[ML^3T^{-2}\right]$$

Let the dimensions of the asked physical quantity (say) I be expressed as:

$$[I] = c^x G^y \left(\frac{e^2}{4\pi\varepsilon_0}\right)^z$$

Or, $[L] = [L^1T^{-1}]^x[M^{-1}L^3T^{-2}]^y[M^1L^3T^{-2}]^z$

$-y + z = 0 \Rightarrow y = z \quad \ldots(i)$

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$$x + 3y + 3z = 1 \quad ...(ii)$$
$$-x - 2y - 2z = 0 \quad ...(iii)$$

Solving equations (i), (ii) and (iii)

$$x = -2$$
$$z = y = \frac{1}{2}$$

Hence, I can be expressed as:

$$\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{\frac{1}{2}}$$

JEE (Main) & Other State Boards for Engineering Entrance

BB1. (a): Surface area of sphere is $S = 4\pi r^2$

$$\frac{\Delta S}{S} = 2\times\frac{\Delta r}{r} = \alpha$$

$$\Rightarrow \quad \frac{\Delta r}{r} = \frac{\alpha}{2} \quad ...(i)$$

Volume of Sphere $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \quad \frac{\Delta V}{V} = 3\times\frac{\Delta r}{r}$$

Substituting from equation (i) we get the following:

$$\frac{\Delta V}{V} = \frac{\mathbf{3}}{\mathbf{2}}\alpha$$

Hence option (a) is correct.

BB2. (d): It moves a distance of 0.25 cm in five rotations and there are 100 divisions on circular scale; hence

$$\text{Least Count of screw gauge} = \frac{0.25}{5\times100}\ \text{cm}$$
$$= 5\times10^{-4}\ \text{cm}$$

One rotation cover linear distance

$$= \frac{0.25\ \text{cm}}{5} = 0.05\ \text{cm}$$

Reading of thickness of wire

= Main Scale Division $\times n$ + Circular Scale Division $\times$ Least Count

$= 4\times0.05\ \text{cm} + 30\times5\times10^{-4}\ \text{cm}$

$= (0.2 + 0.0150)\ \text{cm} = \mathbf{0.2150\ cm}$

Hence option (d) is correct.

BB3. (a): Density d of the material can be written as mass (m) divided by volume (V).

$$d = \frac{m}{V} = \frac{m}{l^3}$$

$$\Rightarrow \quad \frac{\Delta d}{d}\times100 = \frac{\Delta m}{m}\times100 + \frac{3\Delta l}{l}\times100$$

$\Rightarrow$ % Error of density $= 1.5 + 3\times1 = \mathbf{4.5\%}$.

Hence option (a) is correct.

BB4. (d): According to given details when circular scale of screw gauge covers a distance of 1 mm, upon two circular turns.

The distance covered by the screw gauge in axial direction upon its one revolution is called the pitch of screw gauge.

Here, pitch $= \frac{1}{2}\text{mm} = 0.5\,\text{mm}$

Least count (L.C.)

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$
$$= \frac{0.5\ \text{mm}}{50} = 0.01\ \text{mm}$$

Main scale reading, M.S.R. = 3 mm

Circular scale reading, C.S.R. = 35

Zero error (Z.E.) is –0.03 mm

Reading noted by student

M.S.R. = 3 mm, C.S.R. = 35

The diameters of wire is

D = M.S.R. + C.S.R. $\times$ L.C. + Z.E.

$= 3\ \text{mm} + 35\times0.01\ \text{mm} + (-0.03)\ \text{mm}$

$= \mathbf{3.38\ mm}$

BB5. (d): Pitch = 0.5 mm

$$\text{L.C.} = \frac{0.5}{100} = 0.005\ \text{mm}$$

Zero error $= -3\times0.005 = -0.015$ mm

Actual value = Main scale reading + circular scale reading $\times$ L.C. – zero error

$= 5.5\ \text{mm} + (48\times0.005)\ \text{mm} + 0.015\ \text{mm}$

$= \mathbf{5.755\ mm}$

BB6. (a): $\frac{\Delta A}{A}\times100 =$

$$\frac{3\Delta P}{P}\times100 + \frac{2\Delta Q}{Q}\times100 + \frac{1}{2}\frac{\Delta R}{R}\times100 + \frac{\Delta S}{S}\times100$$
$$= 3\times0.5 + 2\times1 + \frac{1}{2}\times3 + 1.5$$
$$= 1.5 + 2 + 1.5 + 1.5$$
$$\frac{\Delta A}{A} = \mathbf{6.5\%}$$

Hence option (a) is correct.

BB7. (a): Given, 30 V.S.D. exactly match with 29 division of main scale division (M.S.D.)

One unit of vernier scale division

$$1\ \text{V.S.D.} = \frac{29}{30}\text{M.S.D.}$$

Least count, L.C. = 1 M.S.D. – 1 V.S.D.

$$= \frac{1}{30}\ \text{M.S.D.}$$

1 M.S.D. is equal to 0.5 degrees. [1° = 60 minutes]

$\frac{1}{30}$ M.S.D. is the $\frac{1}{30}\times30$ minutes

So, Least count is 1 minute

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BB8. (d): It moves a distance of 0.25 cm in five rotations and there are 100 divisions on circular scale; hence

$$\text{Least Count of screw gauge} = \frac{0.25}{5\times100}\ \text{cm}$$

$$= 5\times10^{-4}\ \text{cm}$$

One rotation cover linear distance

$$= \frac{0.25\ \text{cm}}{5} = 0.05\ \text{cm}$$

Reading of thickness of wire

= Main Scale Division $\times n$ + Circular Scale Division $\times$ Least Count

$= 4\times0.05\ \text{cm} + 30\times5\times10^{-4}\ \text{cm}$

$= (0.2 + 0.0150)\ \text{cm}$

$= \mathbf{0.2150\ cm}$

Hence option (d) is correct.

BB9. (d): Least count of vernier scale

L.C. = 1 M.S.D. – 1 V.S.D.

30 V.S.D. = 29 M.S.D.

$$1\ \text{V.S.D.} = \frac{29}{30}\text{M.S.D.}$$

So, $\text{L.C.} = 1\ \text{M.S.D.} - \frac{29}{30}\text{M.S.D.} = \frac{1}{30}\text{M.S.D.}$

One division in main scale has a measure of 0.5 degrees

$$1\ \text{M.S.D.} = \frac{1}{2}\ \text{degrees}$$

$$\frac{1}{30}\text{M.S.D} = \frac{1}{2}\times\frac{1}{30}\ \text{degrees}$$

$$= \frac{1}{60}\ \text{degrees}$$

L.C. = 1 minute

So, $\text{L.C.} = \frac{1}{60}$ degrees

Total reading = M.S.R. + V.S.R. $\times$ L.C.

$$= (58.5 + 9\times\frac{1}{60})\ \text{degrees}$$

$= (58.5 + 0.15)$ degree

$= \mathbf{(58.65)\ degree}$

BB10. (b): $R = \frac{V}{I}$

$$\frac{\Delta R}{R}\times100 = \pm\left[\frac{\Delta V}{V}\times100 + \frac{\Delta I}{I}\times100\right]$$

% error in R = ± [3% + 3%]

$= \mathbf{\pm\ 6\%}$

BB11. (a): From the Coulomb's law

$$\varepsilon_0 = \frac{1}{4\pi}\left(\frac{q^2}{Fr^2}\right)$$

$$= \frac{1}{4\pi}\left(\frac{I^2\cdot t^2}{F\cdot r^2}\right)$$

$$= \frac{A^2T^2}{MLT^{-2}L^2}$$

$$\varepsilon_0 = \mathbf{[M^{-1}L^{-3}T^4A^2]}$$

BB12. (d): $[t] = [G]^a\ [h]^b\ [c]^d$

$[T]' = [M^{-1}\ L^3\ T^{-2}]^a\ [M^1\ L^2\ T^{-1}]^b\ [L^1\ T^{-1}]^d$

$[T]' = [M^{-a+b}\ L^{3a+2b+d}\ T^{-2a-b-d}]$

$-a + b = 0$ ...(i)

$3a + 2b + d = 0$ ...(ii)

$2a + b + d = -1$ ...(iii)

From equation (i) (ii) and (iii) we get,

$$a = \frac{1}{2}, b = \frac{1}{2}, d = \frac{5}{2}$$

$$t = G^{\frac{1}{2}}h^{\frac{1}{2}}c^{-\frac{5}{2}} = \sqrt{\frac{Gh}{c^5}}$$

BB13. (a): We can write the acceleration due to gravity as:

$$g = \frac{4\pi^2L}{T^2}$$

Time period,

$$T = \frac{90}{100}\text{s}$$

Accuracy in time period,

$$\Delta T = \frac{1}{100}\text{s}$$

Accuracy in length, ΔL = 1 mm = 0.1 cm

$$\frac{\Delta g}{g}\times100 = \frac{2\Delta T}{T}\times100 + \frac{\Delta L}{L}\times100$$

$$= \frac{2\times0.01}{0.9}\times100 + \frac{0.1}{20}\times100$$

$= 2.23 + 0.5$

≈ 2.73

Accuracy in determination of g is approximately **3%**

BB14. (c): Mean time $T = \frac{90+91+95+92}{4} = 92$ sec.

The absolute errors are as follows:

$T_1 = 90 \Rightarrow |\Delta T_1| = |T_0 - T_1| = 2$

$T_2 = 91 \Rightarrow \Delta T_2 = |T - T_2| = 1$

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$T_3 = 95 \Rightarrow \Delta T_3 = |T - T_3| = 3$

$T_4 = 92 \Rightarrow \Delta T_4 = |T - T_4| = 0$

Mean error $\Delta T = \dfrac{2+1+3+0}{4} = 1.5$ seconds

Least count of measuring clock is 1 second.

The reported mean time

$= T \pm \Delta T$

$= \mathbf{(92 \pm 1.5)\ s}$

Minimum division in the measuring clock is 1 second, thus the reported mean time should be (92 ± 2)s

BB15. (*b*): Least count of screw gauge

$$\text{L.C.} = \frac{\text{Pitch}}{\text{Number of division on circular scale}} = \frac{0.5}{50}$$

$= 0.01$ mm

$= 0.001$ cm

Negative zero error $= -5 \times$ L.C.

$= -5 \times 0.01$ mm

$= -0.05$ mm

Measured thickness of aluminium sheet is

Δx = M.S.R. + C.S.R. × L.C. − zero error

$\Delta x = 0.5 \text{ mm} + 25 \times 0.01 - (-0.05)$

$\Delta x = \mathbf{0.80\ mm}$

BB16. (*b*): The quantity that is dimensionless and unitless has same value in all the systems of units.

Dimensional formulae:

$[e] = [M^0L^0T^1A^1]$

$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

$[G] = [M^{-1}L^3T^{-2}]$

$[m_e] = [M^1L^0T^0]$

$$\frac{e^2}{2\pi\varepsilon_0 G m_e^2} = \frac{[M^0L^0T^1A^1]^2}{[M^{-1}L^{-3}T^4A^2][M^{-1}L^3T^{-2}][M^1L^0T^0]^2}$$

$$= \frac{[M^0L^0T^2A^2]}{[M^0L^0T^2A^2]} = \mathbf{[M^0L^0T^0A^0]}$$

BB17. (*c*)

BB18. (*b*): Time period of simple pendulum on earth having effective length is l

$$T^2 = \frac{4\pi^2 l}{g} \Rightarrow g = 4\pi^2 \frac{l}{T^2}$$

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta l}{l} \times 100\right) + 2\left(\frac{\Delta T}{T} \times 100\right)$$

$$= \left(\frac{0.1}{20} \times 100\right) + 2\left(\frac{0.01}{0.9} \times 100\right)$$

$$= 0.5 + 2 \times \frac{10}{9} = 0.5 + 2.2 = \mathbf{2.7\%}$$

BB19. (*d*)

BB20. (*c*): Let the dimensional formula of μ_0 be expressed as:

$[\mu_0] = [e]^w\ [m]^x[h]^y[c]^z$

Writing dimensional formulae of the above given quantities:

$[MLT^{-2}A^{-2}] = [A^1T^1]^w[M]^x[ML^2T^{-1}]^y[LT^{-1}]^z$

$= [M^{x+y}L^{2y+z}T^{w-y-z}A^w]$

On comparing:

$x + y = 1$

$2y + z = 1$

$w - y - z = -2$

$w = -2$

We get: $w = -2, x = 0, y = 1, z = -1$

Thus, $\mu_0 = \left(\dfrac{\boldsymbol{h}}{\boldsymbol{ce^2}}\right)$

BB21. (*c*): The relation between current density J, electrical conductivity σ and electric field E is given by:

$J = \sigma E$

$$\sigma = \frac{J}{E} = \frac{I^1L^{-2}}{\dfrac{M^1L^1T^{-2}}{I^1T^1}}$$

$\sigma = M^{-1}L^{-3}T^3I^2$

BB22. (*b, d*): The product of B and D is dimensionless and the product of A and D has same dimensions as C.

Dimensionally we can also write:

$$B \propto \frac{1}{D}$$

$AD = C$

(*i*) $A^2 - (BC)^2 \Rightarrow \left(\dfrac{C}{D}\right)^2 - \left(\dfrac{C}{D}\right)^2$ meaningful

(*ii*) $\dfrac{(A-C)}{D}$ The dimensions of A and C are not same, hence this expression is meaningless

(*iii*) $\dfrac{A}{B} - C \Rightarrow \dfrac{AD}{BD} - C$

Or $\dfrac{C}{BD} - C$

BD is dimensionless thus both the terms C/BD and C have the same dimensions

(*iv*) $\dfrac{C}{BD} - \dfrac{AD^2}{C}$

$\Rightarrow \dfrac{C}{BD} - \dfrac{AD^2}{AD}$

$\Rightarrow \dfrac{\boldsymbol{C}}{\boldsymbol{BD}} - \boldsymbol{D}$

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The dimensions of C and D are not same, thus this combination does not give a meaningful quantity.

BB23. (*a*): Let us express the dimensions of the mass in terms of the time (T), velocity (C) and angular momentum (h)

$$[M^1L^0T^0] = [T^xC^yh^z]$$
$$[M^1L^0T^0] = [T^x][(LT^{-1})^y][(M^1L^2T^{-1})^z]$$
$$[M^1L^0T^0] = [M^zL^{y+2z}T^{x-y-z}]$$
$$z = 1 \quad \ldots(i)$$
$$y + 2z = 0 \quad \ldots(ii)$$
$$x - y - z = 0 \quad \ldots(iii)$$

Solving equations (*i*), (*ii*), and (*iii*)

$$x = -1, y = -2, z = 1$$

Hence, the correct dimensional formula is given by:

$$[M] = \mathbf{[T^{-1}C^{-2}h^1]}$$

BB24. (*c*): Given that: $P = a^{\frac{1}{2}} b^2c^3d^{-4}$

The maximum possible relative error in P can be found as follows:

$$\frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}$$
$$\frac{\Delta P}{P} \times 100\% = \frac{1}{2} \times 2\% + 2 \times 1\% + 3 \times 3\% + 4 \times 5\% = \mathbf{32\%}$$

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BB25. (*a*):

$$\text{Stress} = \frac{\text{Force}}{\text{area}}$$

The stress on the legs of the person can be calculated as:

$$\text{Stress} = \frac{mg}{A} = \frac{\text{Volume } (V) \times \text{density } (\pi) \times g}{\text{area } (A)}$$

Dimensionally, $V = L^3$, $A = L^2$

$$\text{Stress} = \frac{\boldsymbol{L^3\rho g}}{\boldsymbol{L^2}}$$

Thus, stress $\propto L$

As the linear dimensions of the person have increased by a factor of 9, thus the stress on the leg will also change by a factor of 9.

JEE (Advanced) for IIT Entrance

BC1. (*d*): 1 Main scale division (MSD) = 1 mm

$$1 \text{ Vernier scale division (VSD)} = \frac{16}{20} = 0.8 \text{ mm}$$

Least count is equal to difference between one main scale division and Vernier scale division. Hence least count (LC) can be written as follows:

LC = 1 MSD – 1 VSD = 1 – 0.8 = 0.2 mm

Hence option (*d*) is correct.

BC2. (*d*): Unit of dipole moment is C-m, unit of electric flux is N-m^2/C^2 and unit of electric field is N/C. Hence dimensions of these physical entities are different.

Hence option (*d*) is correct.

BC3. (*d*): $x = \varepsilon_0 L\frac{\Delta V}{\Delta t}$

$$\Rightarrow [x] = [\varepsilon_0][L]\frac{[\Delta V]}{[\Delta t]}$$

$$\Rightarrow [x] = [M^{-1}L^{-3}T^4I^2][L]\frac{[ML^2T^{-3}I^{-1}]}{[T]}$$

$$\Rightarrow [x] = \mathbf{[I]}$$

Hence dimension of given expression is same as that of current.

Hence option (*d*) is correct.

BC4. (*d*): C_1:

1 Main scale division (MSD) = 0.1 cm

$$1 \text{ Vernier scale division (VSD)} = \frac{0.9}{10} = 0.09 \text{ cm}$$

Least count (LC) = 1 MSD – 1 VSD = 0.1 – 0.09 = 0.01 cm.

Main scale reading (MSR) = 2.8 cm

Number of Vernier division matching main scale division (n) = 7

Reading (R_1) = MSR + $n \times$ LC = 2.8 + 7 × 0.01 = **2.87 cm**

C_2:

1 Main scale division (MSD) = 0.1 cm

$$1 \text{ Vernier scale division (VSD)} = \frac{1.1}{10} = 0.11 \text{ cm}$$

Least count (LC) = 1 VSD – 1 MSD = 0.11 – 0.1 = 0.01 cm.

Main scale reading (MSR) = 2.8 cm

Here we should note that size of Vernier scale division is bigger than main scale division; hence number of Vernier scale division matching main scale division will be counted from back. In given figure we can see that 7^{th} Vernier scale division is matching so we shall consider $10 - 7 = 3^{rd}$.

Number of Vernier division matching main scale division (n) = 3

Reading (R_2) = MSR + $n \times$ LC = 2.8 + 3 × 0.01 = **2.83 cm**

Noting down R_1 and R_2 we can see that option (*d*) is correct.

Hence option (*d*) is correct.

BC5. (*a*): Measured value of $r = (10 \pm 1)$ mm

$$\Delta r = 1 \text{ mm}$$
$$\text{Percentage error} = \frac{\Delta r}{r} \times 100 = \frac{1}{10} \times 100 = 10\%$$

We can see that option (*a*) is correct.

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Mean value of time period

$$T = \frac{(0.52 + 0.56 + 0.57 + 0.54 + 0.59)}{5}\text{s}$$

$\Rightarrow T = 0.556\text{ s} = 0.56\text{ s}$

Percentage error in time period $\simeq \frac{\Delta T}{T} \times 100$

$$= \frac{0.01}{0.56} \times 100 = 1.78\%$$

We can see that options (*b*) and (*c*) are wrong.

We know that absolute errors are added in case addition and subtraction hence we can write the following:

Measured value of $(R - r) = (50 \pm 2)$ mm

$$\text{Percentage error in } (R-r) = \frac{\Delta(R-r)}{R-r} \times 100 = \frac{2}{50} \times 100 = 4\%$$

$$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

$$\Rightarrow \quad \frac{\Delta g}{g} \times 100 = 2\left(\frac{\Delta T}{T} \times 100\right) + \frac{\Delta(R-r)}{(R-r)} \times 100$$

$$\Rightarrow \quad \frac{\Delta g}{g} \times 100 = 2 \times 1.78 + 4 = \mathbf{7.56\%}$$

So option (*d*) is also wrong.

Hence option (*a*) is correct.

BC6. (*c*): Let l_1 is the distance of null point from left and l_2 be the same from right end. Note that sum of l_1 and l_2 is 100 cm and that we can use later on. Using condition of balanced Wheatstone's bridge we can write the following equation:

$$\frac{R}{90} = \frac{l_1}{l_2}$$

$$\Rightarrow \quad R = 90\frac{l_1}{l_2} \qquad \ldots(i)$$

We can substitute l_1 and l_2 in above equation (*i*) to calculate value of R as follows:

$$R = 90 \times \frac{40}{60} = 60\Omega$$

From equation (*i*) we can write the following:

$$\frac{\Delta R}{R} = \frac{\Delta l_1}{l_1} + \frac{\Delta l_2}{l_2}$$

$$\Rightarrow \quad \frac{\Delta R}{60} = \frac{0.1}{40} + \frac{0.1}{60} \Rightarrow \Delta R = 6\left(\frac{1}{40} + \frac{1}{60}\right)$$

$$\Rightarrow \quad \Delta R = 6\left(\frac{6+4}{240}\right) = \frac{1}{4} = \mathbf{0.25\Omega}$$

Hence unknown resistance is: $(60 \pm 0.25)\Omega$. Option (*c*) is correct.

BC7. (*b*): 1 Main scale division (MSD) = 5.15 – 5.10 = 0.05 cm

50 divisions of Vernier scale are equivalent to 2.45 cm. Hence we can write the following:

$$\text{1 Vernier scale division (VSD)} = \frac{2.45}{50} = 0.049\text{ cm}$$

Least count of the Vernier caliper can be written as follows:

Least count (LC) = 1 MSD – 1 VSD = 0.05 – 0.049 = 0.001 cm

Main scale reading (MSR) = 5.10 cm

24^{th} division of Vernier scale matches with some main scale division. Final reading can be written as follows:

Reading = MSR + $n \times$ LC = $5.10 + 24 \times 0.001 = 5.124$ cm

Hence option (*b*) is correct.

BC8. (*d*): Using the given expression we can write the following:

$$d = \frac{\lambda}{2}\text{cosec}\,\theta$$

By differentiating the above relation we can write the following:

$$\frac{\Delta d}{\Delta\theta} = -\frac{\lambda}{2}\text{cosec}\,\theta\cot\theta$$

Hence absolute error in d can be written as follows:

$$|\Delta d| = \frac{\lambda}{2}\text{cosec}\,\theta\cot\theta\Delta\theta \qquad \ldots(i)$$

Fractional error in d can be written as follows:

$$\left|\frac{\Delta d}{d}\right| = \frac{\frac{\lambda}{2}\text{cosec}\,\theta\cot\theta\Delta\theta}{\frac{\lambda}{2}\text{cosec}\,\theta}$$

$$\Rightarrow \quad \left|\frac{\Delta d}{d}\right| = \cot\theta\Delta\theta \qquad \ldots(ii)$$

From equation (*i*) we can see that absolute error in d is not constant when θ is varied from 0 to 90°. Hence option (*a*) is wrong.

When θ is increased from 0 to 90° then value of cosec θ cot θ decreases from infinity to zero hence option (*b*) is wrong.

From equation (*ii*) we can see that fractional error in d changes when θ is changed hence option (*c*) is wrong.

When θ is increased from 0 to 90° then value of cot θ decreases hence option (*d*) is correct.

BC9. (*d*): Let L be the depth of well. Let t_1 is the time taken by the stone to touch the surface of water then we can write the following:

$$L = \tfrac{1}{2}gt^2 \Rightarrow t_1 = \sqrt{\frac{2L}{g}} = \sqrt{\frac{2L}{10}} = \sqrt{\frac{L}{5}}$$

Now sound takes time t_2 to reach the top and the same can be written as: $t_2 = L/300$ because sound moves with a speed 300 m/s.

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Total time taken by the sound to reach the person can be written as $t = t_1 + t_2$;

$$t = \sqrt{\frac{L}{5}} + \frac{L}{300}$$

On differentiating above equation we get the following:

$$dt = \frac{1}{\sqrt{5}}\frac{1}{2}L^{-1/2}\,dL + \left(\frac{1}{300}dL\right)$$

$$\Rightarrow \qquad \Delta t = \frac{1}{\sqrt{5}}\frac{1}{2}\frac{\Delta L}{\sqrt{L}} + \frac{\Delta L}{300}$$

We can substitute $L = 20$ m and $\Delta t = 0.01$ s in above expression to get the following:

$$\Rightarrow \qquad 0.01 = \Delta L\left(\frac{1}{\sqrt{20}}\frac{1}{\sqrt{20}} + \frac{1}{300}\right)$$

$$\Rightarrow \qquad 0.01 = \Delta L\left(\frac{1}{20} + \frac{1}{300}\right) = \Delta L\frac{15+1}{300}$$

$$\Delta L = \frac{0.01\times 300}{16} = \frac{3}{16}$$

$$\frac{\Delta L}{L}\times 100 = \frac{3}{16}\times\frac{1}{20}\times 100 = \frac{15}{16} \simeq \mathbf{1\%}$$

BC10. (*b*): Young's modulus for the wire can be written as follows:

$$Y = \frac{FL}{Al}$$

Here F is applied load ($F = mg$), L is original length of wire, l is extension and A is the area which can be written as $A = \pi d^2/4$, where d is diameter of the wire. Hence above expression can be rewritten as follows:

$$\Rightarrow \qquad Y = \frac{4mgL}{\pi d^2 l} \qquad \ldots(i)$$

First let us calculate value of Young's modulus from equation (*i*) and later we shall calculate error.

$$\Rightarrow Y = \frac{4\times 1\times 9.8\times 2}{3.14\times(4\times 10^{-4})^2\times(8\times 10^{-4})} = 0.195\times 10^{12}\ \text{N/m}^2$$

$$\Rightarrow \qquad Y = 2\times 10^{11}\ \text{N/m}^2$$

In equation (*i*) error is in the measurements of d and l.

$$\frac{\Delta Y}{Y} = 2\frac{\Delta d}{d} + \frac{\Delta l}{l}$$

$$\Rightarrow \qquad \frac{\Delta Y}{2\times 10^{11}} = 2\frac{0.01}{0.4} + \frac{0.05}{0.8}$$

$$\Rightarrow \qquad \frac{\Delta Y}{2\times 10^{11}} = 2\frac{0.01}{0.4} + \frac{0.05}{0.8} = 0.05 + 0.0625 = 0.1125$$

$$\Rightarrow \qquad \Delta Y = 2\times 10^{11}\times 0.1125 = 0.225\times 10^{11}\ \text{N/m}^2$$

$$= \mathbf{0.2\times 10^{11}\ N/m^2}$$

Hence we can see that option (*b*) is correct.

BC11. (*b*): $\dfrac{dr}{da} = \dfrac{(1+a)(-1)-(1-a)}{(1+a)^2}$

$$\Rightarrow \qquad dr = \frac{-2da}{(1+a)^2}$$

$$\Rightarrow \qquad |\Delta r| = \frac{\mathbf{2\Delta a}}{\mathbf{(1+a)^2}}$$

BC12. (*c*): Pitch of the screw gauge is 0.5 mm and there are 50 divisions on circular scale; hence least count of the instrument can be written as follows:

$$\text{Least count (LC)} = \frac{0.5}{50} = 0.01\ \text{mm}$$

Diameter of the ball can be written as

$$d = \text{MSR} + \text{CSR}\times\text{LC} = 2.5 + 20\times 0.01 = 2.5 + .20$$
$$= 2.70\ \text{mm}$$

Error in the measurement of diameter is equal to least count and can be written as

$$\Delta d = 0.01\ \text{mm}$$

Density (ρ) of the material can be written as follows:

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} = \frac{6\ m}{\pi d^3}$$

From the above expression we can write the following:

$$\frac{\Delta\rho}{\rho}\times 100 = \frac{\Delta m}{m}\times 100 + 3\frac{\Delta d}{d}\times 100$$

$$\frac{\Delta\rho}{\rho}\times 100 = 2 + 1.11 = \mathbf{3.11\%}$$

Correct answer is (*c*).

BC13. (*a*): Least count of the screw gauge and micrometer both are same and can be written as follows:

$$\text{Least count (LC)} = \frac{\text{pitch}}{\text{number of divisions on CS}} = \frac{0.5}{100}$$

Error in the measurement of d and l will be equal to least count.

$$\Delta d = \Delta l = 0.005\ \text{mm}$$

From the given relation $\left(Y = \dfrac{4MLg}{\pi l d^2}\right)$ we can write the following:

$$\frac{\Delta Y}{Y} = \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$$

Contribution to total relative error due l can be written as

$$\frac{\Delta l}{l} = \frac{0.005}{0.25} = 0.02$$

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Contribution in total error due to d can be written as

$$2\frac{\Delta d}{d} = 2\frac{0.005}{0.5} = \mathbf{0.02}$$

We can see that both the contributions are equal and thus option (a) is correct.

C. *MCQs with more than one correct answer*

C1. (a, c, d): Both Reynolds number and strain are dimensionless entities. Torque is force multiplied with length hence torque and force have different dimensions. Curie is disintegrations per unit time and frequency is number of cycles per unit time hence both have same dimensions. Gravitational potential is defined as change in potential energy per unit mass and latent heat is energy per unit mass to change the state of a material; hence both have same dimensions.

C2. (a, c): It is given that main scale divisions are made at 0.05 cm separation.

1 MSD = 0.05 cm

50 divisions of Vernier scale is equivalent to 2.45 cm; hence length of one Vernier scale division can be written as follows:

$$1\text{ VSD} = \frac{2.45}{50} = 0.049\text{ cm}$$

Least count is defined as difference in length of main scale and Vernier scale division.

LC = 1 MSD – 1 VSD = 0.05 – 0.049 = 0.001 cm.

Main scale reading is given to be 4.15 and it is given that 24th division of Vernier scale matches with some division of main scale; hence reading of the Vernier caliper can be written as follows:

$$\text{reading} = \text{MSR} + \text{VSR} \times \text{LC}$$

$$\Rightarrow \text{reading} = 4.15 + 24 \times 0.001 = \mathbf{4.174\text{ cm}}$$

C3. (a, b): Physical quantities having different dimensions cannot be added or subtracted but can be multiplied or be divided.

C4. (a, d): Least count is defined as difference between one main scale division and one Vernier scale division. Refer to solution of C2.

C9. (b, d)

C10. (b, c, d): Mass, length and time should be involved in each set, only then these can be used as fundamental units in any system of units.

C11. (b, d): Permittivity: $[\varepsilon] = [M^{-1}L^{-3}T^4I^2]$

Boltzmann constant: $[k_B] = [ML^2T^{-2}\theta^{-1}]$

Absolute temperature: $[T] = [\theta]$

Number per unit volume: $[n] = [L^{-3}]$

Charge: $[\theta] = [TI]$

Option (b): $\sqrt{\frac{\varepsilon k_B T}{nq^2}} = \sqrt{\frac{L^{-1}T^2I^2}{L^{-3}T^2I^2}} = [L]$

Option (d): $\sqrt{\frac{q^2}{\varepsilon n^{1/3} k_B T}} = \sqrt{\frac{A^2T^2}{[L^{-1}T^2A^2]\times[L^{-3}]^{\frac{1}{3}}}}$

$$= \sqrt{\frac{1}{[L^{-1}]\times[L^{-1}]}} = [L]$$

We can easily understand that dimensions of remaining options are not equal to that of length.

Hence options (b) and (d) are correct.

C12. (a, c): We can see that time period is $T = \frac{40}{20} = 2\text{s}$

A total error of 1 s will get divided among 20 oscillations; hence error in the time period is given by

$$\Delta T = \frac{1}{20} = 0.05\text{s}$$

We can see that option (a) is correct.

Further, we know that time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = \frac{4\pi^2 l}{T^2}$$

Here length is measured to be exact; hence it will not be included for error calculation. For above relation for acceleration due to gravity we can write the following:

$$\frac{\Delta g}{g}\times 100 = 2\frac{\Delta T}{T}\times 100 = 2\frac{0.05}{2}\times 100 = 5\%$$

We can see that option (c) is also correct.

C13. (c, d): We can use the mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ to verify the results. We can use given value of u to calculate the value of v using given focal length of mirror. Then we can compare calculated value of v with the given value of v in the question. We can verify that in last two readings recorded values are not within the range of maximum permissible error of 0.2 cm. Hence options (c) and (d) are found to be correct.

D. *MCQs based on passage / comprehension*

Passage 1

D1. (a): 1 MSD = 1 mm.

10 Vernier scale divisions match with 9 main scale divisions hence

1 VSD = 9/10 = 0.9 mm

Least count of the Vernier caliper can be written as follows:

LC = 1 MSD – 1 VSD = 1 – 0.9 = **0.1 mm**.

D2. (b): Main scale reading (MSR) is given to be 10 and Vernier scale reading (VSR) reading is 1, because 1st division of Vernier scale matches with some division of main scale. Let x represent edge length then we can write the following:

$x = \text{MSR} + \text{VSR} \times \text{LC} = 10 + 1 \times 0.1 = 10.1$ mm.

$\Rightarrow x = \mathbf{1.01\text{ cm}}$

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D4. (a): Volume of the cube can be written as follows:

$V = x^3 = (1.01)^3 = 1.0303\ \text{cm}^3$.

Since edge length is measured only up to 3 significant digits; hence volume must also be expressed in 3 significant digits. On rounding off the volume of cube to 3 significant digits we can write

$V = \mathbf{1.03\ cm^3}$

D5. (b): Density of material $= \dfrac{m}{v} = \dfrac{2.736}{1.03} = 2.65631\ \text{g/cm}^3$

Number of significant digits in this value should be 3; hence we can round it off accordingly.

Thus density of material = $\mathbf{2.66\ g/cm^3}$.

INTEGER TYPE

1. [2]: Moment of inertia is mass × (distance)^2

Dimensional formula becomes ML^2T^0

We can see that dimension of length is **2**.

2. [1]: $[a\sqrt{x}] = [F] \Rightarrow [a] = \dfrac{[F]}{[\sqrt{x}]}$

$\Rightarrow \quad [a] = \dfrac{MLT^{-2}}{L^{1/2}} = ML^{1/2}T^{-2}$

Similarly $[bt^2] = [F] \Rightarrow [b] = \dfrac{[F]}{[t^2]}$

$\Rightarrow \quad [b] = \dfrac{MLT^{-2}}{T^2} = MLT^{-4}$

$\left[\dfrac{a}{b}\right] = \dfrac{ML^{1/2}T^{-2}}{MLT^{-4}} = L^{-1/2}T^2$

On comparing with the given dimensional formula.

$q = -1/2,\ r = 2$

$-qr = \mathbf{1}$

3. [6]: Let $M \propto v^a \rho^b g^c$

$\Rightarrow \quad [M] \propto [v]^a [\rho]^b [g]^c$

$\Rightarrow \quad [M] = [LT^{-1}]^a [ML^{-3}]^b [LT^{-2}]^c$

$\Rightarrow \quad [M^0L^0T^0] = [M^bL^{a-3b+c}T^{-a-2c}]$

Comparing the dimensions on both the sides we get the following:

$b = 1$

$a - 3b + c = 0 \Rightarrow a - 3 + c = 0$

$\Rightarrow \quad a + c = 3$

Also,

$-a - 2c = 0$

Adding the above two equations we get $-c = 3 \Rightarrow c = -3$.

$\Rightarrow \quad -a - 2 \times -3 = 0 \Rightarrow a = \mathbf{6}$.

Hence the value of $n = 6$.

4. [4]: Given energy of the system can be written as follows:

$E = A^2 e^{-\alpha t}$

We can differentiate the above to get the following:

$dE = 2A(dA)(e^{-\alpha t}) + A^2(-\alpha e^{-\alpha t})(dt)$

$\Rightarrow \quad \dfrac{dE}{E} = \dfrac{2A(dA)(e^{-\alpha t}) + A^2(-\alpha e^{-\alpha t})(dt)}{A^2 e^{-\alpha t}}$

$\Rightarrow \quad \dfrac{dE}{E} = \dfrac{2(dA)}{A} + (-\alpha)(dt)$

In the calculation of error, all terms are added so that we may get maximum possible error. Hence using just magnitudes of each term we can write the above expression as follows:

$\Rightarrow \quad \dfrac{\Delta E}{E} = 2\dfrac{\Delta A}{A} + \alpha \Delta t$

$\Rightarrow \quad \dfrac{\Delta E}{E} \times 100 = 2\dfrac{\Delta A}{A} \times 100 + 100\alpha\Delta t \qquad \ldots(i)$

Given: $\dfrac{\Delta A}{A} \times 100 = 1.25 \qquad \ldots(ii)$

Given: $\dfrac{\Delta t}{t} \times 100 = 1.5$

Hence at $t = 5$ s we can write the following:

$\Delta t = \dfrac{1.5 \times 5}{100} \qquad \ldots(iii)$

Given: $\alpha = 0.2 s^{-1} \qquad \ldots(iv)$

Substituting values from equations (ii), (iii) and (iv) in equation (i) we get the following:

$\Rightarrow \quad \dfrac{\Delta E}{E} \times 100 = 2 \times 1.25 + 100 \times 0.2 \times \dfrac{1.5 \times 5}{100}$

$\Rightarrow \quad \dfrac{\Delta E}{E} \times 100 = \mathbf{4}$

Hence percentage error in the value of energy at $t = 5$ s is 4.

5. [8]: In both the measurements of Vernier, main scale reading (MSR) is 3.20×10^{-2} m.

Least count of Vernier (LC) = 1.0×10^{-5} m

In first case 20$^{\text{th}}$ division of Vernier matches with one main scale division.

First reading of Vernier

$L_1 = \text{MSR} + n \times \text{LC} = 3.20 \times 10^{-2} + 20 \times 1.0 \times 10^{-5} = 3.220 \times 10^{-2}$ m

In second case 45$^{\text{th}}$ division of Vernier matches with one main scale division.

Second reading of Vernier

$L_2 = \text{MSR} + n \times \text{LC} = 3.20 \times 10^{-2} + 45 \times 1.0 \times 10^{-5} = 3.245 \times 10^{-2}$ m

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Extension in the wire due to 2 kg load can be written as:

$l = L_2 - L_1 = 0.025 \times 10^{-2}$ m

Error in the measurement of L_1 and L_2 both is same as least count (1×10^{-5} m) hence for their difference errors will be added. So error in the measurement of elongation will be two times the LC.

$$\Delta l = 2 \times 10^{-5} \text{m}$$

Young's modulus can be written as follows:

$$Y = \frac{FL}{Al}$$

Here force (F), length of wire (L) and area of cross section (A) are given to be exact; hence error in the calculation of Y can be written as follows:

$$\frac{\Delta Y}{Y} \times 100 = \frac{\Delta l}{l} \times 100 = \frac{2 \times 10^{-5}}{0.025 \times 10^{-2}} \times 100 = \mathbf{8\%}$$

Hence answer is 8.

6. [3]: Density: $[\rho] = [ML^{-3}]$

Intensity: $[S] = \left[\frac{ML^2T^{-3}}{L^2}\right] = [MT^{-3}]$

Frequency: $[f] = [T^{-1}]$

Let us assume that

$$d \propto \rho^a S^b f^c$$

$\Rightarrow$ $[d] = [\rho]^a [S]^b [f]^c$

$\Rightarrow$ $[L] = [ML^{-3}]^a [MT^{-3}]^b [T^{-1}]^c$

$\Rightarrow$ $[L] = [M^{a+b} L^{-3a} T^{-3b-c}]$

On comparing dimensions on both the sides we get the following:

$-3a = 1 \Rightarrow a = -1/3$

$a + b = 0 \Rightarrow b = -a = 1/3$

We can see that $d \propto S^{1/3}$

Hence value of $n = \mathbf{3}$.

Answer is 3.

NCERT Exemplar Problems: MCQs Type-I

2.1. (b): The number of significant figure in 0.06900 is four.

The correct option is (b).

2.2. (b): Here, 0.301 has least number of significant figures, equal to three.

$\therefore$ The sum of 436.32, 227.2 and 0.301 must also have three significant figures.

$436.32 + 227.2 + 0.301 = 663.821$

$= 664$ (3 significant figures)

The correct option is (b).

2.3. (c): Given, mass = 4.237 g

Volume = 2.5 cm^3

$\therefore$ Density $= \frac{\text{Mass}}{\text{Volume}} = \frac{4.237}{2.5} = 1.6948$ gcm^{-3}

Here, volume has least number of significant figures, equal to two.

$\therefore$ Density = 1.7 g/cm^3 (rounded off to two significant figures)

The correct option is (c).

2.4. (d): When rounded off to 3 significant figures both 2.745 and 2.735 with give the value 2.74

The correct option is (d).

2.5. (a): Given, length, $l = (16.2 \pm 0.1)$ cm

Breadth, $b = (10.\ 1 \pm 0.1)$ cm

$\therefore$ Area, $A = l \times b = (16.2 \times 10.1)$ cm^2

$= 163.62$ cm^2

$= 164$ cm^2 (3 significant figures)

Relative error in A is

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

$$= \frac{0.1}{16.2} + \frac{0.1}{10.1}$$

$$= (6.173 + 9.901) \times 10^{-3}$$

$$= 16.074 \times 10^{-3} = 0.016074$$

$$= 0.016$$

$\therefore$ $\Delta A = 0.016 \times A$

$= 0.016 \times 164$ cm^2

$= 2.636$ cm^3

$= 3$ cm^2 (1 significant figure)

$\therefore$ Area, $A = \mathbf{(164 \pm 3)\ cm^2}$

The correct option is (a).

2.6. (c): [Tension] = [Force] = $[MLT^{-2}]$

$$[\text{Surface Tension}] = \frac{[\text{Force}]}{[\text{Length}]} = \frac{[MLT^{-2}]}{L}$$

$= \mathbf{ML^0 T^{-2}}$

The correct option is (c).

2.7. (a): Given, $A = 2.5$ $ms^{-1} \pm 0.5$ ms^{-1}

B = 0.10 s ± 0.01 s

Let X = AB

$= 2.5 \times 0.10 = 0.25$ m

Relative error is

$$\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

$$= \frac{0.5}{2.5} + \frac{0.01}{0.10}$$

$= 0.2 + 0.1 = 0.3$

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$\Delta X = 0.3 \times X$

$= 0.3 \times 0.25 \text{ m} = 0.075 \text{ m}$

$= 0.08 \text{ m}$ (2 significant figures)

$\therefore$ $X = \mathbf{(0.25 \pm 0.08)\ m}$

The correct option is (*a*).

2.8. (*d*): Given, $A = 1.0 \text{ m} \pm 0.2 \text{ m}$

$B = 2.0 \text{ m} \pm 0.2 \text{ m}$

Let $X = \sqrt{AB}$

$= \sqrt{(1.0)(2.0)} = \sqrt{2.0} = 1.414 \text{ m}$

$= 1.4$ (2 significant figures)

Relative error is

$$\frac{\Delta X}{X} = \frac{1}{2}\frac{\Delta A}{A} + \frac{1}{2}\frac{\Delta B}{B}$$

$$= \frac{1}{2}\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

$$= \frac{1}{2}\left(\frac{0.2}{1.0} + \frac{0.2}{2.0}\right)$$

$$= \frac{1}{2}(0.2 + 0.1) = \frac{0.3}{2} = 0.15$$

$\Rightarrow$ $\Delta x = 0.15X = 0.15 \times 1.414$

$= 0.2121 \text{ m}$

$= 0.2$ (1 significant figure)

$\Rightarrow$ $X = \mathbf{1.4 \text{ m} \pm 0.2 \text{ m}}$

The correct option is (*d*).

2.9. (*a*): The measurement, which is measured with an instrument having minimum least count, is the most precise.

Here 5.00 mm is the most precise measurement.

The correct option is (*a*).

2.10. (*a*): The measurement with the least difference with mean value.

The correct option is (*a*). Because it is closest to 5 cm.

2.11. (*c*): Given $1 \text{ N} = 10^5$ dyne

and $1 \text{ m}^2 = 10^4 \text{ cm}^2$

Young's modulus, $Y = 1.9 \times 10^{11} \text{ N m}^2$

$$= 1.9 \times 10^{11} \times \frac{10^5 \text{ dyne}}{10^4 \text{ cm}^2}$$

$= \mathbf{1.9 \times 10^{12} \text{ dyne/cm}^2}$

The correct option is (*c*).

2.12. (*d*): We know that

$[P] = [MLT^{-1}]$

$[A] = [L^2]$

$\Rightarrow$ $[L] = [A^{1/2}]$

Also $[P]\ [T] = [ML]$

$\Rightarrow$ $M = \frac{[PT]}{[L]} = \frac{[PT]}{[A^{\frac{1}{2}}]} = [PTA^{-\frac{1}{2}}]$

$\therefore$ Energy $= [ML^2 T^{-2}] = [PTA^{-1/2} AT^{-2}]$

$= \mathbf{[PA^{1/2} T^{-1}]}$

The correct option is (*d*).

NCERT Exemplar Problems: MCQs Type-II

2.13. (*b, c*): In option (*b*) and (*c*), the arguments of sine are not dimensionless (vt and t/a). So, options (*b*) and (*c*) are dimensionally incorrect.

The correct options are (*b*) and (*c*).

2.14. (*a, e*): Given that P, Q and R are physical quantities with different dimensions. So, neither of the three quantities can be added or subtracted (any two out of three).

Thus, combination (*a*) and (*c*) can never be a meaningful quantity.

In options (*b*) and (*d*), product of PQ or PR can be equal to R or Q^2, respectively, so this can be a meaningful relation.

The correct options are (*a*) and (*e*).

2.15. (*b, d*): $E = h\nu$

$\Rightarrow$ $h = \frac{E}{\nu}$

or $[h] = \frac{[E]}{[\nu]} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

Linear Impulse = Force × change in time

$= F \times t$

$= [MLT^{-2}] \times [T]$

$= [MLT^{-1}]$

Angular impulse = Torque × Time

$= \tau \times t$

$= [ML^2T^{-2}] \times [T]$

$= [ML^2T^{-1}]$

Linear momentum = mass × velocity

$= m \times v$

$= [M]\ [LT^{-1}]$

$= [M\ LT^{-1}]$

Angular momentum $= m\ v\ r$

$= [M]\ [LT^{-1}]\ [L]$

$= \mathbf{[ML^2T^{-1}]}$

So, the dimensions of angular impulse and angular momentum are same as that of h.

The correct options are (*b*) and (*d*).

2.16. (*a, b, d*): Here, $[h] = [ML^2T^{-1}]$

Speed of light $[c] = [LT^{-1}]$

Mass of electron, $[m_e] = [M]$

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Universal gravitational constant, $[G] = [M^{-1} L^3 T^{-2}]$
Charge of electron, $[e] = [AT]$
Mass of proton, $[m_p] = [M]$

Now, $\left[\frac{hc}{G}\right] = \frac{[ML^2T^{-1}][LT^{-1}]}{[M^{-1}L^3T^{-2}]} = [M^2]$

$\Rightarrow$ $[M] = \left[\frac{hc}{G}\right]^{\frac{1}{2}}$

Also, $\left[\frac{h}{c}\right] = \frac{[ML^2T^{-1}]}{[LT^{-1}]} = [ML]$

$\Rightarrow$ $[L] = \left[\frac{h}{cm}\right] = \left[\frac{h}{c}\right]\left[\frac{G}{hc}\right]^{\frac{1}{2}} = \frac{[hG]^{\frac{1}{2}}}{[c]^{\frac{3}{2}}}$

As $[c] = [LT^{-1}]$

$\Rightarrow$ $[T] = \left[\frac{L}{c}\right] = \frac{[hG]^{\frac{1}{2}}}{[c]^{\frac{5}{2}}}$

Any of the options (a), (b) and (d) can be used to express L, M and T in terms of the given fundamental quantities.

The correct options are (a), (b) and (d).

2.17. (a, b): Pressure $= \frac{\text{Force}}{\text{Area}}$

$[\text{Pressure}] = [MLT^{-2}] = [ML^{-1}T^{-2}]$

Now, $\frac{\text{Energy}}{\text{Volume}} = \frac{\text{Force}}{\text{Area} \times \text{distance}} = \frac{[ML^2T^{-2}][L]}{[L^2][L]}$

$= [ML^{-1}T^{-2}]$

$= [\text{Pressure}]$

The correct options are (a) and (b).

2.18. (b, d): Parsec and light year are units of distance not time.

The correct options are (b) and (d).

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Chapter Practice Test

for Board Examination

Time Allowed: 2 hours **Maximum Marks: 35**

1. What are different systems of units used for measuring all kinds of physical quantities? (1)

2. How many astronomical units are equal to one light year? (1)

3. Define errors in a measurement of any physical quantity. (1)

4. Define precision and accuracy. (1)

5. How many significant digits are there in 7521.05890? (1)

6. Convert 1° and 1′ to radians. (2)

7. Check the dimensional consistency of the equation $t = 2\pi\sqrt{\frac{l}{g}}$ (2)

8. How can we minimise the random errors in an experiment? (2)

9. Express Newton in terms of dyne. (2)

10. A physical balance of least count 0.1 gram is used to measure the mass of an object. Write the results if mean value of the observed reading is 125.4 g. (3)

11. The diameter of cylinders are $d_1 = (2.02 \pm 0.01)$ cm and $d_2 = (3.21 \pm 0.03)$ cm respectively. Find the difference between their diameters along with the error limits. (3)

12. Explain three types of errors observed in measurement. (3)

13. Compute the following with regard to significant figures. (3)

(*a*) 5.3×0.185

(*b*) $\frac{0.9251 \times 1.03}{1.285}$

14. How do we measure the standard international unit for mass, length and time? Also explain why they are also called fundamental units? (5)

15. Write a short note on uses of dimensional analysis. Also mention the limitations of dimensional analysis. (5)

► To check your performance, see HINTS AND SOLUTIONS TO SOME QUESTIONS at the end of Part I of the book.

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CHAPTER

3

MOTION IN A STRAIGHT LINE

1. INTRODUCTION

We know that the motion is an integral part of our life. If we observe ourselves, then we find that we are moving here and there to perform our routine tasks. We can walk and run or we can use some vehicle to change our position from one place to another. We are living on the planet earth, which itself is moving around the sun in a big circular path. Apart from the revolution around the sun, earth is also rotating about its own axis which passes through the centre of the earth. There are other planets which are also revolving around the sun. But it is not the end, because this whole planetary system around the sun including sun is also revolving around the centre of the Milky Way galaxy, which in turn is moving within the group of galaxies. In nature we may observe the motion as translation, rotation and vibration. It is not only at a big scale that motion is taking place. There is a continuous motion at the microscopic scales also. For example, structure of an atom is highly dynamic. We know that electrons are moving around the nucleus at such a high speed. Many times motion at microscopic scale is beyond our comprehension. For example, there is a glass filled with water kept on a table and we may not appreciate any kind of motion in water inside the glass. But the fact is that molecules of water are randomly moving within the volume of water and that too very fast. Similarly, if we observe an iron block kept on table then we may not observe any motion but at the microscopic levels we know that particles like electrons, protons and neutrons within the material are in a continuous state of motion.

Firstly, let us define motion without any discussions about the cause of motion. **Such a description of motion, without considering the cause of motion, is studied under kinematics.** ***Motion is change in position of an object with the passage of time.*** In this chapter we shall learn the concepts of velocity and acceleration. To learn these concepts we shall first confine ourselves to the motion in a straight line. Motion of an object in a straight line is called rectilinear motion. In this chapter we shall also derive standard equations to describe the motion of object moving with uniform acceleration.

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Learning & More

Mechanics is the branch of science which deals with the study of motion of objects. It is further classified into statics, kinematics and dynamics. In statics, we study about the mechanics related to objects at rest. In kinematics, we study about the object in motion irrespective of the cause of the motion. However, in dynamics, we study about the mechanics related to the objects in motion taking into account the causes of motion too.

Motion is caused by forces but in this chapter cause of the motion will not be discussed. We shall discuss only about the motion or changes in motion. In this chapter we shall discuss about the objects in motion which are speeding up or slowing down to stop or reversing its direction of motion and most importantly how time is involved in these changes.

In this chapter we shall discuss the motion of a particle or we may call it a point object. A point object is something which does not occupy any space. To consider object as point object is often useful in describing motion of objects. If an object is moving without changing its orientation then its motion is purely translation and object can be considered as point object for describing its motion. There are cases where considering object like point object is very much justified. For example, when we talk about motion of earth around the sun then earth can be treated like point object because the size of earth is too small in comparison to its orbital radius and the size of the sun. In later chapters we shall learn about the centre of mass of object where entire mass of the object can be assumed to be concentrated and in many cases we assume the location of centre of mass as the location of equivalent point object. When we shall study about rotational dynamics in mechanics then only size and shape of object will play an important role but before that most of the mechanics' concepts are not affected by the size and shape of the objects but are based on location of centre of mass and its motion. Hence, we can understand the practical importance of this concept of point object. Concept of point object is not just an approximation and it is much more than that.

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1.1. Vectors and Scalars

In physics we study several physical quantities. ***The physical quantities which require only magnitude for their complete description are known as scalar quantities. Examples of scalar quantities are mass, speed, distance, energy, etc. But there are some physical quantities which are direction oriented. Physical quantities which require magnitude as well as direction for their complete description are called vector quantities***. Examples of vector quantities are displacement, velocity, acceleration, force, linear momentum, etc. We shall learn about these and many other vector quantities in other chapters of this book. Whenever we talk about a vector quantity then we need to mention about its direction also. Calculations involving vector quantities are affected by their directions and we need to learn vector algebra to completely understand the mathematics behind the vector quantities. Vector algebra is a part of next chapter and there we shall learn about it in detail. In the context of this chapter, only a little information about vector algebra is required.

Time is an important scalar quantity in kinematics. Vector quantities involved in kinematics are displacement, velocity and acceleration. In this chapter we shall learn about motion in a straight line and in this case all involved vectors are directed along the same straight line. These vectors may be parallel or anti-parallel to each other. We assign one direction along the line as positive and hence its opposite is treated as negative. Magnitudes of vectors along positive direction are written with positive sign and for those along negative direction are written with negative sign. While performing calculations we need to take into account these positive and negative signs on vector quantities. Hence, we can understand that in vector algebra, positive and negative

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signs are used to represent two opposite directions on a straight line. Generally, vectors are represented with a small line or an arrowhead on the symbol of physical quantity, for example velocity can be represented as $\vec{v}$ or $\bar{v}$.

2. POSITION, PATH LENGTH AND DISPLACEMENT

A particle is said to be in a state of motion when it changes its position in space with time. How to measure the position of particle? We need a reference point to measure the position of particle. Let some particle be located at a point in space and we want to describe its position. We are living in a three-dimensional world and in general we can use X-Y-Z coordinate system to describe the position of a point in space. This rectangular coordinate system consists three mutually perpendicular axes which are known as X, Y and Z axes. These three axes perpendicularly intersect each other at a common point called origin of this coordinate system. We always need a reference to specify position of some point and here origin serves as a reference point. Positive direction along each axis is specified and hence opposite will be treated as negative direction. The coordinates (x, y, z) can be used to describe the position of particle in space. If there is a clock to measure the time then ***coordinate system along with the clock is called frame of reference***. Three axes of coordinate system represent three dimensions and we should always remember that there are two opposite directions in every dimension.

There are two types of frames of reference, inertial frame of reference and non-inertial frame of reference. The one in which Newton's law of motion is followed is inertial frame of reference. A frame of reference which contains a passenger sitting in a car either at rest or moving with constant velocity along the straight line is an example of inertial frame of reference. The one in which Newton's law of motion is not followed is called non-inertial frame of reference. A frame of reference which contains a passanger sitting in a car moving with variable velocity is an example of non-inertial frame of reference.

If there is change in one or more coordinates of particle with the passage of time then we say that particle is in the state of **motion**. If coordinates of particle do not change with time then it is said to be in the state of **rest**. Rest and motion are relative terms and depend on the choice of frame of reference. Same object can be in a state of rest with respect to one particular frame of reference and in a state of motion when observed from some other frame of reference. For example, we can say that car is in state of motion with respect to frame of reference attached with the road, if frame of reference is assumed to be attached with the passenger of car then from this frame of reference, car is in a state of rest, because car is not changing its position with respect to the passenger. Similarly you can see that many objects kept inside your room are in a state of rest, because your observations are with respect to frame of reference attached with the room. But we know that earth is moving and hence all these objects including you are in a state of motion when observed from the space. So it is easy to understand that the rest and motion are relative terms and it depends on choice of frame of reference to describe them.

In a particular frame of reference, we need to select a set of axes and the selection of the axes and the origin is based on the situation. Let us take an example of a particle moving along some straight line. In this case we can assume one of the three axes along the path and the origin

KEY NOTE

Mainly motion can be classified into three categories:

Rectilinear motion: when a point object is moving along a straight line it is said to be in rectilinear motion.

Circular motion: when a point object is moving along a circle it is said to be in circular motion.

Oscillatory motion: an object moving to and fro or back and forth repeatedly about a fixed point in a given interval of time, then it is said to be in oscillatory or vibratory motion.

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somewhere on this direction of motion. Let us say that X-axis is aligned with the straight line path of the particle. We can easily understand that when the particle moves then only X-coordinate changes with time. The type of motion in which only one of the coordinates of particle changes with time is called **one-dimensional motion**. Hence, with the appropriate choice of axes we can always treat the motion in a straight line as one-dimensional motion. Let us now take an example of a particle moving along a circular path. We can select two coordinate axes lying on the plane of circle and hence third axis will be perpendicular to its plane. Let us assume that X and Y-axes are on the plane of circle and the Z-axis is perpendicular to the plane of circle. We can understand that when the particle moves on its circular path then its Z-coordinate will not change and only X and Y coordinates will change. Hence, the motion of particle in a circular path can always be treated as a two-dimensional motion. The type of motion in which any two coordinates of the moving particle change is called two-dimensional motion. Similarly, when we throw an object at a particular angle (not equal to 90°) with the horizontal then the motion of the object remains restricted to a vertical plane. Hence, only two axes are sufficient to describe the motion in such a situation. Such type of motion of object under gravity is called projectile motion and this is also one example of two-dimensional motion. When a particle is moving on a plane and its path is not straight then also it is a two-dimensional motion. Most of our discussions will be restricted to one-dimensional and two-dimensional motion. In this chapter we shall learn about one-dimensional motion and in the next chapter we shall learn about two-dimensional motion.

Let us take an example of an air balloon floating in air. Its position is described by using all the three coordinates, thus it can be said to be in three-dimensional motion. Thus, the type of motion in which all the three coordinates change with time is called three-dimensional motion. A kite or a bird flying in the sky is also an example of three-dimensional motion. Selection of frame of reference is first and very important step towards the quantitative analysis of the motion of particle in space. In this chapter we are concerned about the motion in a straight line; hence only one axis is required to describe the motion. And this axis coincides with the straight line path of the particle. Let us select origin at point O as shown in Fig. 2.1. This origin can be shifted anywhere on the X-axis since the distance between any two positions will not change with the shift in origin.

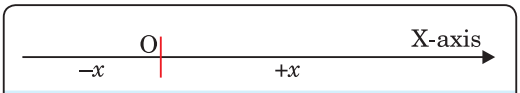

Fig. 2.1. X-axis to represent motion in a straight line.

Selection of positive direction depends on our choice and for now we are assuming positive direction towards right, as indicated by an arrow in Fig. 2.1. We use arrow on the axis to indicate its positive direction. Direction opposite to it is treated as negative direction. Origin O is used to measure the position of the particle. The position of an object at origin is assumed to be zero. Since we have assumed rightward direction as positive, hence position of the points to the right of origin will be positive and if a point is on the left side of the origin then position of the point is taken as negative. This is same as writing X-coordinate of the point.

Path length is also called the distance. Distance and displacement are two important terms of kinematics. Distance is a scalar quantity.

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When a particle moves from one point in space to another not necessarily in a straight line in a given time then the actual length of path travelled by the particle is called **distance**. In Fig. 2.2 a particle moves from initial point A towards the final point B. It can follow any random path to move from A to B and length of the path is equal to the distance travelled by particle. We can see that the minimum distance will be covered when the particle moves straight from A to B. On the other hand displacement is defined as a vector quantity. ***Magnitude of the displacement vector is equal to the shortest distance between initial point and final point***. Direction of displacement vector is taken from initial point towards the final point as shown by the straight arrow AB in Fig. 2.2. We know that the displacement is the shortest distance between initial and final points, hence distance travelled by the particle between two points in space can be greater than or equal to the magnitude of displacement between those two points. Here displacement of the object can be represented as AB.

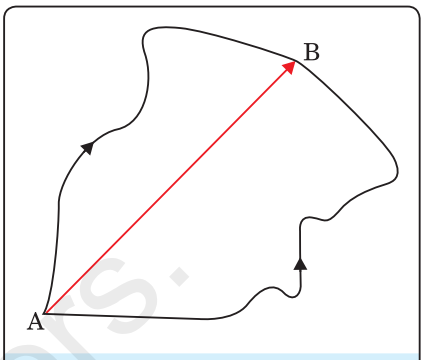

Fig. 2.2. Representation of distance and displacement.

In this chapter we are mainly concerned about straight line motion and let us discuss about distance and displacement in the context of straight line motion. Let us take an example of a particle moving along the straight line and assume that X-axis of reference coincides with the line of motion of particle. Point O, as shown in Fig. 2.3, is the origin of our reference. Rightward direction is selected as positive direction for X-axis. Scale to measure the position of particle is shown in Fig. 2.3 and the values mentioned are in metre.

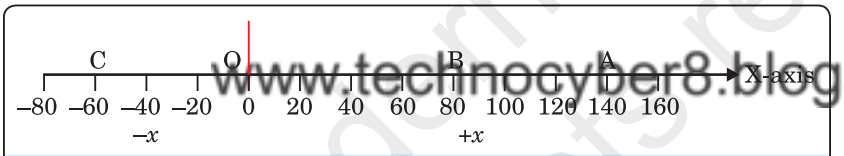

Fig. 2.3. Representation of motion in one-dimension.

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Let us consider that a particle starts from the origin O at time $t = 0$. Other points marked as A, B and C are the positions of the particle at different instants of time. Position coordinates of A, B and C can be written as follows:

$$x_A = +140 \text{ m}, x_B = +80 \text{ m and } x_C = -60 \text{ m}.$$

Note that rightward direction is selected as positive, hence points to the right of the origin O have positive position coordinates and those on left have negative position coordinates. Let us consider the interval of motion when the particle moves from O to A. In this interval the distance travelled by the particle is 140 m. We know that the magnitude of displacement for this interval is the shortest distance between initial and final points, which is also the equal to 140 m. Direction of displacement is taken from initial point towards the final point of motion, hence must be directed along OA which is positive direction. So we can say that the displacement for this interval is +140 m. Positive sign indicates the direction of displacement which is along the positive X direction. Displacement is change in position and hence it can also be calculated by subtracting initial position coordinate from the final position coordinate ($\Delta x = x_f - x_i$). Hence, displacement for the given interval can be written as $\Delta x = x_A - x_O = (+140 \text{ m}) - (0) = +140 \text{ m}$.

Let us now consider the interval of motion when the particle moves from point O to A and then back to point B. Distance travelled by the

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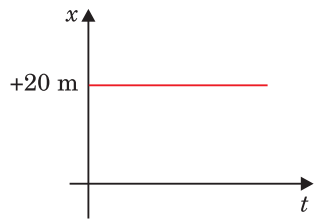

Fig. 2.4. Position-time graph for a particle at rest.

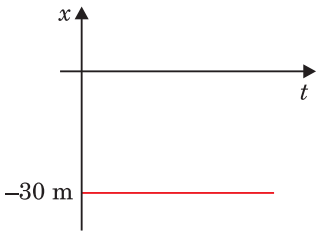

Fig. 2.5. Position-time graph for a particle at rest.

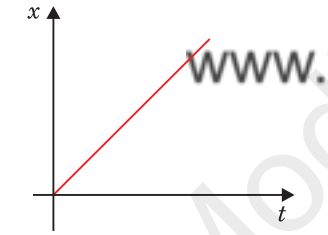

Fig. 2.6. Position-time graph for a particle in uniform motion.

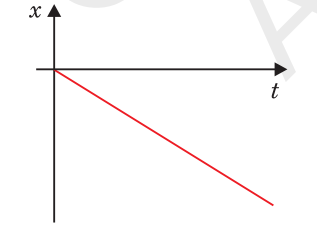

Fig. 2.7. Position-time graph for a particle in uniform motion in negative direction.

KEY NOTE

An object which covers unequal displacements in equal interval of time is said to be in non-uniform motion.

particle is actual length of the path which is equal to 140 m + 60 m = 200 m. Initial point is O and final point is B for the given motion. Magnitude of displacement is the shortest distance, hence 80 m. Its direction is from initial point O towards the final point B hence along the positive direction. Hence, displacement for the motion is + 80 m. In case of straight line motion the distance and the magnitude of displacement will be different when the particle reverses its direction of motion during that interval.

Let us now consider the interval when the particle moves from point B to point C. Distance travelled by the particle in this interval is 80 m + 60 m = 140 m. We can see that the magnitude of displacement is also 140 m. Direction of displacement is from initial point towards the final point and hence displacement in this interval is directed from B to C and hence along negative direction. So displacement for this interval can be written as –140 m. Here, negative sign indicates the direction of displacement which is along negative X-direction. We can also calculate the displacement as $\Delta x = x_C - x_B = (-60\text{ m}) - (+80\text{ m}) = -140\text{ m}$.

When a particle moves along the straight line which is assumed as X-axis, then X-coordinate of the particle changes with time. We can draw a graph between X-coordinate and time. This is called position-time graph and is one powerful tool to represent and analyse the motion. We can start with the simplest cases to understand how graphs can help to represent the motion. Let us take an example of a particle in a state of rest at $x = +20$ m. Then its position-time graph will be a line parallel to the time axis because position coordinate of the particle does not change with time. Position-time graph of a particle at rest at $x = +20$ m is shown in Fig. 2.4.

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Similarly, if the particle is at rest at $x = -30$ m, then its position-time graph will be represented as shown in Fig. 2.5.

A particle is said to be in uniform motion, if it covers equal distances in equal time intervals. Let us take an example of a particle at $x = 0$ at time $t = 0$. Let the particle be moving uniformly along positive X direction then its position-time $(x - t)$ graph will be represented as shown in Fig. 2.6.

In the above case if particle starts moving along negative direction uniformly then its position-time $(x - t)$ graph will be as shown in Fig. 2.7.

If the motion of a particle is not uniform then its position-time graph can be a curve. For example, when a lift moves between two floors then it does not move uniformly at the beginning and at the time it stops. Only for some time in between the floors, lift moves uniformly and position-time $(x - t)$ graph of this situation is as shown in Fig. 2.8. But we need to learn more concepts to understand this graph. These concepts are covered in subsequent sections in the same chapter and there we shall revisit the position-time graph again.

Note that the position or displacement can be negative or positive. Hence, the position-time graph can be above the time axis and as well as below the time axis. Similarly, position coordinate can increase or decrease and hence graph can rise or go down as time passes. But if we try to make the distance-time graph then we should keep in mind that it is always positive and hence graph will always be above the time axis. Moreover, for a particle the distance may remain constant with time or it will increase with the time and hence graph will be

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parallel to time axis or it will rise. **Distance-time graph will never go down with the passage of time.** We should remember that the distance is equal to the total length of the path followed by the particle. It is always positive and never decreases with the time. In Table 1 some key differences between the distance and displacement are given.

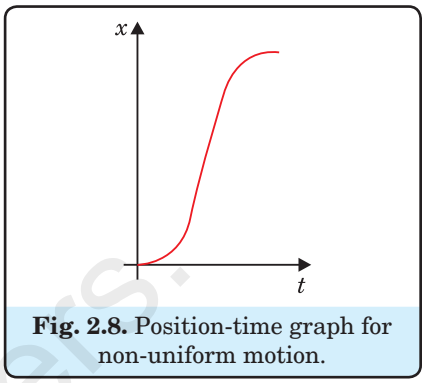

Fig. 2.8. Position-time graph for non-uniform motion.

Table 1. Difference between distance (path length) and displacement

	Distance	Displacement
1	It is the actual length of the path travelled by an object in the given time interval.	It is the shortest distance between the final and the initial position of an object in a given time interval.
2	It is a scalar quantity.	It is a vector quantity.
3	There can be many paths between two positions and so are the values of the distance travelled by an object in given time between the two positions.	The value of displacement of an object travelling between two position in given time is always same.
4	The value of distance can only be positive.	Displacement of an object can be zero, negative or positive.
5	The value of distance travelled by an object between two positions in given time interval can be either greater or equal to the displacement of the object. It can never be less than the value of displacement.	The value of displacement of an object between two positions in given interval of time can never be greater than the distance travelled. However, it can be less than the distance travelled by the object.

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3. AVERAGE VELOCITY AND AVERAGE SPEED

When position of an object changes with time it is said to be in a state of motion. Time taken by the object to change the position by a certain amount is also an important factor. We define velocity to indicate how fast the position is changing with time. First let us understand average velocity which is defined for a certain interval of time. ***Average velocity is defined as the total change in position or the total displacement (Δx) of an object divided by the time interval (Δt) taken by the object for this change in position or displacement***.

$$v_{av} = \frac{\Delta x}{\Delta t}$$

If position of an object at an instant of time t_1 is x_1 and at time t_2 position is x_2 then the average velocity for this interval can be written as follows:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \quad \ldots(1)$$

SI unit of velocity is m/s. It is a vector quantity and its direction is same as the direction of displacement or change in position.

Let us now consider the position-time graph of a particle as shown in Fig. 3.1.

Let at time t_1 the position of particle be x_1 and the corresponding point on the graph is shown as A. Similarly, let at time t_2 the position of particle be x_2 and the corresponding point on the graph is shown as B. Line joining points A and B is chord of the graph for selected interval

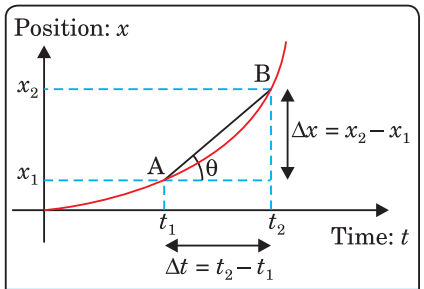

Fig. 3.1. Position-time graph.

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of time. Here, θ is the angle made by the chord with the time axis, and tan θ is the slope of this chord. Slope of the chord AB can be written as follows:

$$\tan\theta = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = v_{av} \qquad \ldots(2)$$

Hence, the slope of the chord in $x - t$ graph for a given interval is equal to the average velocity of object for that interval. Average velocity can be positive or negative depending on the displacement. We should always remember that ***if x – t graph is rising with the passage of time in some time interval then the average velocity remains positive for that interval. Similarly, when x – t graph is going down with the passage of time in some time interval then the average velocity in that interval of time will be negative.***

Average velocity gives information of change in position or displacement with respect to time. And we know that the actual length of the path may be different from the displacement. We define average speed to describe the rate of motion along actual path of object.

Average speed in a time interval of motion is defined as distance covered by the body divided by time interval in which this distance is covered.

$$\text{Average Speed} = \frac{\text{Total path length}}{\text{Total time interval}} \qquad \ldots(3)$$

Speed is a scalar quantity and it does not tell about the direction of motion. Average speed is always positive unlike the average velocity which can be positive or negative. Its unit is same as that of velocity which is m/s. In general we know that distance can be greater or equal to the displacement and hence average speed for a given time interval can also be greater than or equal to magnitude of average velocity in the same time interval. If for straight line motion in a time interval, object does not reverse its direction of motion, then magnitude of its average velocity is same as average speed. But if particle reverses its direction of motion during the straight line motion then average speed becomes greater than magnitude of average velocity in that time interval.

Let us take an example of particle moving along X-axis as shown in Fig. 3.2. Values marked in the figure are in metres.

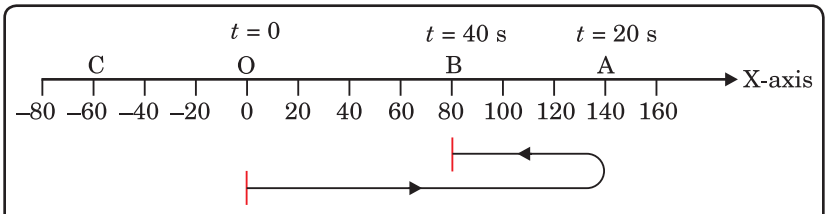

Fig. 3.2. Motion of an object along X-axis.

In this example particle starts from origin O at time $t = 0$ and then reaches the point A at $t = 20$ s and then to the point B at $t = 40$ s.

To calculate average velocity we can write initial and final positions of particle as follows:

$$x_1 = 0; x_2 = +80 \text{ m}.$$

Change in position or displacement can be written as follows:

$$\Delta x = x_2 - x_1 = 80 - 0 = 80 \text{ m};$$

REMEMBER

For an object moving along straight line in a given time interval, the average speed can be greater than or equal to the average velocity of the object in the same time interval.

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Time interval $\Delta t = 40$ s.

$$\text{Average velocity} = \frac{\Delta x}{\Delta t} = \frac{80}{40} = 2 \text{ m/s}.$$

To calculate the average speed for the same time interval we can write distance covered as follows:

Total distance covered = 140 m + 60 m = 200 m.

Time interval $\Delta t = 40$ s.

$$\text{Average speed} = \frac{\text{distance}}{\text{time interval}} = \frac{200}{40} = 5 \text{ m/s}$$

If we consider the interval OA then average speed and magnitude of average velocity, both will be same and equal to 140/20 = 7 m/s. We can easily understand that the average velocity and average speed depend on the selected time interval.

KEY NOTE

The value of average speed and magnitude of average velocity need not to be the same. However, the value of instantaneous speed is equal to the magnitude of instantaneous velocity.

FORMULAE USED

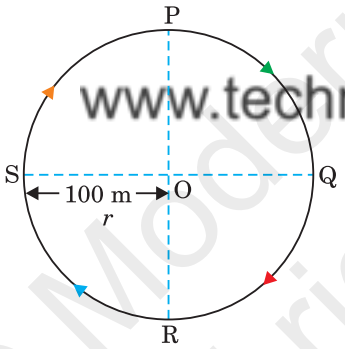
Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

Average velocity = $\frac{\text{Total displacement}}{\text{Total time}}$

SOLVED EXAMPLES

Example 1.

An athlete is running on a circular track PQRS of radius 100 m shown below



He starts from point P and moves clockwise.

(*i*) *Find the distance travelled by him and displacement when he*

(*a*) *reaches Q*

(*b*) *moves from Q to S*

(*c*) *reaches P again after one revolution*

(*ii*) *If he completes one revolution in 5 minutes, find his*

(*a*) *average speed*

(*b*) *average velocity*

Solution: (*i*) (*a*) Total perimeter of circular track = $2\pi r$
$= 2\pi(100)$ m

$\therefore$ Distance covered = $\frac{1}{4} \times 2\pi(100) = \mathbf{50\pi}$ **m**

Displacement $= |\overrightarrow{PQ}| = \sqrt{PO^2 + OQ^2} = \sqrt{r^2 + r^2}$

$= \sqrt{2}r = \mathbf{100\sqrt{2}}$ **m**

(*b*) From Q to S:

Distance covered = $\frac{1}{2} \times 2\pi r = \pi r = \mathbf{100\pi}$ **m**

Displacement = $|\overrightarrow{QR}| = 2r = \mathbf{200}$ **m**

(*c*) From P to P:

Distance covered = $2\pi r = 2 \times \pi \times 100 = \mathbf{200\pi}$ **m**

Displacement = **0** (Final position and initial position are same)

(*ii*) (*a*) Distance covered in one revolution

$= 2\pi r = 200\pi$ m

Time taken, $t = 5$ minutes

$\therefore$ Average speed = $\frac{\text{Distance}}{\text{time}} = \frac{200\,\pi}{5 \text{ min}}$

$= \mathbf{40\pi}$ **metre/min**

Displacement in one revolution = 0

(*b*) $\therefore$ Average velocity = $\frac{\text{Displacement}}{\text{time}} = \mathbf{0}$

Example 2.

A car travels from X to Y at a speed of 50 km/h and returns back from Y to X at a speed of 60 km/h. Find the average speed and average velocity of the car.

Solution: Let x be the distance between X and Y. Time taken by the car to travel from X to Y is

$$t_1 = \frac{x}{50}$$

Time taken by the car to travel from Y to X is

$$t_2 = \frac{x}{60}$$

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Total time taken by the car to travel from X to Y and then Y to X is

$$t_1 + t_2 = \frac{x}{50} + \frac{x}{60} = x\left(\frac{1}{50} + \frac{1}{60}\right) = \frac{11x}{300}\text{ s}$$

Total distance covered in the journey = $2x$

$$\therefore \text{ Average speed} = \frac{2x}{\frac{11x}{300}} \quad \frac{300 \times 2}{11} = \frac{600}{11}\text{ m/s}$$

$$= \mathbf{54.55\ m/s}$$

In the total journey, the car started from point X and returns back to X. So, the net displacement of the car is zero.

$\therefore$ Average velocity = **0**

Example 3.

On a journey of 80 km, a car covers the first 40 km with a uniform speed of 80 km / hr. How fast it should travel in the next 40 km so that an average speed of 100 km / hr is maintained for the entire journey?

Solution: Let total distance, $2s$ = 80 km

Time taken to cover a distance s with a speed v_1 is

$$t_1 = \frac{s}{v_1}$$

Let v_2 be the speed of the car in the next 40 km of journey. Then, time taken to cover another 40 km with a speed v_2 is

$$t_2 = \frac{s}{v_2}$$

Average speed over entire journey is given by

$$v_{av} = \frac{2s}{t_1 + t_2} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2s}{s\left(\frac{1}{v_1} + \frac{1}{v_2}\right)}$$

$$= \frac{2v_1v_2}{v_1 + v_2}$$

Given, v_{av} = 100 km/hr

and v_1 = 80 km/hr

$$\therefore \quad 100 = \frac{2 \times 80 \times v_2}{80 + v_2}$$

$$\Rightarrow \quad 80 + v_2 = \frac{160v_2}{100}$$

$$\Rightarrow \quad 80 + v_2 = 1.6v_2$$

or $0.6v_2 = 80$

$$\Rightarrow \quad v_2 = \frac{80}{0.6} = \mathbf{133.3\ km/hr}$$

Example 4.

A cyclist moves with a speed of 30 km / hr in the first 10 minutes, with speed of 15 km / hr in the next 20 minute and with a speed of 40 km / hr in the last 15 minutes. Calculate the average speed of the cyclist.

Solution: Distance covered by the cyclist in t_1 = 10 min = $\frac{10}{60}$ hr

v_1 = 30 km/hr

d_1 = Speed × time

$$= v_1 \times t_1 = \frac{30 \times 10}{60}\text{ km} = 5\text{ km}$$

Distance covered by the cyclist in t_2 = 20 min = $\frac{20}{60}$ hr

v_2 = 15 km/hr

$d_2 = v_2 \times t_2$

$$= \frac{15 \times 20}{60} = 5\text{ km}$$

Distance covered by the cyclist in t_3 = 15 min = $\frac{15}{60}$ hr

$d_3 = v_3 \times t_3$ v_3 = 40 km/hr

$$= \frac{40 \times 15}{60} = 10\text{ km}$$

Total distance covered by the cyclist

$$d = d_1 + d_2 + d_3$$

$$= 5 + 5 + 10 = 20\text{ km}$$

Total time taken, t = (10 + 20 + 15) min

$$= 45\text{ min} = \frac{45}{60}\text{ hr}$$

$$\text{Average speed of cyclist} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{20}{\frac{45}{60}} = \frac{20 \times 60}{45} = \frac{80}{3}$$

$$= \mathbf{26.6\ km/hr}$$

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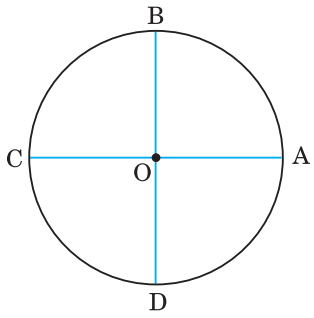
PRACTICE PROBLEMS 1

1. The minute hand of a table clock is 5 cm long. Find the average velocity of the tip of the minute hand between 3.00 pm and 3.30 pm.
2. A particle is moving in anticlockwise direction along a circle of radius R.

Answers to Practice Problems

1. 5.5×10^{-3} cm/s
2. (*i*) πR, $2R$ (*ii*) $\sqrt{\frac{3\pi R}{2}}$, $\sqrt{2}R$



Calculate the distance and displacement covered by the particle (*i*) from A to C (*ii*) from A to D.

3. A bus travelled a total distance *s*. It covered its journey in consecutive one-fourth distances with speeds 60 km/hr, 50 km/hr, 70 km/hr and 80 km/hr, respectively. Calculate the average speed of bus for entire journey.

4. A body is travelling due east at a speed of 15 km/h for one hour and then turns towards south at a speed of 10 km/h for one hour. Find the average velocity of the body.

5. A boy is taking rounds along a 20 m wall back and forth. He completed 10 rounds in 45 minutes. Calculate the average speed and average velocity of the boy.

6. A car is travelling along a straight line and covers one-fourth of the total distance with a velocity of 10 m/s. The remaining distance is covered with a velocity of 5 m/s. Calculate the average velocity of the car.

Answers to Practice Problems

3. 63.03 km/h
4. 9.015 km/h
5. 0.148 m/s; 0
6. 5.71 m/s

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4. INSTANTANEOUS VELOCITY AND INSTANTANEOUS SPEED

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When we calculate average velocity or speed then selected time interval can be large or small. ***But when time interval selected is infinitesimally small then it is known as instantaneous velocity or speed.***

We can define velocity at a particular instant as limiting value of average velocity when time interval Δt becomes infinitesimally small or when time interval Δt approaches zero. Mathematically we can write the same as follows:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad \ldots(1)$$

Let us take an example of a particle whose X-coordinate changes with time as $x = t^2$, where t is measured in seconds and x is measured in metres.

Let us calculate the instantaneous velocity of the particle at time $t = 3$ s.

Here, $t_1 = 3$ s

$\Rightarrow$ $x_1 = (3)^2 = 9$ m

Let $t_2 = (3 + \Delta t)$

$\Rightarrow$ $x_2 = (3 + \Delta t)^2 = 9 + \Delta t^2 + 6\Delta t$

$\Rightarrow$ $\Delta x = x_2 - x_1 = (9 + \Delta t^2 + 6\Delta t) - (9)$

$\Rightarrow$ $\Delta x = 6\Delta t + \Delta t^2$

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

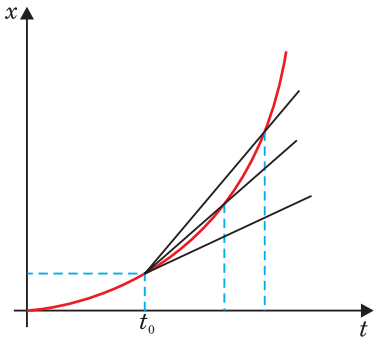

Fig. 4.1. Determining instantaneous velocity of a moving object.

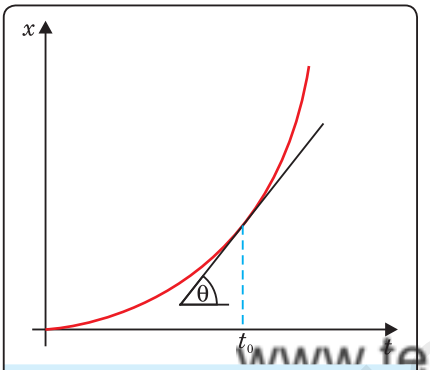

Fig. 4.2. Determining the instantaneous velocity of moving object.

KEY NOTE

For a moving object the instantaneous speed at any point is equal to the magnitude of instantaneous velocity but average speed in a given time interval is always greater or equal to the magnitude of average velocity in that interval.

KEY NOTE

The slope of the tangent in a graph plotted between two physical quantities is rate of change of quantity represented by Y-axis with respect to the change in the quantity represented by X-axis. In Fig. 4.2, the slope of displacement-time graph determines the velocity and its unit is m s^{-1}.

$$\Rightarrow \quad v = \lim_{\Delta t \to 0} \frac{(6\Delta t + \Delta t^2)}{\Delta t}$$

$$\Rightarrow \quad v = \lim_{\Delta t \to 0} (6 + \Delta t)$$

$$\Rightarrow \quad v = 6 \text{ m/s}$$

Mathematics of differential calculus is developed for the calculation of instantaneous rate of change. In differential calculus we use letter d instead of 'Δ' to represent change. Here, 'dt' indicates the time interval that is infinitesimally small, so that it is approximated to be equal to zero. Hence, instantaneous velocity can be written as follows:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad \ldots(2)$$

Here, $\frac{dx}{dt}$ represents instantaneous rate of change of position or the instantaneous velocity of the particle. Here, $\frac{dx}{dt}$ is also known as differential coefficient of x with respect to time t.

Graphically, we have understood that the slope of chord in $x - t$ graph, in an interval represents average velocity for that interval of time. From Fig. 4.1 we can easily understand that when width of the interval approaches zero then chord becomes tangent to the graph at a point and slope of the tangent at a point of time on $x - t$ graph is equal to instantaneous velocity of the particle at that instant of time.

If θ is the angle made by tangent on $x - t$ graph corresponding to an instant $t = t_0$ then tan θ is instantaneous velocity of particle at $t = t_0$ as shown in Fig. 4.2.

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From Fig. 4.2, we can write instantaneous velocity at $t = t_0$ as follows:

$$v = \frac{dx}{dt} = \tan\theta = \text{Slope of the tangent at } t = t_0$$

In a large time interval generally the distance is greater than or equal to the amount of displacement in that interval. But when the interval of time approaches zero then difference between distance and displacement vanishes. That is why instantaneous speed at any point is equal to the magnitude of velocity at that instant of time but we know that average speed in a time interval is always greater or equal to magnitude of average velocity in that interval.

5. ACCELERATION

As we have studied earlier that the rate of change of displacement with respect to time is defined as velocity of the particle but velocity of the particle can also change with time. ***Rate of change of velocity with respect to time is defined as acceleration of object.*** It is a vector quantity and its unit is m/s^2.

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

Average acceleration of a particle in a time interval is defined as the change in velocity divided by the time interval. Let v_1 and v_2 be the instantaneous velocities of particle at times t_1 and t_2, the average acceleration can be written as follows:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \qquad \ldots(1)$$

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We can also draw a graph between velocity of particle and time as we have drawn the displacement-time graph. We can see in Fig. 5.1 slope of the chord of $v - t$ graph in a time interval represents average acceleration of the particle in that time interval.

$$\tan\theta = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = a_{av} \quad \ldots(2)$$

Instantaneous acceleration is defined as limiting value of average acceleration when time interval is infinitesimally small.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \quad \ldots(3)$$

Slope of the tangent of $v - t$ graph at a certain instant represents instantaneous acceleration of the particle at that instant of time. In Fig. 5.2 slope of the tangent at $t = t_0$ represents instantaneous acceleration of the particle at that instant.

$$a = \frac{dv}{dt} = \tan\theta = \text{Slope of the tangent at } t = t_0.$$

We can understand that $v - t$ graph of a particle will be a straight line when it is moving with a constant acceleration.

Hence, finally we know that the slope of the $x - t$ graph represents velocity and the slope of $v - t$ graph represents acceleration. One important point regarding $v - t$ graph is that the area under the graph projected on the time axis represents displacement of the particle. In case of $v - t$ graph area above the time axis is taken as positive and that below the time axis is taken as negative. If we consider the positive and negative signs to calculate the total area under the graph then we get the net displacement and if we just add the magnitudes without considering their sign then we get the total distance covered by the particle.

Instantaneous acceleration is given as the first derivative of the velocity at the given instant.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Since, $$\vec{v} = \frac{d\vec{x}}{dt}$$

$$\Rightarrow \quad \vec{a} = \frac{d\left(\frac{d\vec{x}}{dt}\right)}{dt} = \frac{d^2\vec{x}}{dt}$$

Thus instantaneous acceleration can also be expressed as the second derivative of the position of the moving object at the instant.

In case of straight line motion when the velocity and acceleration both are directed along the same direction or have the same sign then magnitude of velocity increases and such kind of motion is called **accelerated motion**. When velocity and acceleration are along the opposite directions or are having opposite signs then magnitude of velocity decreases with time and such kind of motion is called **retarding motion**. Let v and a represent velocity and acceleration at some instant of time then we can write the following:

- If v and a, both are positive then particle is accelerated. If v and a, both are negative then also particle is accelerated.

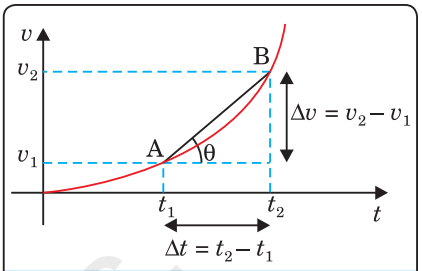

Fig. 5.1. Determining average acceleration using velocity-time graph.

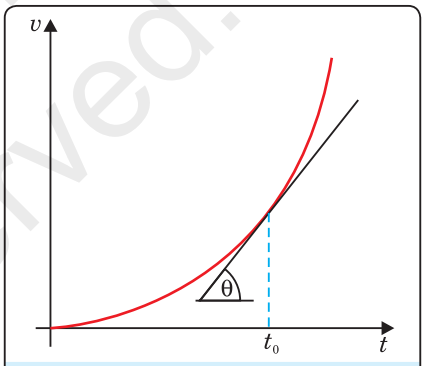

Fig. 5.2. Determining instantaneous acceleration using velocity–time graph.

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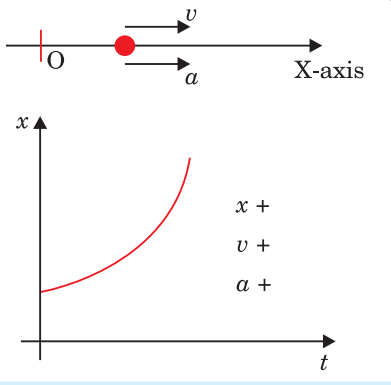

Fig. 5.3. Position-time graph for accelerated motion.

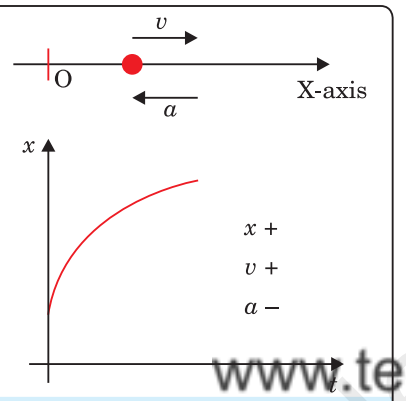

Fig. 5.4. Position-time graph for retarding motion.

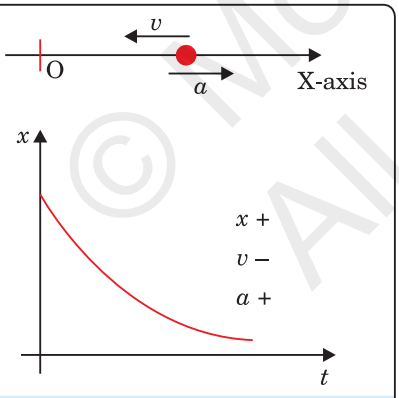

Fig. 5.5. Position-time graph for retarding motion.

- If v is positive and a is negative then particle is retarded.
- If v is negative and a is positive then also particle is retarded. We know that the slope of $x - t$ graph represents velocity and hence for accelerated motion, magnitude of slope of $x - t$ graph increases with time and that for retarding motion, magnitude of slope of $x - t$ graph decreases with time.

Let x and v represent position and velocity of the particle at some instant of time then we can write the following:

- If x and v both have same sign then particle moves away from the origin.
- When they have opposite signs then particle moves towards the origin.

Figure 5.3 shows the $x - t$ graph of a particle which is placed in the positive side of the X-axis (x +) and moving along the positive direction (v +). Applied acceleration is also in the positive direction (a +). It is a case of the accelerated motion and thus the magnitude of the slope of $x - t$ graph is increasing with the time.

Figure 5.4 shows the $x - t$ graph of a particle on the positive side of the X-axis (x +) and moving along the positive direction (v +). Applied acceleration is in the negative direction (a –). It is a case of the retarding motion and thus the magnitude of the slope of $x - t$ graph is decreasing with the time.

Figure 5.5 shows the $x - t$ graph of a particle on the positive side of the X-axis (x +) and moving along the negative direction (v–). Applied acceleration is in the positive direction (a +). It is a case of the retarding motion and thus the magnitude of the slope of $x - t$ graph is decreasing with the time.

Figure 5.6 shows the $x - t$ graph of a particle on the positive side of the X-axis (x +) and moving along the negative direction (v –). Applied acceleration is also in the negative direction (a –). It is a case of the accelerated motion and thus the magnitude of the slope of $x - t$ graph is increasing with the time.

Figure 5.7 shows the $x - t$ graph of a particle on the negative side of the X-axis (x –) and moving along the positive direction (v +). Applied acceleration is also in the positive direction (a +). It is a case of the accelerated motion and thus the magnitude of the slope of $x - t$ graph is increasing with the time.

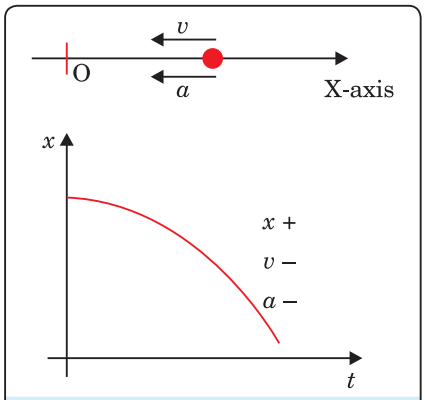

Fig. 5.6. Position-time graph for accelerating motion.

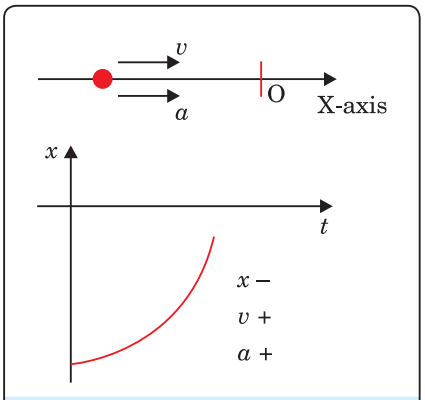

Fig. 5.7. Position-time graph for accelerating motion.

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Figure 5.8 shows the $x - t$ graph of a particle on the negative side of the X-axis ($x -$) and moving along the positive direction ($v +$). Applied acceleration is in the negative direction ($a -$). It is a case of the retarding motion and thus the magnitude of the slope of $x - t$ graph is decreasing with the time.

Figure 5.9 shows the $x - t$ graph of a particle on the negative side of the origin ($x -$) and moving along the negative direction ($v -$). Applied acceleration is in the positive direction ($a +$). It is a case of the retarding motion and thus the magnitude of the slope of $x - t$ graph is decreasing with the time.

Figure 5.10 shows the $x - t$ graph of a particle on the negative side of the origin ($x -$) and moving along the negative direction ($v -$). Applied acceleration is also in the negative direction ($a -$). It is a case of accelerated motion and thus the magnitude of the slope of $x - t$ graph is increasing with the time.

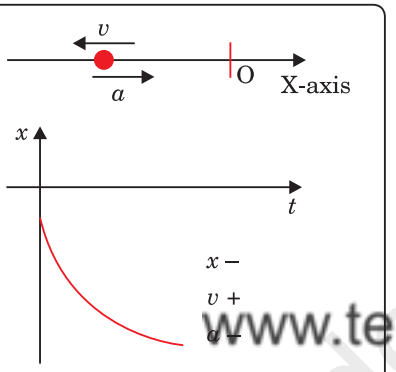

Fig. 5.9. Position-time graph for retarding motion.

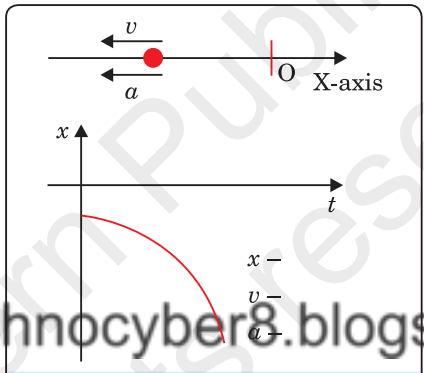

Fig. 5.10. Position-time graph for acceleration motion.

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Graph between velocity and time ($v - t$) is very useful in the numerical analysis. We know that the slope of $v - t$ graph is equal to the acceleration of a particle in motion and the area under the graph projected on the time axis represents the displacement. Fig. 5.11 shows the $v - t$ graph for a particle moving along the positive direction of X-axis with a constant velocity and hence without any acceleration ($a = 0$).

Figure 5.12 shows the $v - t$ graph for a particle moving along the negative direction with a constant velocity and hence without any acceleration ($a = 0$).

Figure 5.13 shows the $v - t$ graph for a particle moving along the positive direction ($v +$) and a constant acceleration is also applied in the positive direction of X-axis ($a +$).

Figure 5.14 shows the $v - t$ graph for a particle moving along the positive direction ($v +$) and a constant acceleration is applied in the negative direction ($a -$).

The graph between acceleration and time ($a - t$) can also be drawn. Slope of $a - t$ graph represents rate of change of acceleration and we do not have any standard term to represent it. **Area under the $a - t$ graph projected on time axis, for an interval of time is equal to change in velocity of particle in that time interval**.

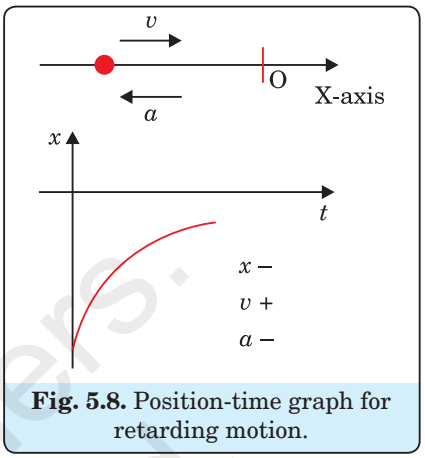

Fig. 5.8. Position-time graph for retarding motion.

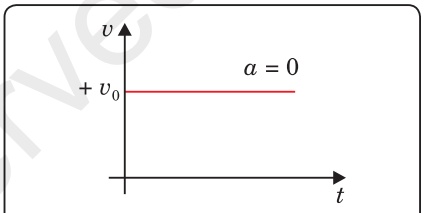

Fig. 5.11. Velocity-time graph of an object with zero acceleration.

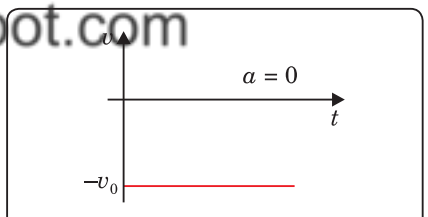

Fig. 5.12. Velocity-time graph of an object with zero acceleration.

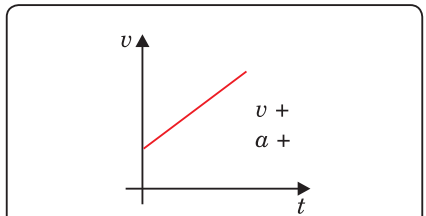

Fig. 5.13. Velocity-time graph of an object with constant acceleration.

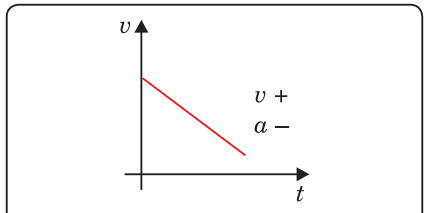

Fig. 5.14. Velocity-time graph of an object with constant acceleration.

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SOLVED EXAMPLES

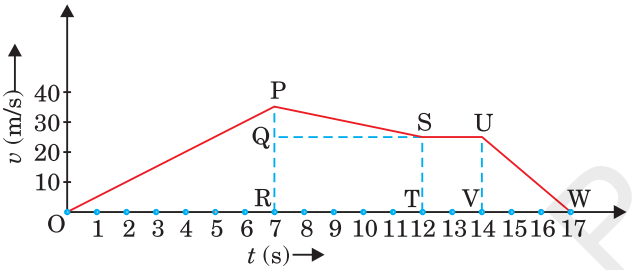
Example 1.

A body starts from rest and accelerates uniformly at 5 m/s² for 7 seconds, along a straight line. It then starts decelerating at a rate of 2 m/s² for the next 5 seconds, moves uniformly with a velocity of 25 m/s for the next 2 seconds and then retards again and comes to rest in another 3 seconds.

(*i*) *Plot the velocity versus time graph for the body.*

(*ii*) *Calculate the total distance travelled by the body with the help of the* graph.

Solution: (*i*) The velocity–time graph of the body is shown below:



(*ii*) Distance travelled by the body = Area under $v - t$ graph

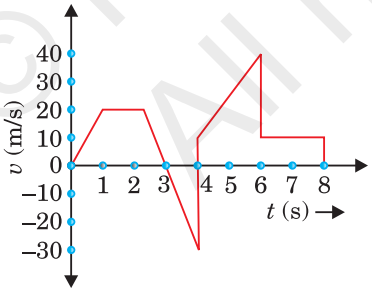
= Area of Δ OPR + Area of ΔPQS + Area of □ QRTS + Area of □ SUTV + Area of ΔUVW

$$= \frac{1}{2} \times 7 \times 35 + \frac{1}{2} \times 10 \times 5 + 25 \times 5 + 25 \times 2 + \frac{1}{2} \times 25 \times 3$$

$$= 360 \text{ m}$$

Example 2.

The velocity versus time graph of a body moving along a straight path is shown below.



(*i*) *Calculate the distance covered by the body from $t = 0$ to $t = 8$ s.*

(*ii*) *Find its net displacement from $t = 0$ to 8 s.*

Solution: (*i*) Distance covered by the body from $t = 0$ to 8 *s*

= Area under the $v - t$ graph

$$= \frac{1}{2} \times 1 \times 20 + 1 \times 20 + \frac{1}{2} \times 1 \times 20 + \frac{1}{2} \times 1 \times 30 + 2 \times 10 + \frac{1}{2} \times 2 \times 30 + 2 \times 10$$

$$= 10 + 20 + 10 + 15 + 20 + 30 + 20 = 95 \text{ m}$$

(*ii*) Displacement is positive from 0 to 3 s, negative for 3 to 4 s and positive from 4 to 8 s.

∴ Total displacement in 0 to 8 s is

$$= \frac{1}{2} \times 1 \times 20 + 1 \times 20 + \frac{1}{2} \times 1 \times 20 - \frac{1}{2} \times 1 \times 30 + 2 \times 20 + \frac{1}{2} \times 2 \times 30 + 2 \times 10$$

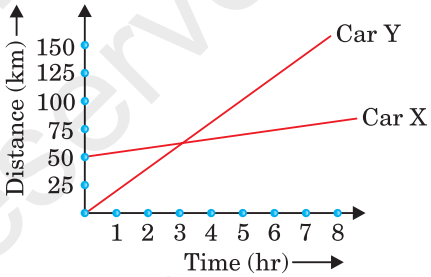
$$= 10 + 20 + 10 - 15 + 20 + 30 + 20 = 95 \text{ m}$$

Example 3.

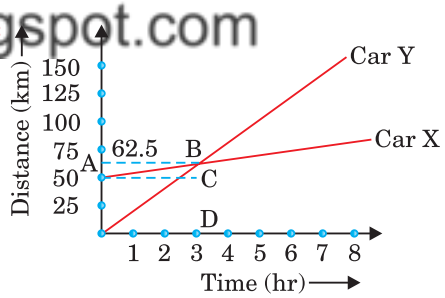
The figure given below shows the distance–time graph of the two cars, which began their journey together, in the same direction, from different points of start.

(*i*) *Find the speed of car X and car Y.*

(*ii*) *When and where will car Y catch car X?*



Solution:



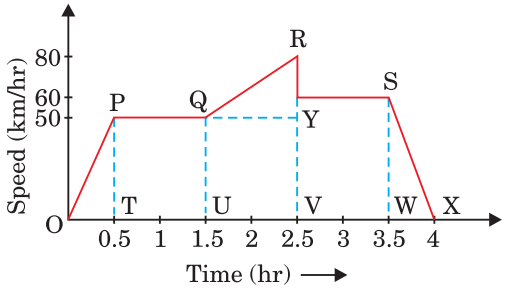
(*i*) Speed of X = $\frac{BC}{AC} = \frac{62.5 - 50}{3 - 0} = \frac{12.5}{3} = 4.17$ km/hr

Speed of Y = $\frac{BD}{CD} = \frac{62.5 - 0}{3 - 0} = 20.83$ km/hr

(*ii*) The two graphs intersect at point B, so car Y will catch car X after 3 hours and at a distance of 62.5 km from the origin.

Example 4.

The plot of speed for the journey of a bus from one bus stop to another is shown below.



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(*i*) *Find the maximum acceleration (or deceleration) of the bus during the journey.*

(*ii*) *Calculate the distance covered by the bus from 0.5 hr to 2.5 hr.*

Solution: (*i*) Slope of part OP $= \frac{50}{0.5\text{ h}} = 100\text{ km/hr}^2$

Slope of part QR $= \frac{80-50}{1} = \frac{30}{1} = 30\text{ km/hr}^2$

Slope of part SX $= \frac{60-0}{4-3.5} = \frac{60}{0.5} = 120\text{ km/hr}^2$

$\therefore$ Maximum acceleration (deceleration) is observed for part SX.

(*ii*) Distance covered by the bus for 0.5 h to 2.5 h

= Area of $\square$ PQUT + Area of $\square$ QUVY + Area of Δ QRY

$= 50 \times 1 + 1 \times 50 + \frac{1}{2} \times (80-50) \times 1$

$= 100 + \frac{1}{2} \times 30 =$ **115 km**

Example 5.

The position of an object moving along X-axis is given by $x = a + bt^2$, where $a = 8.5$ m, $b = 2.5$ m/s^2 and t is measured in seconds. What is its velocity at $t = 0$ and $t = 2$ s? What is the average velocity between $t = 2$ s and $t = 4$ s? *(N.C.E.R.T. Solved Example)*

Solution: Given, $x = a + bt^2$

Instantaneous velocity is

$$v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 0 + 2bt = 2bt$$

Now, at $t = 0$, $v = 0$

$t = 2$ s, $v = 2 \times 2.5 \times 2 = 10$ m/s ($b = 2.5$ ms^{-2})

At $t = 2$ s, $x = a + 4b$

And $t = 4$ s, $x = a + 16b$

Average velocity between 2 s and 4 s

$$= \frac{(a + 16\,b) - (a + 4\,b)}{4 - 2}$$

$$= \frac{12b}{2} = 6b$$

$= 6 \times 2.5$ m/s

$=$ **15.0 m/s**

Example 6.

The acceleration of a particle varies with time t as

$$a = t^2 + t + 2$$

where t is in seconds.

The particle starts with an initial velocity $v = 3$ m/s at $t = 0$. Find the velocity of the particle at the end of 5 s.

Solution: $a = t^2 + t + 2 = \frac{dv}{dt}$

$\Rightarrow$ $dv = (t^2 + t + 2)dt$

Integrating both sides, we get

$$\int dv = \int (t^2 + t + 2)\, dt$$

$\Rightarrow$ $v = \frac{t^3}{3} + \frac{t^2}{2} + 2t + C$

At $t = 0$, $v = 3$ m/s

$\therefore C = 3$

At $t = 5$ s, we have

$$v = \frac{(5)^3}{3} + \frac{5^2}{2} + 2 \times 5 + 3$$

$= 41.67 + 12.5 + 13$

$= 67.17$ m/s $\simeq$ **67.2 m/s**

Example 7.

The displacement x of a particle varies as

$$\sqrt{x} = t - 2$$

(*i*) *What is the velocity and acceleration of the particle at $t = 0$?*

(*ii*) *When will the velocity of the particle become zero?*

(*iii*) *What is the displacement of the particle when its velocity is zero?*

(*iv*) *Is the motion of the particle uniformly accelerated or not?*

Solution: Displacement, $\sqrt{x} = t - 2$

$\Rightarrow$ $x = (t-2)^2 = t^2 - 4t + 4$

Velocity, $v = \frac{dx}{dt} = 2t - 4$

Acceleration, $a = \frac{dv}{dt} = 2$

(*i*) At $t = 0$,

$v = 2 \times 0 - 4 = -4$ m/s

$a = 2$ m/s^2

(*ii*) Velocity will become zero when

$2t - 4 = 0$

$\Rightarrow$ $t = \frac{4}{2} = 2$ s

(*iii*) Displacement at $t = 2$ s is

$x = (2)^2 - 4 \times 2 + 4$

$= 4 - 8 + 4 = 0$

(*iv*) Since a is constant and independent of time. So the particle has a uniform acceleration.

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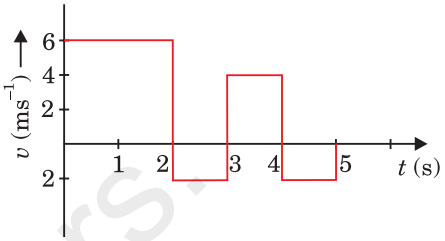
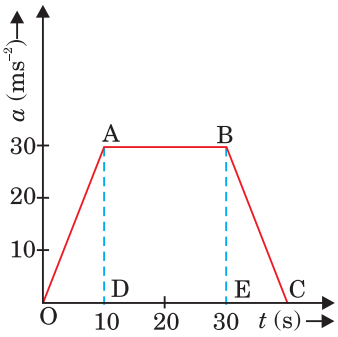
PRACTICE PROBLEMS 2

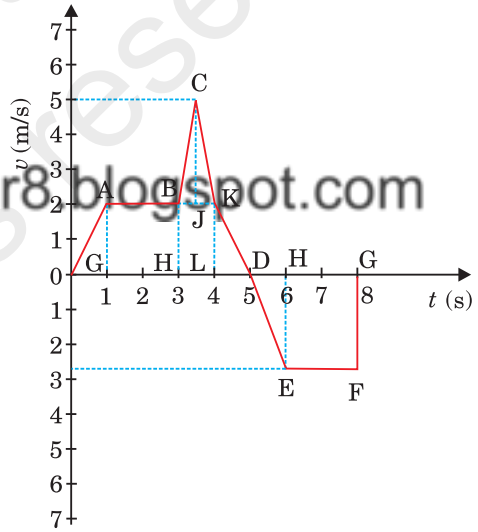
1. Calculate the displacement and distance travelled by the body in 5 s, from the adjoining velocity-time graph.



2. From the adjoining acceleration-time graph for a particle, find the average acceleration in initial 30 s.



3. Calculate the displacement covered by a moving body in 8 s. The velocity-time graph of the body is shown below.



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4. A body is projected in a vertical direction. The altitude y is given by

$$y = 10\,t^2 - 9t + 5$$

Calculate the initial velocity of the body.

5. The displacement of a particle moving along a straight line is given by

$$s = 3t^3 + 2t^2 - 10t + 9$$

Calculate the velocity at $t = 2$ s and acceleration at $t = 3$ s.

6. A particle is moving with an initial velocity of 5 ms^{-1}. What will be the velocity of the particle at $t = 3$ s, if the acceleration of the particle is given by

$$a = 16\,t^3 - 12\,t^2 + 6t - 25$$

7. The velocity of an object varies as $v = k\frac{t^3}{4}$. Calculate the change in position of the ball in time t.

Answers to Practice Problems

1. 12 m and 20 m, respectively
2. 25 m/s^2
3. +2 m
4. –9 m/s
5. 34 m/s and 58 m/s^2
6. 173 m/s
7. $k\frac{t^4}{16}$

6. EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

Let a particle be moving with uniform acceleration 'a' along a straight line with initial velocity 'u'. Let 'v' be the velocity of the particle after time t. Velocity-time graph for this interval of time is represented by AC in Fig. 6.1.

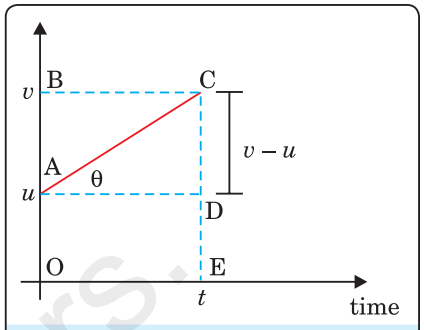

Fig. 6.1. Velocity-time graph for uniformly accelerated motion.

We know **that the slope (tan θ) of *v-t* graph gives the acceleration (*a*)**. For the given $v - t$ graph we can write the following:

$$\tan\theta = \frac{CD}{AD}$$

$$\Rightarrow \quad a = \frac{v-u}{t}$$

$$\Rightarrow \quad v = u + at \qquad \ldots(1)$$

Above obtained equation is the first equation of uniformly accelerated motion. We can use the above equation as a formula for a particle moving with constant acceleration.

As we have already studied that **area under *v* – *t* graph gives the displacement for the particle**. Let a particle placed at origin start moving along the X-axis, let at time t its position coordinate be x. Therefore, the displacement of the particle at time t is x. Hence, we can write the following:

x = Area under the graph AC = Area within ACDEOA

$$\Rightarrow \quad x = \frac{1}{2} \times (OA + CE) \times OE$$

$$\Rightarrow \quad x = \frac{1}{2} \times (u + v) \times t$$

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Substituting value of v from equation (1) we get the following:

$$\Rightarrow \quad x = \frac{1}{2} \times (u + u + at) \times t$$

$$\Rightarrow \quad x = \frac{1}{2} \times (2u + at) \times t$$

$$\Rightarrow \quad x = ut + \frac{1}{2}at^2 \qquad \ldots(2)$$

Equation (2) is the second equation of uniformly accelerated motion. We can use this equation as a formula for a particle moving with constant acceleration.

Now we can substitute time t calculated from equation (1) in equation (2) to get the third equation of motion. Or as an alternate approach we can square the equation (1) and proceed as follows:

$$v^2 = (u + at)^2$$

$$\Rightarrow \quad v^2 = u^2 + 2uat + a^2t^2$$

$$\Rightarrow \quad v^2 = u^2 + 2a\left(ut + \frac{1}{2}at^2\right)$$

We can now substitute the value from equation (2) in above equation to get the following:

$$\Rightarrow \quad v^2 = u^2 + 2ax \qquad \ldots(3)$$

Equation (3) is the third equation of uniformly accelerated motion. We can use this as a formula for a particle moving with a constant

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acceleration. It is only a scalar equation. However, equation (1) and (2) are vector in nature.

To derive the equations of motion here, we have assumed that the particle is at $x = 0$ at time $t = 0$. If position coordinate of the particle is x_0 at time $t = 0$ then these equations of motion can be written as follows:

$$\Rightarrow \quad v = u + at \qquad \ldots(1a)$$

$$\Rightarrow \quad x = x_0 + ut + \frac{1}{2}at^2 \qquad \ldots(2a)$$

$$\Rightarrow \quad v^2 = u^2 + 2a(x - x_0) \qquad \ldots(3a)$$

6.1. Deriving Equations Using Method of Integration

Let a particle be moving with a uniform acceleration 'a' along a straight line with an initial velocity 'u'. Let 'v' be the velocity of the particle after time t.

(1) We know that instantaneous acceleration can be written as follows:

$$a = \frac{dv}{dt}$$

$$\Rightarrow \quad dv = adt$$

On integrating both sides for the time interval between 0 and t, in which the velocity changes from u to v, we get the following:

$$\Rightarrow \quad \int_u^v dv = a\int_0^t dt$$

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$$\Rightarrow \quad [v]_u^v = a[t]_0^t$$

$$[\,v - u] = a[t - 0]$$

$$\Rightarrow \quad v - u = at$$

$$\Rightarrow \quad v = u + at$$

Hence, we have got first equation of motion.

(2) We know that the instantaneous velocity can be written as follows:

$$v = \frac{dx}{dt}$$

$$\Rightarrow \quad dx = vdt$$

Substituting the value of v from the first equation of motion we get the following:

$$\Rightarrow \quad dx = (u + at)dt$$

$$\Rightarrow \quad dx = udt + atdt$$

On integrating both sides we get the following:

$$\Rightarrow \quad \int_{x_0}^{x} dx = u\int_0^t dt + a\int_0^t tdt$$

(Here x_0 is the position of the object at time, $t = 0$ and x is the position of the object at time t)

$$\Rightarrow \quad [x]_{x_0}^{x} = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

KEY NOTE

The equations of motion derived in this section are only for uniformly accelerated motion. These equations are also not valid for circular motion or simple harmonic motion since the acceleration is not uniform in these cases.

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$$\Rightarrow \quad x - x_0 = u(t-0) + a\left(\frac{t^2}{2} - 0\right)$$

$$\Rightarrow \quad x = x_0 + ut + \frac{1}{2}at^2$$

Hence, we have got the second equation of motion.

(3) We can write instantaneous acceleration and velocity as follows:

$$a = \frac{dv}{dt} \ \& \ v = \frac{dx}{dt}$$

Dividing the above equations we get the following relation.

$$\frac{a}{v} = \frac{dv}{dx}$$

$$\Rightarrow \quad vdv = adx$$

On integrating both sides of above equations, we get

$$\int_u^v vdv = a\int_{x_0}^x dx$$

(Here, x_0 is the position of the object at time, $t = 0$ and x is the position of the object at time t and u and v are initial and final velocities)

$$\Rightarrow \quad \left[\frac{v^2}{2}\right]_u^v = a[x]_{x_0}^x$$

$$\Rightarrow \quad \left[\frac{v^2}{2} - \frac{u^2}{2}\right] = a[x - x_0]$$

$$\Rightarrow \quad v^2 = u^2 + 2a(x - x_0)$$

Hence, we have got the third equation of motion.

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KEY NOTE

Displacement in *n*th second

*n*th second is interval from $(n-1)$ s to n s.

Displacement in $(n-1)$ s can be written as

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

Displacement in n second can be written as.

$$S_n = un + \frac{1}{2}an^2$$

Displacement in *n*th second can be written as

$$S_{n^{\text{th}}} = S_n - S_{n-1}$$

$$\Rightarrow \quad S_{n^{\text{th}}} = \left[un + \frac{1}{2}an^2\right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

Solving above equation we get the following:

$$S_{n^{\text{th}}} = u + \frac{a}{2}(2n-1)$$

FORMULAE USED

(*i*) $v = u + at$

(*ii*) $x = x_0 + ut + \frac{1}{2}at^2$

(*iii*) $v^2 = u^2 + 2a(x - x_0)$

(*iv*) $S_{n^{\text{th}}} = u + \frac{a}{2}(2n-1)$

SOLVED EXAMPLES

Example 1.

A motorcyclist starts from rest and accelerates to a speed of 60 km/hr in 50 s, on a straight road. Calculate the distance covered by the motorcyclist in this time.

Solution: Here, initial velocity, $u = 0$

Time, $t = 50$ s

Final velocity, $v = 60$ km/hr

$$= \frac{60 \times 5}{18} = 16.67 \text{ m/s}$$

Using, $v = u + at$

$$\Rightarrow \quad a = \frac{v-u}{t} = \frac{16.67 - 0}{50}$$

$$= 0.3334 \text{ m/s}^2$$

Distance, $s = ut + \frac{1}{2}at^2$

$$= 0 \times 50 + \frac{1}{2} \times 0.3334 \times (50)^2$$

$$= \mathbf{416.75\ m}$$

Example 2.

An electron enters an electric field of width 40 μm with a speed of 6×10^3 m/s and takes 6 ns to cross it. Find its acceleration and velocity just after it exits the field.

Solution: Initial velocity, $u = 6 \times 10^3$ m/s

Time taken, $t = 6$ ns $= 6 \times 10^{-9}$ s

Distance travelled by the electron,

$$s = 40\ \mu\text{m} = 40 \times 10^{-6} \text{ m}$$

$$= 4 \times 10^{-5} \text{ m}$$

We know that,

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad a = \frac{2(s - ut)}{t^2}$$

$$= \frac{2(4 \times 10^{-5} - 6 \times 10^3 \times 6 \times 10^{-9})}{(6 \times 10^{-9})^2}$$

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$$= \frac{2(4 - 3.6) \times 10^{-5}}{36 \times 10^{-18}}$$

$$= 0.022 \times 10^{13} \text{ m/s}^2$$

$$= \mathbf{0.22 \times 10^{12} \text{ m/s}^2}$$

Also, Final velocity, $v = u + at$

$$= 6 \times 10^3 + 0.22 \times 10^{12} \times 6 \times 10^{-9}$$

$$= (6 + 1.32) \times 10^3 = \mathbf{7.32 \times 10^3 \text{ m/s}}$$

Example 3.

On seeing red light ahead, a driver takes 0.3s to apply the brakes. This is called the reaction time of the driver. If he was initially driving at a speed of 36 km/hr and application of the brakes causes a deceleration of 4.0 m/s^2, calculate the distance travelled by the car after he sees the red light.

Solution: The driver takes 0.3 *s* to apply the brakes after he sees the red light.

So, during this reaction time, he continues to move with a uniform speed of 36 km/hr $= \frac{36 \times 5}{18} = 10$ m/s

$\therefore$ Distance covered during 0.3 s $= 10 \times 0.3 = 3$ m

Now, for the motion with deceleration, we have

Initial velocity, $u = 10$ m/s

Final velocity, $v = 0$

deceleration, $a = -4.0$ m/s^2

As $v^2 - u^2 = 2as$

$\Rightarrow$ $0^2 - 10^2 = 2 \times (-4.0) \times s$

$\Rightarrow$ $s = \frac{100}{8} = 12.5$ m

$\therefore$ Total distance travelled by the car = 3 m + 12.5 m

= 15.5 m

Example 4.

Two trains travelling along a straight track are heading towards each other, travelling at 90 km/hr and 100 km/hr. Drivers of both the trains apply brakes together when the train are 1.0 km apart, decelerating each train at 2.0 m/s^2. Determine that whether the train accident will be averted.

Solution: For the 1st train:

$u_1 = 90$ km/hr $= 25$ m/s

$u_1 = 0$; $a_1 = -2.0$ m/s^2

$\therefore$ $v_1^2 - u_1^2 = 2a_1 s_1$

$\Rightarrow$ $0^2 - (25)^2 = 2 \times (-2) \times s_1$

$\Rightarrow$ $s_1 = \frac{625}{4} = 156.25$ m

Distance covered by the first train before coming to halt = $s_1 = 156.25$ m

For the second train:

$u_2 = 108$ km/hr $= 30$ m/s

$v_2 = 0$; $a_2 = -2.0$ m/s^2

$\therefore$ $v_2^2 - u_2^2 = 2a_2 s_2$

$\Rightarrow$ $0^2 - (30)^2 = 2 \times (-2) s_2$

$\Rightarrow$ $s_2 = \frac{900}{4} = 225$ m

Distance covered by the second train before coming to halt = $s_2 = 225$ m

Total distance covered by both the trains

= 156.25 + 225 m

= 381.25

This distance is less than 1 km (= 1000 m), which was the distance between the trains before applying brakes.

So, the accident (collision) will be averted.

Example 5.

A truck starts from rest and accelerates uniformly to a velocity of 40 m/s in 10 s. It then runs with this uniform velocity and is finally brought to rest in 50 m by a uniform deceleration. If the total run of the truck is 500 m, find its acceleration, deceleration and total time of journey.

Solution: When the truck has uniform acceleration after starting from rest:

$u = 0$, $t = 10$ s, $v = 40$ m/s

$v = u + at$

$\Rightarrow$ $a = \frac{v - u}{t} = \frac{40 - 0}{10} = 4$ m/s^2

Distance travelled, $s_1 = ut + \frac{1}{2}at^2$

$= \frac{1}{2} \times 4 \times (10)^2 = 200$ m

When the truck runs with uniform velocity:

Let distance covered is s_2 in this case. If s_3 is the distance covered during deceleration, then

As $s_1 + s_2 + s_3 = 500$ [given $s_3 = 50$ m]

$\therefore$ $s_2 = 500 - s_1 - s_3$

$= 500 - 200 - 50$

$= 250$ m

This distance is covered with a uniform velocity of 40 m/s

$\therefore$ Time taken $= \frac{250}{40} = 6.25$ s

When the truck is finally brought to rest:

$u = 40$ m/s, $v = 0$, $s = s_3 = 50$ m

$\therefore$ $v^2 - u^2 = 2as$

$\Rightarrow$ $0^2 - 40^2 = 2 \times a \times 50$

$\Rightarrow$ $a = -\frac{1600}{100} = -16$ m/s^2

Retardation $= 16$ m/s^2

Also, $v = u + at$

$0 = 40 - 16 \times t$

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$$\Rightarrow \quad t = \frac{40}{16} = 2.5 \text{ s}$$

$\therefore$ Total time taken for the journey = 10 + 6.25 + 2.5

= **18.75 s**

❑ Example 6.

A thief starts his car from a point with an acceleration of 2 m/s^2. The police chasing the thief arrives at the point 10 s later and continues to chase the thief with a uniform velocity of 40 m/s in their car. In what time the police's car will overtake the thief's car?

Solution: Given, acceleration of thief's car, $a = 2$ m/s^2

Initial velocity, $u = 0$

Uniform velocity of police's car, $v = 40$ m/s

Let t be the time in seconds taken by the police's car to overtake the thief's car.

Distance covered by the police's car in t seconds = vt

$= 40\,t$

Distance covered by the thief's car in $(t + 10)$ seconds

$$s = u(t + 10) + \frac{1}{2}a(t + 10)^2$$

$$= \frac{1}{2} \times 2(t + 10)^2 = (t + 10)^2$$

Now, $40t = (t + 10)^2$

$\Rightarrow \quad 40t = t^2 + 20t + 100$

$t^2 - 20t + 100 = 0$

$\Rightarrow \quad (t - 10)^2 = 0$

or $t =$ **10 s**

❑ Example 7.

A man is 10 m behind the bus stop when the bus left the stop, with an acceleration of 2 m/s^2. What should be the minimum uniform speed of man so that he may catch the bus? If he runs with required minimum speed then how much time it will take to catch the bus?

Solution: Let v be the speed of the man and t be the time taken by man to catch the bus. Condition for man to catch the bus can be written as follows:

Distance moved by man = 10 + distance moved by bus

$$\Rightarrow \quad vt = 10 + \frac{1}{2} \times 2 \times t^2$$

$$\Rightarrow \quad t^2 - vt + 10 = 0 \qquad \ldots(i)$$

Above equation must have real roots.

$\Rightarrow \quad \Delta \geq 0$

$\Rightarrow \quad v^2 - 40 \geq 0$

$\Rightarrow \quad v \geq 2\sqrt{10}$

Hence minimum speed for man is $2\sqrt{10}$ m/s.

Substituting minimum value of v in equation (i) we get the following:

$t^2 - 2\sqrt{10}\,t + 10 = 0$

$\Rightarrow \quad (t - \sqrt{10})^2 = 0$

$\Rightarrow \quad t = \sqrt{10}$ **s**

❑ Example 8.

A train initially at rest accelerates at a constant rate x for time t_1 and then decelerates at a constant rate y for the remaining time. If the total time of its journey is t seconds, then find in terms of t, x and y

(i) the maximum velocity attained

(ii) total distance travelled

Solution: (i) For the time t_1:

$$v = u + at$$

$$= 0 + xt_1$$

$$\Rightarrow \quad t_1 = \frac{v}{x} \qquad \ldots(i)$$

Here, v will be the maximum velocity attained by the train because after that, it will start to decelerate

For the time $t_2 = (t - t_1)$

Using $v = u + at$

we can write

$$0 = v - y\,(t - t_1)$$

or $t - t_1 = \frac{v}{y} \qquad \ldots(ii)$

Adding (i) and (ii), we get

$$t_1 + t - t_1 = \frac{v}{x} + \frac{v}{y}$$

$$\Rightarrow \quad t = v\left(\frac{1}{x} + \frac{1}{y}\right) = v\left(\frac{x + y}{xy}\right)$$

or $\quad v = \dfrac{xyt}{x + y}$

This is the required expression for maximum velocity.

(ii) Distance travelled by the train in time t_1 is

$$s_1 = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2}xt_1^2 = \frac{1}{2}x \cdot \frac{v^2}{x^2} = \frac{v^2}{2x}$$

$$= \frac{x^2y^2t^2}{2x(x + y)^2}$$

$$= \frac{xy^2t^2}{2(x + y)^2}$$

Distance travelled by the train in time $t_2 = (t - t_1)$ is

$$s_2 = ut + \frac{1}{2}at^2$$

$$= v(t - t_1) - \frac{1}{2}y(t - t_1)^2$$

$$= v \cdot \frac{v}{y} - \frac{1}{2}y\frac{v^2}{y^2} = \frac{v^2}{2y} = \frac{x^2yt^2}{2(x + y)^2}$$

$\therefore$ Total distance travelled by the train

$$s = s_1 + s_2 = \frac{xy^2t^2}{2(x + y)^2} + \frac{x^2yt^2}{2(x + y)^2}$$

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$$= \frac{xyt^2}{2(x+y)^2}(y+x)$$

$$= \frac{\boldsymbol{xyt^2}}{\boldsymbol{2(x+y)}}$$

Example 9.

A body covers a distance of 15 m in the 5th second and 20 m in the 7th second. How much it will cover in the 12th second?

Solution: Here, $s_{5th} = 15$ m and $s_{7th} = 20$ m

As $s_{5th} = u + \frac{1}{2}a(2\times 5 - 1) = 15$

$\Rightarrow u + \frac{9a}{2} = 15$ ...(*i*)

Also, $s_{7th} = u + \frac{1}{2}a(2\times 7 - 1) = 20$

$\Rightarrow u + \frac{13}{2}a = 20$ ...(*ii*)

Subtracting (*i*) from (*ii*), we get

$$\frac{4a}{2} = 5 \qquad \Rightarrow a = \frac{10}{4} = 2.5 \text{ m/s}^2$$

Using (*i*), $u = 15 - \frac{9a}{2} = 15 - \frac{9\times 2.5}{2}$

$= 3.75$ m/s

Now, $S_{12th} = u + \frac{1}{2}a(2\times 12 - 1)$

$= 3.75 + \frac{1}{2}\times 2.5 \times 23$

$= 3.75 + 28.75 = \mathbf{32.5\ m}$

Example 10.

An object moving with uniform acceleration has a velocity of 20 m/s after 4 seconds and 28 m/s after 6 seconds. Calculate the distance travelled by the object in 10th second.

Solution: For a velocity of 20 m/s:

$v = u + at$

$\Rightarrow 20 = u + 4a$ ...(*i*)

For a velocity of 28 m/s

$28 = u + 6a$ ...(*ii*)

Subtracting (*i*) from (*ii*), we get

$8 = 2a$

$\Rightarrow a = \frac{8}{2} = 4$ m/s^2

Substituting this value in (*i*), we get

$20 = u + 4\times 4$

$\Rightarrow u = 20 - 16 = 4$ m/s

$\therefore$ Distance travelled by the object in 10th second

$S_{10th} = u + \frac{1}{2}\times a(2n-1)$

$= u + \frac{1}{2}\times 4(2\times 10 - 1)$

$= 4 + 2(19)$

$= \mathbf{42\ m}$

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PRACTICE PROBLEMS 3

1. A particle is moving with an initial velocity of 2 m/s and covers a distance of 20 m in 5 s. Find (*a*) acceleration of particle (*b*) time taken by the particle to attain a velocity of 10 m/s.
2. A particle is moving under a constant acceleration of 3 m/s^2. The initial velocity of the particle is 10 m/s. Calculate the distance travelled by the particle in 5 s and also in the 5th second of its motion.
3. A particle moves 20 m in 3rd second and 30 m in 5th second of its journey. What will be the distance covered by the body in 4 s after 6th second?
4. A particle is initially at rest. It starts moving with a constant acceleration. The speed of the particle in t seconds is 50 m/s and after one second the speed becomes 75 m/s. Calculate the acceleration of the particle and distance covered in $(t+1)^{th}$ second of its motion.
5. What will be the minimum stopping distance required by a car to stop if it is moving with a velocity of 40 m/s with a deceleration of 20 m/s^2? How much time will it take to stop the car?

Answers to Practice Problems

1. (*a*) 0.8 m/s^2 (*b*) 10 s
2. 87.5 m, 23.5 m
3. 190 m
4. 25 m/s^2, 62.5 m
5. 40 m, 2 s

7. MOTION UNDER GRAVITY

We know that earth applies gravitational force on all objects and that is why objects fall on the surface of earth when released from a height. If the height of the object is not much (negligible as compared to the radius of earth) then we can assume that the force and resulting acceleration due to gravity remain nearly constant near the surface of earth. Acceleration due to gravity near the surface of earth is found to be approximately **9.8 m/s^2**. We know that the acceleration is a vector quantity and the direction of acceleration due to gravity is always vertically downwards (towards the centre of earth). If an object is released from the rest from a certain height or it is projected vertically upwards or downwards with some initial velocity then in these cases motion of the object remains restricted to vertical straight line. Acceleration due to gravity is represented by 'g' and remains constant near the surface of earth. Hence, we can use three equations of motion for uniformly accelerated motion. To analyse the motion under gravity along vertical straight line we assume one of the coordinate axes along the vertical direction. We can select upward or downward direction as positive direction. Let us understand the procedure with the help of a few examples.

(1) An object is thrown vertically upwards with an initial velocity u. Acceleration due to gravity is downwards and hence is directed opposite to the initial velocity given to the object. When the acceleration (rate of change of velocity) is in the direction opposite to that of the velocity of a particle, the motion of the particle is retarded. Hence, in this case the motion of the object is first retarded and then the object comes to rest for an instant at a certain height (H) above the point of projection.

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Let the time taken by the particle to reach this point be t_1. At time $t = t_1$, the particle comes to rest but the acceleration is still acting downwards and hence particle is again moved downwards. During the downward journey the velocity and acceleration are in the same direction and hence, **speed of the particle increases as it moves down**. Let the particle reaches starting point O again at time $t = t_2$. Let us assume point of projection as origin O and upward direction as the positive direction as shown in Fig. 7.1.

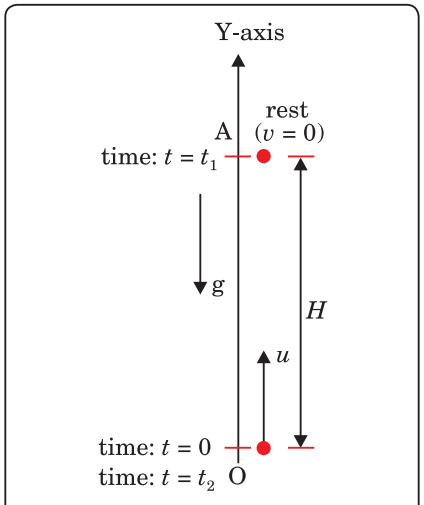

Fig. 7.1. Motion of an object thrown vertically upwards.

Let us consider the time interval when the particle starts from O and reaches the highest point A.

$$v = u + at$$

$$\Rightarrow \quad 0 = (+u) + (-g)\, t_1$$

Note that the upward direction is taken as positive and hence initial velocity u is taken as positive because it is upwards and acceleration due to gravity is negative because it is directed vertically downwards. Solving the above equation we get the following:

$$\Rightarrow \quad t_1 = \frac{u}{g} \qquad \ldots(1)$$

Hence, the time taken by an object to reach the top-most point when it is thrown upwards with initial velocity u is $\frac{u}{g}$.

We should understand that the selection of positive direction is a matter of choice as we can see that by changing our choice for

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positive direction, signs of all the vector quantities get reversed and thus overall equation remains unaffected.

Let us now apply third equation of motion for the motion of the object between points O and A.

$$v^2 = u^2 + 2ax$$

Here direction of displacement (x) is upwards (from initial point towards the final point) and hence will be treated as positive.

$$\Rightarrow \quad 0 = (+u)^2 + 2(-g)(+H)$$

$$\Rightarrow \quad H = \frac{u^2}{2g} \qquad \ldots(2)$$

Above expression can be used for calculating the maximum height attained by the object when thrown vertically upwards with an initial velocity u.

Let us now consider the time interval when the particle starts from O and reaches A and then again comes back to point O. For this interval the net displacement is zero. We can apply second equation of motion as follows:

$$x = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad 0 = (+u)t + \frac{1}{2}(-g)t^2$$

$$\Rightarrow \quad 0 = \left(u - \frac{1}{2}gt\right)t$$

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$$\Rightarrow \quad t = 0 \ \& \ t = \frac{2u}{g}$$

Here, $t = 0$ represents the beginning of motion and $t = 2u/g$ represents the time when the particle is again back to point O. We have assumed t_2 as the time to the come back to the starting point O.

$$t_2 = \frac{2u}{g} \qquad \ldots(3)$$

When an object is thrown vertically upwards with an initial velocity u then it comes back to the starting point after time $2u/g$. **Hence particle takes *u/g* time in upward journey and same time for the downward journey**. When no air resistance is considered then the upward and downward journeys are symmetrical. Particle comes back to the starting point with the same speed but the direction of motion is opposite.

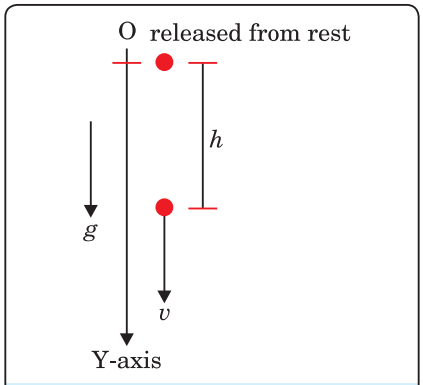

Fig. 7.2. Motion of an object released from rest.

(2) An object is released from rest from a point at a certain height from the earth's surface (refer to Fig. 7.2) and due to gravity it accelerates downwards. Let us consider starting point as the origin (O) and downward direction as the positive direction. Let the particle descend height h in time t. Acceleration due to gravity is downwards and hence will be taken as positive. Direction of displacement is taken from initial point towards the final point, hence displacement h is also directed downwards and hence positive.

We can write velocity attained by the object in terms of time using first equation of motion.

$$v = u + at$$

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$$\Rightarrow \quad v = 0 + (+g)\,t$$

$$v = gt \qquad \ldots(4)$$

We can also write velocity attained in terms of height descended by the object using third equation of motion.

$$v^2 = u^2 + 2ax$$

$$\Rightarrow \quad v^2 = 0 + 2(+g)(+h)$$

$$\Rightarrow \quad v = \sqrt{2gh} \qquad \ldots(5)$$

We can write height descended by the object in terms of time using second equation of motion.

$$x = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad (+h) = 0 + \frac{1}{2}(+g)t^2$$

$$\Rightarrow \quad h = \frac{1}{2}gt^2 \qquad \ldots(6)$$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}} \qquad \ldots(7)$$

Thus, the time taken by an object released from rest from a height h can be calculated by using equation (7).

(3) Let us now take an example of particle thrown upwards with initial velocity u, from the top of a building of height H. We have to calculate the time taken by the particle to reach the ground. In Fig. 7.3, A is the point at the top of building from where the object is projected. In above cases we have taken the initial point as the origin. In this case we are assuming bottom of building as the origin and that is shown as O in the figure. Here we have assumed upward direction as positive and marked vertical axis as Y-axis. The second equation of motion in this case can be written as follows:

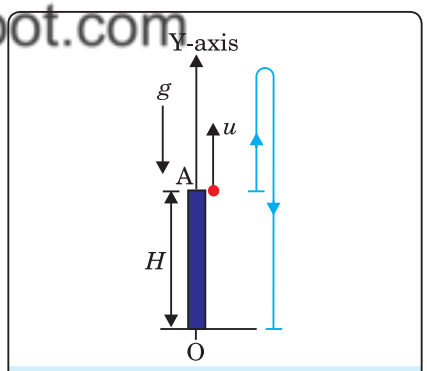

Fig. 7.3. Motion of an object thrown upward from a height.

$$y = y_0 + ut + \frac{1}{2}at^2$$

We have to calculate the time taken by the particle to reach the bottom of the building. Hence, final y-coordinate of the particle will be 0.

$$y = 0, y_0 = +H, u = +u, a = -g.$$

$$\Rightarrow \quad 0 = H + (+u)t + \frac{1}{2}(-g)t^2$$

$$\Rightarrow \quad gt^2 - 2ut - 2H = 0$$

$$\Rightarrow \quad t = \frac{2u \pm \sqrt{4u^2 + 8gH}}{2g}$$

$$\Rightarrow \quad t = \frac{u}{g} \pm \sqrt{\frac{u^2}{g^2} + \frac{2H}{g}}$$

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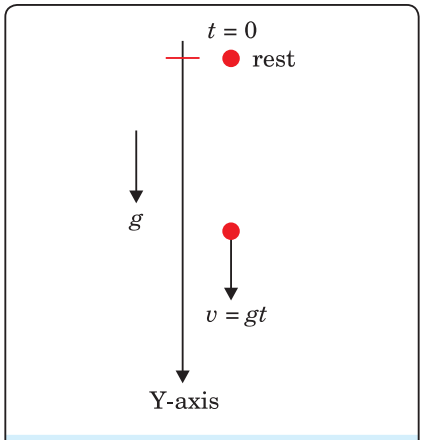

Fig. 7.4. Free fall.

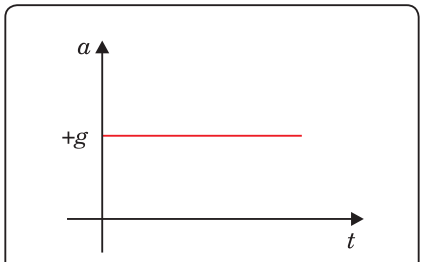

Fig. 7.5. Acceleration-time graph for free fall.

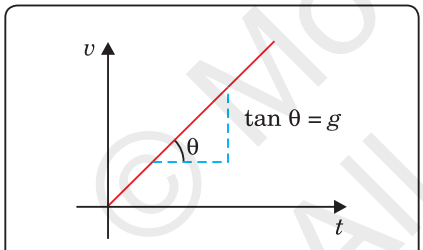

Fig. 7.6. Velocity-time graph for free fall.

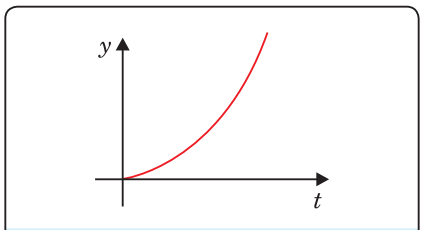

Fig. 7.7. Position-time graph for free fall.

Out of the two solutions of the quadratic equation, one is positive and the other is negative. Positive value of t is the valid solution. Hence, the time taken by the particle to reach the bottom of building is given by the following relation:

$$\Rightarrow \qquad t = \frac{u}{g} + \sqrt{\frac{u^2}{g^2} + \frac{2H}{g}}$$

(4) Graphical representation of motion of an object under free fall

If we release an object near the surface of earth then it is accelerated vertically downwards due to gravitational attraction of earth. Here we are neglecting air resistance in our analysis. If the height of object is small as compared to the radius of the earth then we can assume acceleration due to gravity as constant. The acceleration due to gravity is represented by g and its value near the surface of earth is approximately equal to 9.8 m/s^2. Hence, given case is an example of the motion with uniform acceleration. To represent a motion in terms of graphs we need to select the origin and the axes for measurement. In this case let us take the point from where object is dropped as the origin for the measurement and downward direction as positive direction for Y-axis as shown in Fig. 7.4.

Acceleration due to gravity is always vertically downwards and here downward direction is taken as the positive direction and hence acceleration remains constant in positive direction and acceleration-time graph will be as shown in Fig. 7.5.

Similarly, we can use first equation of motion to get the velocity in terms of time.

$$v = u + at$$

$$\Rightarrow \qquad v = 0 + (+g)\, t$$

$$\Rightarrow \qquad v = gt$$

Hence, **velocity increases linearly with time in positive direction** and graph between velocity and time will be as shown in Fig. 7.6.

If $v - t$ graph makes an angle θ with the time axis then its slope is tan θ. Slope of $v - t$ graph represents the acceleration and hence we can write tan θ = g.

We can use second equation of motion to write y-coordinate of the object in terms of time.

$$y = ut + \frac{1}{2} at^2$$

$$\Rightarrow \qquad y = 0 + \frac{1}{2} (+g)\, t^2$$

$$\Rightarrow \qquad y = \frac{1}{2} gt^2$$

Position-time graph for the object under free-fall can be shown as in Fig. 7.7. It is a parabolic curve.

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SOLVED EXAMPLES

Example 1.

A ball is thrown vertically upwards with a velocity of 4.9 m/s. Find

(i) the maximum height reached by the ball

(ii) time taken to reach the maximum height

Solution: (*i*) Given, initial velocity, $u = 4.9$ m/s

Acceleration, $a = 9.8$ m/s^2

If we assume upward direction as positive direction then initial velocity will be treated as positive and acceleration due to gravity as negative.

At maximum height, final velocity, $v = 0$

Using $v^2 - u^2 = 2ah$, we have

$$0^2 - (4.9)^2 = 2 \times (-9.8)h$$

$$\Rightarrow \quad h = \frac{(4.9)^2}{2 \times 9.8} = \mathbf{1.225\ m}$$

(*ii*) Using, $v = u + at$

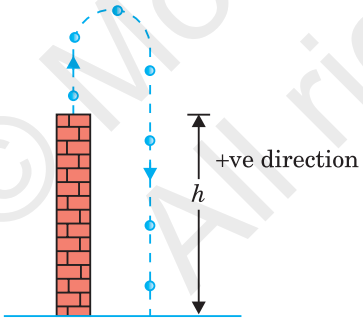
$$0 = 4.9 - 9.8t$$

$$\Rightarrow \quad t = \mathbf{0.5\ s}$$

Example 2.

A stone is thrown vertically upwards from the top of a building, with a speed of 14.7 m/s. If it returns to the earth in 8 s, calculate the height of the building.

Solution: Let us assume upward direction as positive direction



Given, initial velocity, $u = 14.7$ m/s

$g = -9.8$ m/s

Time taken to reach earth, $t = 8$ s

Net displacement of the stone, $s = -h$

(Vertically upward direction here is taken as positive and the net displacement from the initial position is in the vertically downward direction)

Now, $\quad s = ut + \frac{1}{2}gt^2$

$$\Rightarrow \quad -h = 14.7 \times 8 + \frac{1}{2} \times (-9.8) \times 8^2$$

$$= 117.6 - 313.6 = -196 \text{ m}$$

$$\Rightarrow \quad h = \mathbf{196\ m}$$

Example 3.

Using the information given in example 2, find

(i) the greatest height reached by the stone above the ground

(ii) the velocity with which it strikes the ground

(iii) time taken to reach the maximum height

Solution:

(*i*) Let s' be the height covered in upward motion then

$$v^2 = u^2 + 2gs'$$

$$0 = (14.7)^2 + 2 \times (-9.8)s'$$

$$\Rightarrow \quad s' = \frac{(14.7)^2}{19.6} = 11.025 \text{ m}$$

$\therefore$ The greatest height above the ground $= s' + s$

$= 11.025 + 196$

$= \mathbf{207.025\ m}$

(*ii*) For the downward motion of the stone,

$u = 0$, $g = 9.8$ m/s^2, $s = 207.028$ m

Using $\quad v^2 = u^2 + 2gs$

$$= 0 + 2 \times (9.8) \times 207.025$$

$$\Rightarrow \quad v = 63.7 \text{ m/s}$$

(*iii*) Let t be the time taken to reach the maximum height. Then, for the upward motion.

$u = 14.7$ m/s, $v = 0$, $g = -9.8$ m/s^2

Using $v = u + gt$, we get

$$t = \frac{v - u}{g} = \frac{0 - 14.7}{-9.8} = \mathbf{1.5\ s}$$

Example 4.

A ball is thrown vertically upwards with a velocity of 20 m/s from the top of a multistorey building. The height of the point from where the ball is thrown is 25.0 m from the ground. (a) How high will the ball rise? and (b) How long will it be before the ball hits the ground?

Take $g = 10$ m/s^2 *(N.C.E.R.T. Solved Example)*

Solution: (*a*) Given, initial velocity, $u = 20$ m/s

$g = -10$ m/s^2 (upward direction is assumed as positive)

At the highest point, final velocity, $v = 0$

Let the ball rise to height h from where it is thrown above the building. Then

$$v^2 - u^2 = 2gh$$

$$\Rightarrow \quad 0^2 - 20^2 = 2 \times (-10) \times h$$

$$\Rightarrow \quad h = \frac{400}{20} = 20 \text{ m}$$

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(*b*) Net displacement of the ball, s = – Height of building
= –25.0 m

Now, $s = ut + \frac{1}{2}gt^2$

$\Rightarrow \quad -25 = 20\,t + \frac{1}{2} \times (-10) \times t^2$

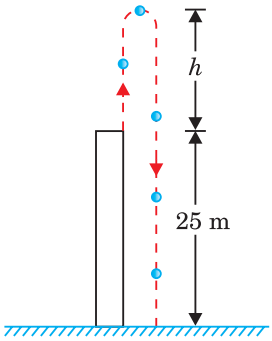
$\Rightarrow \quad 5t^2 - 20t - 25 = 0$

$\Rightarrow \quad t^2 - 4t - 5 = 0$

$(t + 1)(t - 5) = 0$

$t \neq -1$

$\therefore \quad t = 5$ s



Example 5.

A boy projects a stone vertically upwards and catches it back after 5 s. How high the stone goes and with what velocity it was thrown? Also calculate the separation between the highest point and the stone 4 s after it was projected upwards.

Take $g = 10\ m/s^2$

Solution: Let us take upward as positive direction. For the stone,

Time of ascent = Time of descent

Total time taken by the stone to come back to the same level of projection = 5 s

$\therefore$ Time of ascent = 2.5 s

For the upward motion of the ball:

Initial velocity, u = ?

Final velocity, $v = 0$

Time taken, t = 2.5 s

$g = -10\ m/s^2$

Using $v = u + gt$, we get

$0 = u - 10 \times 2.5$

$\Rightarrow \quad u = 25$ m/s

This is the velocity with which the stone was projected. Maximum height attained by the ball is given by

$s = ut + \frac{1}{2}gt^2$

$= 25 \times 2.5 - \frac{1}{2} \times 10 \times (2.5)^2$

$= 62.5 - 31.25 =$ **31.25 m**

After 2.5 s, the stone is at the highest point.

So, distance of the stone from the highest point 4 s after it was thrown = Distance covered by the stone in 1.5 s from the highest point

$\therefore \quad s = ut + \frac{1}{2}gt^2$

$= 0 \times 1.5 + \frac{1}{2} \times 10 \times (1.5)^2$

$=$ **11.25 m**

Example 6.

A hot air balloon is projected vertically with a net vertical acceleration of 4 m/s² upwards, by lighting its burner. The burner is blown out in 2 minutes but the balloon continues to move up. Find the maximum height reached by the balloon before it starts to come down again.

Solution: Given, net vertical acceleration, $a = 4\ m/s^2$

Height covered by the balloon in 2 minutes (120 s) is

$h_1 = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 4 \times (120)^2$

$= 28{,}800$ m

Velocity attained by the hot air balloon after 2 min (120 s),

$v = u + at$

$= 0 + 4 \times 120$

$= 480$ m/s

After the burner is blown out,

$u = 480$ m/s and final velocity, $v = 0$

Using $v^2 - u^2 = 2gh_2$, we get

$0^2 - (480)^2 = -2 \times 9.8 \times h_2$

$\Rightarrow \quad h_2 = \frac{(480)^2}{2 \times 9.8} \simeq 11{,}755$ m

$\therefore$ Maximum height reached by the balloon is

$h = h_1 + h_2 = (28{,}800 + 11{,}755)$ m

$= 40{,}555$ m $= 40.555$ km

$\simeq$ **41 km**

Example 7.

A packet is dropped from a helicopter, which is ascending at a rate of 16 m/s, when it is at a height of 14.4 m above the ground.

(*i*) *How much time it will take the packet to reach the ground?*

(*ii*) *With what velocity it will hit the ground?*

Solution: (*i*) Initially the velocity of packet is same as that of the upward velocity of helicopter. So,

$u = 16$ m/s, $g = -9.8\ m/s^2$, $s = -14.4$ m

Note that here we have assumed upward as positive direction.

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Using, $s = ut + \frac{1}{2}gt^2$, we get

$$-14.4 = 16\,t - \frac{1}{2} \times 9.8\,t^2$$

$\Rightarrow$ $-14.4 = 16t - 4.9\,t^2$

$\Rightarrow$ $4.9\,t^2 - 16t - 14.4 = 0$

Solving, we get $t = \mathbf{4\ s}$

(*ii*) Using $v = u + gt$

$= 16 - 9.8 \times 4 = \mathbf{-23.2\ m/s}$

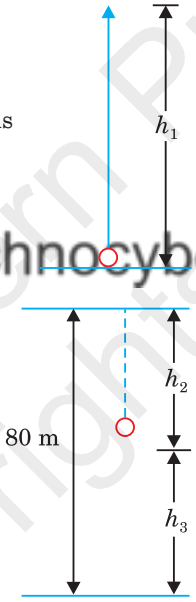
(Negative because it is directed vertically downwards)

Example 8.

A ball is thrown upwards with a speed of 20 m/s from the ground. Another ball is dropped from rest from a building 80 m high at the same time,

(*i*) *Find the difference in their height after they have fallen for 2 s?*

(*ii*) *If the balls are along the same line of motion, when will they collide?*



Solution: (*i*) For the first ball:

$u = 20$ m/s, $g = -9.8$ m/s^2

Height reached by the ball in $t = 2$ s is

$$h_1 = ut + \frac{1}{2}gt^2$$

$$= 20 \times 2 + \frac{1}{2} \times 9.8 \times 4$$

$$= 20.4 \text{ m}$$

For the second ball:

$u = 0$ m/s, $g = 9.8$ m/s^2

Distance travelled in $t = 2s$ is

$$h_2 = ut + \frac{1}{2}gt^2$$

$$= 0 \times 2 + \frac{1}{2} \times 9.8 \times 4$$

$$= 19.6 \text{ m}$$

$\therefore$ Height of the second ball from the ground,

$h_3 = 80$ m $- 19.6$ m

$= 60.4$ m

$\therefore$ Difference in the height of two balls $= h_3 - h_1$

$= 60.4 - 20.4$ m $= \mathbf{40\ m}$

(*ii*) Let after time t s, the balls will collide, then for the first ball:

$$h = 20t - \frac{1}{2}9.8t^2$$

$$= 20\,t - 4.9\,t^2 \qquad ...(i)$$

For the second ball, $80 - h = \frac{1}{2} \times 9.8\,t^2$

$\Rightarrow$ $80 - h = 4.9\,t^2$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$h + 80 - h = 20t - 4.9\,t^2 + 4.9\,t^2$

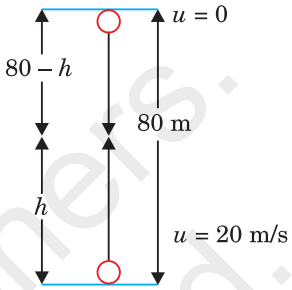
$\Rightarrow$ $80 = 20t$

$\Rightarrow$ $t = 4$ s

Substituting this in eq. (*ii*), we get

$80 - h = 4.9 \times 4^2$

$\Rightarrow$ $h = 80 - 78.4 = 1.6$ m



So, the two balls will collide after **4 s**, at a height of 1.6 m from the ground.

Example 9.

A stone is dropped from a stationary hot air balloon in the air and travels 14.7 m in the last second before it hits the ground. Find the height of the balloon from the ground.

Solution: Here, initial velocity of the stone, $u = 0$

$g = 9.8$ m/s^2 (downward motion)

Let the last second be the nth second. Then

$$S_{n\text{th}} = u + \frac{g}{2}(2n - 1)$$

$\Rightarrow$ $14.7 = 0 + \frac{9.8}{2}(2n - 1)$

$\Rightarrow$ $14.7 = 4.9(2n - 1)$

or $2n - 1 = \frac{14.7}{4.9} = 3$

$\Rightarrow$ $2n = 4$ or $n = 2$

$\therefore$ $t = 2$ s

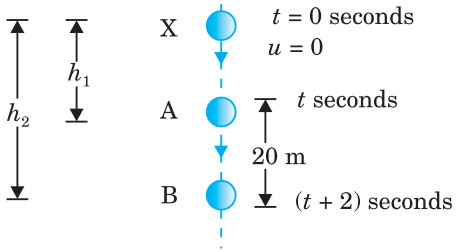
$\therefore$ Height of the balloon from the ground is

$$h = 0 + \frac{1}{2} \times 9.8 \times 2^2 = \mathbf{19.6\ m}$$

Example 10.

A body is falling freely under gravity. It passes two points A and B (A being higher than B) 20 m apart vertically, in 2s. Find the elevation of the point above A, from where it began to fall.

Solution:



Let X be the point above A from where the body began to fall. Then,

$$h_2 - h_1 = \frac{1}{2}g(t + 2)^2 - \frac{1}{2}gt^2$$

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$$= \frac{1}{2} g(4t + 4)$$

Given, $h_2 - h_1 = 20$ m

$$\therefore \quad 20 = \frac{1}{2} \times 9.8(4t + 4)$$

$$= 9.8(2t + 2)$$

$$\Rightarrow \quad 2t + 2 = \frac{20}{9.8} \quad \Rightarrow 2t + 2 = 2.04$$

$$\Rightarrow \quad 2t = 0.04 \quad \Rightarrow t = 0.02 \text{ s}$$

$$\therefore \quad h_1 = \frac{1}{2} gt^2 = \frac{1}{2} \times 9.8 \times (0.02)^2$$

$$= \mathbf{0.001\ m}$$

PRACTICE PROBLEMS 4

1. A ball is thrown vertically upwards from ground with such a speed so that it just reaches the top of a building and then falls back. If it takes 7 s for the ball to return to the hands of thrower, calculate the height of the building, approximately.

2. A ball A is dropped from the top of a tower 50 m high and at the same instant another ball B is projected upwards with the velocity of 10 m/s. After how much time and where the two balls will meet?

3. Two bodies are dropped from rest from two different buildings; one at a height 200 m and another at a height of 150 m above the ground. What will be the difference in height of the bodies after they have fallen for 2 s? (g = 10 m/s^2)

4. Two balls are released from the same height at an interval of 2 s. When will the separation between the balls be 20 m after the first ball is released? Take, g = 10 m/s^2.

Answers to Practice Problems

1. 60 m

2. After 5 s at 122.5 m from the top of tower

3. 50 m

4. 2 s

8. RELATIVE VELOCITY

Concept of motion is always relative. You always need a reference to measure the position of an object. Same is the reference for velocity as well as for the acceleration. In fact, all the measurements are relative in physics. In this section we will consider the objects moving with uniform velocities. Most of the time we consider the velocity with respect to the frame of reference attached to the ground. And many times we do not even mention the frame of reference because it is supposed to be understood. When we take the ground or earth as our frame of reference then for local movements we can treat this frame of reference to be at rest. But we should always remember that there is nothing in state of absolute rest or motion. Concept of relative motion is very important in physics and here we shall learn to express position and velocity of one object with respect to another object when positions and velocities of both of them are known in one common frame of reference which is usually fixed reference attached to the ground.

Vector quantities are denoted in a certain way and those things we shall learn in next chapter. But here in case of straight line motion we are not using any vector notations because in case of one-dimensional motion we can take care about directions by using positive (+) and negative (–) signs. Let there be two objects A and B moving along X-axis. v_A and v_B are the velocities of A and B respectively, with respect to the reference attached to ground or any other stationary object. Let x_{0A} and x_{0B} are position coordinates of A and B respectively at time t = 0. And let x_A and x_B be the position coordinates of A and B respectively at some instant of time t. We can write the following equations:

$$x_A = x_{0A} + v_A t \quad \ldots(1)$$

$$x_B = x_{0B} + v_B t \quad \ldots(2)$$

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Position of B with respect to A can be written as follows:

$$x_{BA} = x_B - x_A$$

$$\Rightarrow \quad x_{BA} = (x_{0B} - x_{0A}) + (v_B - v_A)t \quad \ldots(3)$$

Rate of change of position with respect to time is defined as velocity. Hence on differentiating above equation we shall get velocity of B with respect to A, because x_{BA} is the position of B with respect to A.

$$\Rightarrow \quad \frac{dx_{BA}}{dt} = v_B - v_A$$

$$\Rightarrow \quad v_{BA} = v_B - v_A \quad \ldots(4)$$

In above equation v_{BA} represents the velocity of B with respect to A.

Similarly we can write the velocity of object A with respect to B as follows:

$$\Rightarrow \quad v_{AB} = v_A - v_B \quad \ldots(5)$$

From equation (4) and (5) we can see that $v_{AB} = -v_{BA}$.

Figure 8.1 shows the position-time graph of two objects A and B moving with the same velocity along the positive direction. We can see that the separation between objects remains constant with time. Relative velocity of one object with respect to another is zero in this case.

$$v_{AB} = v_{BA} = 0.$$

Figure 8.2 shows the position-time graph of two objects A and B moving along positive direction. In given case $v_A > v_B$. Initial positions are such that these two objects do not meet and separation between them grows with time. Velocity of A with respect to B ($v_{AB} = v_A - v_B$) is along the positive direction and velocity of B with respect to A ($v_{BA} = v_B - v_A$) is along the negative direction. Let $v_A = 40$ m/s and $v_B = 20$ m/s.

$$v_{AB} = v_A - v_B = 40 - 20 = 20 \text{ m/s}.$$

$$v_{BA} = v_B - v_A = 20 - 40 = -20 \text{ m/s}.$$

Here, v_{AB} = Relative velocity of object A with respect to B

v_{BA} = Relative velocity of object B with respect to A

Figure 8.3 shows position-time graph of two objects A and B moving along positive direction. In given case $v_A < v_B$. Initial positions are such that these two objects meet at a certain instant. Let $v_A = 30$ m/s and $v_B = 50$ m/s.

$$v_{AB} = v_A - v_B = 30 - 50 = -20 \text{ m/s}.$$

$$v_{BA} = v_B - v_A = 50 - 30 = 20 \text{ m/s}.$$

In Fig. 8.4 object A is moving along negative direction and object B is moving along positive direction. Initial positions of the objects are such that they meet at a certain instant. Let $v_A = -40$ m/s and $v_B = 30$ m/s.

$$v_{AB} = v_A - v_B = -40 - 30 = -70 \text{ m/s}.$$

$$v_{BA} = v_B - v_A = 30 - (-40) = 70 \text{ m/s}.$$

When two objects are moving along the same direction then the magnitude of relative velocity is equal to the difference in magnitudes of their velocities. And when they are moving along opposite directions then magnitude of relative velocity is equal to the sum of magnitudes of their velocities. If you are sitting inside the train near the window and there is a parallel railway track and another train is running on that track too. If train on the parallel track is running in the same direction as that of yours then you will feel that other train is moving with slower speed. But when train on the parallel track is running along opposite direction to that of yours then you will feel that other train is running with a high speed.

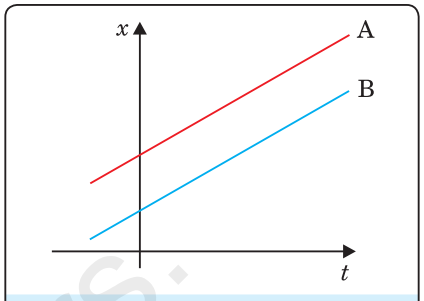

Fig. 8.1. Position-time graph for objects moving with same velocity.

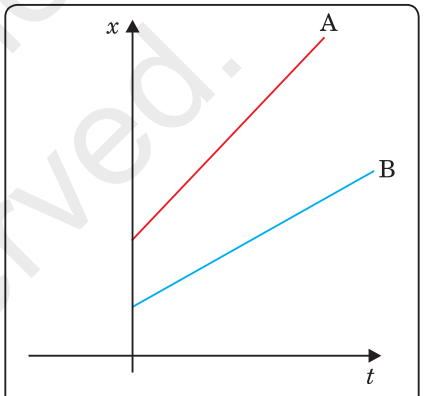

Fig. 8.2. Position-time graph for objects moving with different velocities.

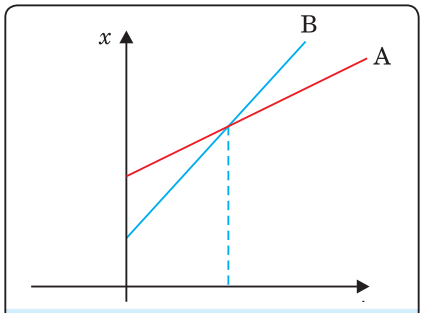

Fig. 8.3. Position-time graph of two objects moving with different velocities meeting at a point.

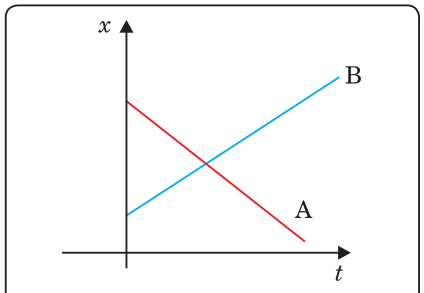

Fig. 8.4. Position-time graph of two objects moving in different directions.

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SOLVED EXAMPLES

Example 1.

On a straight road, two cars X and Y are moving in the same direction. Velocity of car X is 20 m/s and that of car Y is 15 m/s. Find the velocity of X relative to Y and vice versa.

Solution: Given, velocity of X, $v_X = 20$ m/s

Velocity of Y, $v_Y = 15$ m/s

Velocity of X relative to Y,

$v_{XY} = v_X - v_Y = 20 - 15 = \mathbf{5\ m/s}$

Positive relative velocity shows that the driver of car Y sees car X moving ahead of him at the rate of 5 m/s.

Now, velocity of Y relative to X,

$$v_{YX} = v_Y - v_X = 15 - 20 = \mathbf{-5\ m/s}$$

Negative relative velocity shows that the driver of car X sees (when looking back) the car Y moving behind him at the rate of **5 m/s**.

Example 2.

Two parallel rail tracks run north-south. Train A moves north with a speed of 54 km/h and train B moves south with a speed of 90 km/hr. What is the

(a) velocity of B with respect to A?

(b) velocity of ground with respect to B? and

(c) velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 km h^{-1} with respect to the train A) as observed by a man standing on the ground?

(N.C.E.R.T. Solved Example)

Solution: Let south to north be the positive direction of X-axis. Then,

Speed of train A, $v_A = +54$ km/hr $= \dfrac{54 \times 5}{18}$ m/s $= 15$ m/s

Speed of train B, $v_B = -90$ km/hr $= \dfrac{-90 \times 5}{18}$ m/s

$= -25$ m/s

(*a*) Relative velocity of B with respect of A is

$$v_{BA} = v_B - v_A = -25 - 15 = -40 \text{ m/s}$$

which means that train B appears to an observer in train A to move with a speed of **40 m/s** from north to south.

(*b*) Relative velocity of ground with respect to B is

$$v_{OB} = 0 - v_B = 0 + 25 = 25 \text{ m/s}$$

which means that to an observer in train B, the earth (or ground) appears to move with a speed of **25 m/s** from south to earth.

(*c*) Let v_M be the velocity of the monkey with respect to the ground. Then, relative velocity of the monkey with respect to A is

$$v_{MA} = v_M - v_A = -18 \text{ km/hr} = -5 \text{ m/s}$$

$\therefore$ $v_M = v_A - 5 = (15 - 5)$ m/s

$= \mathbf{10\ m/s}$

Example 3.

A train, 100 m in length, is moving with a velocity of 54 km/hr in one direction. Another train, 80 m in length, is moving with a velocity of 36 km/hr in the opposite direction. Find the time when the two trains will completely cross each other.

Solution: Relative velocity of first train w.r.t. to second train = (54 – (–36) km/hr = 90 km/hr

$= \dfrac{90 \times 5}{18}$ m/s = 25 m/s

In order to cross other train, total distance to be travelled by each train = 100 + 80 = 180 m

Time taken by each train to cross other train

$= \dfrac{180}{25} = \mathbf{7.2\ s}$

Example 4.

A boat rescued people drowning in a river during a flood, with maximum of 5 people at a time 7 people were holding a rock in middle of water flow. The boat arrives and picks 5 of them and left 2 people holding the rock. When the boat began to travel upstream, 2 people holding the rock, left hold of it and start flowing with the water flow. The boat travels 2 km upstream, left the 5 people, turned about and caught with the 2 people drowning. How long will it take the boat to reach those 2 people?

Take, speed of boat w.r.t. to still water = 10 m/s

Speed of water flow = 2 m/s

Solution: For upstream motion of boat:

Relative velocity w.r.t. to water flow = 10 – 2 = 8 m/s

Distance travelled = 2 km = 2000 m

Time taken, $t_1 = \dfrac{2000}{8} = 250$ s

For downstream motion of 2 people:

Distance moved by them in 250 $s = 2 \times 250 = 500$ m

Distance between the boat after it turned about and 2 people in the water = 2000 + 500 = 2500 m

This distance is to be covered by the boat with its own velocity (w.r.t. to still water) because the velocity of the water flow is shared by both (boat and the 2 people in water)

$\therefore$ Time taken, $t_2 = \dfrac{2500}{10} = 250$ s

Total time taken to reach them, $t = t_1 + t_2$

$= (250 + 250)$ s

$= 500$ s

$= \dfrac{500}{60} = \mathbf{8.33\ mins}$

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PRACTICE PROBLEMS 5

1. A bus A travelling at a speed of 70 km/h just overtakes another bus B at a speed of 60 km/h. The length of each bus is 10 m. Calculate the time taken by the bus A to overtake bus B.
2. A boy starts running on a straight line path with speed of 20 km/h and a girl is running on the same path with a speed of 15 km/h starting from same point. What is the position of the girl w.r.t. boy in 20 minutes?
3. Two trains A and B each of length 200 m are running on parallel tracks. A overtakes B in 30 s and A crosses B in 15 s. What are the velocities of trains A and B?
4. The speed of a man along with the river is 20 km/h and speed of the man against the river current is 10 km/h. Calculate the speed of the man in still water and speed of the river.
5. Two cars A and B are moving in same direction with a speed of 40 km/h. The distance between cars A and B is constant and is 5 km. Another car C is also moving in the same direction. If at a certain instant car B is overtaking C, then car A takes 20 minutes to overtake C. Find speed of C?
6. A thief's car is moving on a straight road with a speed of 80 km/h. It is followed by a police car moving at a constant speed of 120 km/h, crossing a T-point 20 s later than the thief's car. At what distance from the T-point, the police car will catch the thief's car?

Answers to Practice Problems

1. 7.2 s
2. 1.67 km behind the boy
3. 20 m/s and 6.6 m/s, respectively
4. 15 km/h and 5 km/h, respectively
5. 25 km/h
6. 1333.3 m

Chapter Summary

KEY TERMS & LAWS

- **Motion:** Motion is change in position of an object with the passage of time. The position of object can be specified with reference to some arbitrarily chosen origin.
- **Scalar quantities:** Physical quantities which require only magnitude for their complete description are known as scalar quantities. Examples of scalar quantities are mass, speed, distance, energy, etc.
- **Vector quantities:** Physical quantities which require magnitude as well as direction for their complete description are called vector quantities. Examples of vector quantities are displacement, velocity, acceleration, force, etc.
- **One-dimensional motion:** The type of motion in which only one of the coordinates of particle changes with time is called one-dimensional motion.
- **Path length:** It is the total length of the path traversed by an object. It is also known as distance.
- **Displacement:** It is the change in position of an object and is equal to the shortest distance travelled by the object between the initial and final points.
- **Uniform motion:** A particle is said to be in uniform motion if it covers equal distances in equal time intervals.
- **Non-uniform motion:** An object which covers unequal displacements in equal interval of time is said to be in non-uniform motion.
- **Average velocity:** Average velocity is defined as the total change in position or the total displacement of an object divided by the time interval taken by the object for this change in position or displacement.
- **Average speed:** Average speed in a given time interval is defined as the total distance covered by the body divided by total duration of the time interval in which this distance is covered.
- **Instantaneous velocity or instantaneous speed or simply velocity:** We can define velocity at a particular instant as limiting value of average velocity when time interval Δt becomes infinitesimally small or when time interval Δt approaches zero.

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- **Acceleration:** Rate of change of velocity with respect to time is defined as the acceleration of object. It is a vector quantity.
- **Average acceleration:** Average acceleration of the particle in a given time interval is defined as the total change in velocity divided by the total duration of the time interval.
- **Instantaneous acceleration:** Instantaneous acceleration is defined as the limiting value of average acceleration when time interval is infinitesimally small.
- **Relative velocity:** Relative velocity of an object with respect to another object is defined as the velocity with which one object moves with respect to another.

FACTS

- Rest and motion are relative terms and depend on the choice of frame of reference.
- Distance is a scalar quantity whereas displacement is a vector quantity.
- Distance travelled by an object is greater or equal to the magnitude of displacement between the two points.
- A distance-time graph is always above the time axis whereas a displacement-time graph can be above or below time axis.
- Slope of the chord in x-t graph for a given time interval is equal to the average velocity of the object for that interval.
- Average speed is always positive unlike the average velocity which can be positive or negative.
- For an object moving along the straight line in a given time interval, the average speed can be greater than or equal to the average velocity of the object in the same time interval.
- The velocity at a particular instant is equal to the slope of the tangent drawn on the position-time graph at that instant.
- Slope of the tangent on v-t graph at a certain instant represents instantaneous acceleration of the particle at that instant of time and area under the graph projected on the time axis represents the displacement.
- V- t graph of a particle will be a straight line when it is moving with a constant acceleration.
- Area under the a-t graph projected on the time axis, for an interval of a time is equal to change in velocity of the particle in that interval.
- If a particle is gaining speed, acceleration is in the direction of the velocity and if it is losing speed, the acceleration is in the direction opposite to that of the velocity.
- If at any instant, velocity of a particle is zero, it does not necessarily implies that its acceleration will also be zero.
- When two objects are moving along the same direction then magnitude of relative velocity is equal to the difference in magnitudes of their velocities. And when they are moving along opposite directions then the magnitude of the relative velocity is equal to the sum of the magnitudes of their velocities.

RELATIONS

- Average velocity of a particle for displacement Δx and time interval Δt is $v_{av} = \dfrac{\Delta x}{\Delta t}$
  - If position of an object at an instant of time t_1 is x_1 and at time t_2 position is x_2 then average velocity for this interval can be written as follows:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- Average Speed $= \dfrac{\text{Total path length}}{\text{total time interval}}$
- Instantaneous velocity: $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$

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- Average acceleration:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

- Instantaneous acceleration:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\left(\frac{d\vec{x}}{dt}\right)}{dt} = \frac{d^2\vec{x}}{dt}$$

- Kinematic equations of motion: When the object is at position $x_0 = 0$ at $t = 0$, kinematic equations of motion for constant acceleration are given by:
  - $v = u + at$
  - $x = ut + \frac{1}{2}at^2$
  - $v^2 = u^2 + 2ax$

  If the particle starts at $x = x_0$, then x will be replaced by $x - x_0$ in the above equations.
- Motion under gravity:
  - Time taken to reach the top most point when an object is thrown upwards with an initial velocity u is

$$t = \frac{u}{g}$$

  - Maximum height attained by object when thrown vertically upwards with initial velocity u.

$$H = \frac{u^2}{2g}$$

  - Time taken to return back to the initial point when some object is thrown upwards with initial velocity u is

$$t = \frac{2u}{g}$$

  - Time taken by an object to reach the ground when released from rest from height h

$$t = \sqrt{\frac{2h}{g}}$$

Units and Dimensional Formulae of the Quantities Discussed

S no.	Physical Quantity	SI Unit	Dimensional Formula
1	Path length (Distance)	m	[L]
2	Displacement	m	[L]
3	Velocity (Average and instantaneous)	m s^{-1}	[LT^{-1}]
4	Speed (Average and instantaneous)	m s^{-1}	[LT^{-1}]
5	Acceleration (Average and instantaneous)	m s^{-2}	[LT^{-2}]

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Conceptual Questions

Q.1. How is the vehicle over speeding determined while crossing the toll intercity?

Ans. When a vehicle enters the toll, the time is mentioned on the slip. On reaching the next toll, the difference in time and distance is used to calculate the speed of the vehicle over the entire trip, which gives an idea of over speeding.

Q.2. For a moving object the distance covered by it is always greater than or equal to the displacement of the object in a given time interval. Comment.

Ans. Distance is actual length of path. Displacement is shortest distance between initial and final point, hence distance is always greater or equal to displacement.

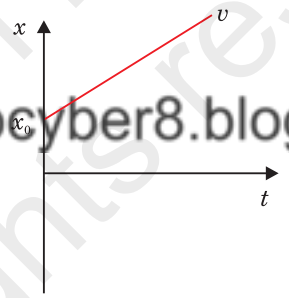
Q.3. A toy cart is designed in such a way that after moving 500 cm forward it reverses 300 cm back at a rate of 100 cm per five seconds. Find the time taken by the cart to cover a distance of 1 m.

Ans. To cover the distance of 1 m, the cart will actually cover $5 \times 500 + 5 \times 300 = 4000$ cm = 40 m

Time taken to cover the distance of 4000 cm will be 200 seconds.

Q.4. Represent the motion of an object in uniform motion along a straight line when both initial distance and velocity of the object is positive.

Ans. here x_0 is the initial distance.



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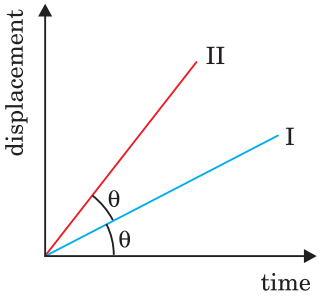
Q.5. From the given displacement-time graph of two cars moving on a straight road, which of the following is moving with greater velocity?



Ans. The slope of displacement-time graph gives the velocity. Here, car II has greater slope than car I and is moving with greater velocity.

Q.6. For a moving object is it possible that magnitude of average speed is less than magnitude of average velocity?

Ans. No. The average velocity is given by total displacement /total time and the average speed is given by total distance/ total time. Since value of distance will be always greater than or equal to the displacement, therefore, the average speed will also be always greater than or equal to the magnitude of average velocity.

Q.7. For a 5 cm long minute hand in the wall clock, find the displacement of the tip of the hand in an interval of 30 minutes. Also find the ratio of distance to displacement.

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Ans. In 30 minutes, the minute hand will reach the diametrically opposite end. Thus the displacement will be 5 + 5 = 10 cm.

The distance covered will be $\pi r = 3.14 \times 5 = 15.7$ cm

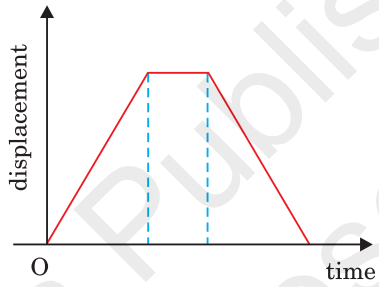
Thus ratio of distance to displacement will be $\frac{10}{15.7} = \frac{100}{157}$

Q.8. The pilot landing or taking off the aero plane is an inertial observer or non-inertial observer?

Ans. While taking off and landing, the plane is under acceleration, thus the observer is in non-inertial frame of reference. Therefore, the pilot is a non-inertial observer here.

Q.9. A man goes to market by walk to purchase some groceries. On the way back home after purchasing he takes an autorickshaw to home. Represent the given journey in form of displacement-time graph.

Ans.



Q.10. From the given position-time graph of two objects comment on their relative velocities.

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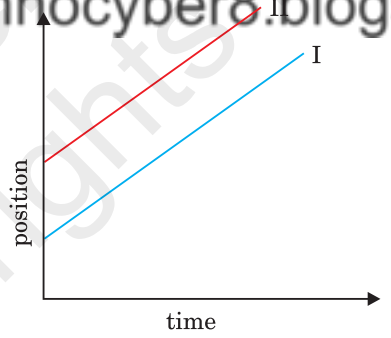

Ans. From the given figure it is clear that at $t = 0$ both the objects are at different positions. But since both the curves are straight and parallel to each other, their relative velocity will be zero.

Q.11. When is a moving object considered as a point object? Give some examples.

Ans. Any moving object is considered as a point object when the size of object is very small in comparison to the distance travelled by it. For example, horizon mission travelling in solar system.

Q.12. What can be concluded from a position-time graph of a moving object with negative slope?

Ans. It means that the velocity of the object is negative.

Q.13. Is it possible for a moving object to have instantaneous velocity equal to the average velocity?

Ans. Yes, it is possible only in motion of an object in one dimension. It is not possible in case when object is moving in two or three dimensions.

Q.14. Give an example when the direction of the acceleration of the moving object is opposite to the direction of the motion.

Ans. In case of an object thrown upward, the object is moving in the upward direction but the acceleration of the object is in downward direction.

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Tough & Tricky PROBLEMS

Problem 1. A stone is thrown up from the top of a tower and it takes time t_1 to reach the ground. A second stone is thrown down with the same speed and it takes time t_2 to reach the ground. How much time a third stone would take to reach the ground, if it is dropped down?

Solution Let H be the height of tower. Let us assume the downward direction as positive direction. Net displacement for all three cases is H (downward), hence it can be written as $s = +H$. Acceleration due to gravity is also in vertically downward direction, so it can be written as $a = +g$.

We can use the following equation for displacement of stones.

$$s = ut + \tfrac{1}{2}at^2$$

For the first stone, initial velocity (let u) is upwards. So, it will be treated as negative. Substituting all values in above equation of motion we get.

$$H = -ut_1 + \frac{1}{2}gt_1^2 \quad \ldots(1)$$

For the second stone, initial velocity is downwards. So, it will be treated as positive. We can write the following equation:

$$H = ut_2 + \frac{1}{2}gt_2^2 \quad \ldots(2)$$

For the third stone, initial velocity is zero. So, we can write the following equation:

$$H = \frac{1}{2}gt^2 \quad \ldots(3)$$

Here t is the time taken by third stone to reach the ground.

Let us multiply equation (1) with t_2 and equation (2) with t_1 and then add both. Then we get,

$$H(t_1 + t_2) = \frac{1}{2}gt_1t_2(t_1 + t_2)$$

$$\Rightarrow \quad H = \frac{1}{2}gt_1t_2 \quad \ldots(4)$$

Comparing equation (3) and (4) we get the following:

$$H = \frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2$$

$$\Rightarrow \quad t = \sqrt{t_1t_2}$$

Problem 2. A particle is moving in a straight line along X-axis and its x-coordinate varies with time t as:

$$x = t^2 - 4t + 6$$

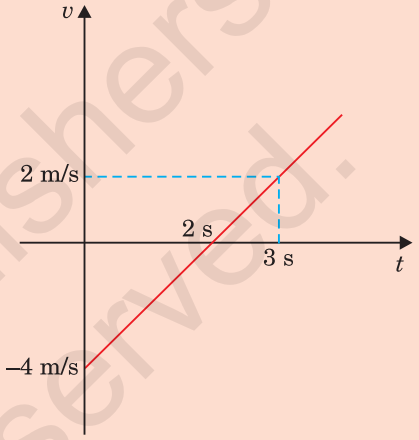
Find the distance and displacement of particle in time interval $t = 0$ to $t = 3$ s.

Solution Velocity of particle can be calculated by differentiating x w.r.t. t.

$$v = \frac{dx}{dt} = \frac{d}{dt}(t^2 - 4t + 6)$$

$$\Rightarrow \quad v = 2t - 4$$

We can see that velocity of particle is zero at $t = 2$ s. A graph between velocity (v) and time (t) is given below:



Area enclosed by the v-t graph below time axis represents displacement along negative x-direction and area above time axis represents displacement along positive x-direction.

Displacement between $t = 0$ to $t = 2$ s is

$$x_1 = -\frac{1}{2} \times 4 \times 2 = -4 \text{ m}$$

Displacement between $t = 2$ s to $t = 3$ s.

$$x_2 = \frac{1}{2} \times 2 \times 1 = 1 \text{ m}$$

Net displacement can be written as follows:

$$x = x_1 + x_2 = -4 + 1 = -3 \text{ m}$$

Hence, displacement is 3 m along negative x-direction.

Distance travelled can be written as follows:

$$x = |x_1| + |x_2| = 4 + 1 = \mathbf{5\ m}$$

Problem 3. A particle is moving along a straight line such that its displacement x and time t are related as follows:

$$\boldsymbol{x^2 = 1 + t^2}$$

Show that acceleration of the particle can be represented as: $a = \frac{1}{x} - \frac{t^2}{x^3}$.

Solution $x^2 = 1 + t^2$

$$\Rightarrow \quad x = (1 + t^2)^{1/2}$$

Velocity of the particle can be written by differentiating displacement.

$$\Rightarrow \quad v = \frac{dx}{dt} = \frac{1}{2}(1 + t^2)^{-1/2} \times 2t$$

$$\Rightarrow \quad v = t(1 + t^2)^{-1/2}$$

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Now we can differentiate velocity to find acceleration as follows:

$$\Rightarrow \quad a = \frac{dv}{dt} = t \times \left(-\frac{1}{2}\right)(1+t^2)^{-3/2} \times 2t + (1+t^2)^{-1/2} \times 1$$

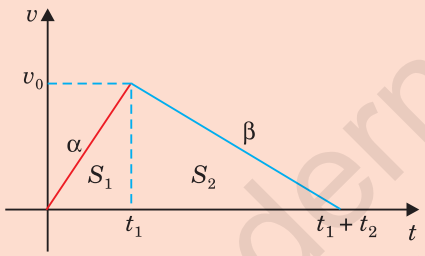
$$\Rightarrow \quad a = \frac{1}{(1+t^2)^{1/2}} - \frac{t^2}{(1+t^2)^{3/2}}$$

$$\Rightarrow \quad a = \frac{\mathbf{1}}{\boldsymbol{x}} - \frac{\boldsymbol{t^2}}{\boldsymbol{x^3}}$$

Problem 4. An elevator accelerates from rest at a constant rate α for time interval t_1 and travels a distance S_1. It then retards at a constant rate β for time interval t_2 and finally comes to rest after travelling a distance S_2 during its retardation. Show that:

$$\frac{S_1}{S_2} = \frac{t_1}{t_2} = \frac{\beta}{\alpha}.$$

Solution Velocity-time graph for the above situation is as follows:



Maximum velocity attained by the elevator is v_0. Slope of the v-t graph represents acceleration of the body; hence we can write the following relation:

$$\alpha = \frac{v_0}{t_1} \text{ and } \beta = \frac{v_0}{t_2}$$

$$\Rightarrow \quad \frac{\alpha}{\beta} = \frac{t_2}{t_1} \qquad \ldots(1)$$

Area under the v-t graph represents displacement; hence we can write the following:

$$S_1 = \frac{1}{2} v_0 t_1 \text{ and } S_2 = \frac{1}{2} v_0 t_2$$

Dividing the above two equations we get

$$\frac{S_1}{S_2} = \frac{t_1}{t_2} \qquad \ldots(2)$$

On combining equations (1) and (2), we get

$$\frac{\boldsymbol{S_1}}{\boldsymbol{S_2}} = \frac{\boldsymbol{t_1}}{\boldsymbol{t_2}} = \frac{\beta}{\alpha}.$$

Problem 5. A driver is driving a car at a speed of 90 km/h. He spots a child standing on his way and decided to apply the brakes but it took him 0.3 s to actually apply the brakes. If the retardation produced by the brakes is 10 m/s^2, calculate the total distance covered by car before coming to rest.

Solution Speed of the car $(u) = 90$ km/h

$$= 90 \times \frac{5}{18} \text{ m/s} = 25 \text{ m/s}$$

The driver took 0.3 s to react. Hence, the car continues to move with constant speed of 25 m/s for 0.3 s. Distance travelled by the car during this interval of time.

$$x_1 = 25 \text{ m/s} \times 0.3 \text{ s} = 7.5 \text{ m}$$

For constant retardation, we can use the following equation:

$$v^2 = u^2 + 2as$$

$$\Rightarrow \quad (0)^2 = (25)^2 + 2(-10)x_2$$

$$\Rightarrow \quad x_2 = \frac{(25)^2}{20} = \frac{625}{20} = 31.25 \text{ m}$$

Total distance travelled by the car before it comes to rest can be written as follows:

$$x = x_1 + x_2 = 7.5 + 31.25 = \mathbf{38.75 \text{ m}}.$$

Problem 6. A truck is moving at a constant speed of 50 km/h on a straight road which terminates on a wall. A fly starts moving with a constant speed of 100 km/h from the wall towards the truck when the truck is at a distance 25 km from the wall. Fly reaches the truck and then turns back towards the wall and then on reaching the wall it again turns towards the truck and so on. It makes several trips between the truck and the wall, before the truck just reaches the wall.

(a) What is the total distance travelled by the fly during this period?

(b) How many trips the fly makes between the truck and the wall?

Solution

(a) Time taken by the truck to reach the wall

$$= \frac{\text{Distance}}{\text{speed}} = \frac{25 \text{ km}}{50 \text{ km/h}} = \frac{1}{2} \text{ h}.$$

Hence with a speed of 100 km/hr, fly will cover 50 km in 1/2 hr.

(b) Let us assume that at a certain instant fly is at the wall and truck is at a distance x_1 from the wall. Fly and truck are moving towards each other with speeds 100 km/h and 50 km/h, respectively. Hence relative speed becomes 100 + 50 = 150 km/h. Time taken by the fly to reach the truck can be written as follows:

$$t_1 = \frac{x_1}{150}$$

Distance travelled by the fly to reach the truck

$$x' = 100 t_1 = 100 \times \frac{x_1}{150} = \frac{2}{3} x_1$$

Distance travelled by truck in this time interval

$$x'' = 50 \times t_1 = 50 \times \frac{x_1}{150} = \frac{x_1}{3}$$

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Further time taken by the fly to reach back the wall

$$t_2 = \frac{x'}{100} = \frac{2x_1}{3\times 100} = \frac{x_1}{150}$$

Further distance travelled by truck in time t_2

$$x''' = 50 \times t_2 = 50 \times \frac{x_1}{150} = \frac{x_1}{3}$$

Distance between truck and the wall after trip of the fly is complete can be written as follows:

$$x = x_1 - x'' - x''' = x_1 - \frac{x_1}{3} - \frac{x_1}{3} = \frac{x_1}{3}$$

$$\Rightarrow \quad x_2 = x_1/3$$

Using the above result we can write the following:

Distance between the truck and the wall at the beginning of 1st trip = 20 km

Distance between the truck and the wall at the beginning of 2nd trip = $\left(\frac{1}{3}\right) \times 20$ km

Distance between the truck and the wall at the beginning of 3rd trip = $\left(\frac{1}{3}\right)^2 \times 20$ km

Similarly

Distance between the truck and the wall at the beginning of n^{th} trip = $\left(\frac{1}{3}\right)^{n-1} \times 20$ km

Distance between the truck and the wall will reduce to zero only when n approaches infinity. Hence, the fly will theoretically make infinite trips between truck and the wall before truck touches the wall.

Problem 7. Height of a tower is 125 m and a particle is dropped from rest from the top of the tower. After two seconds of its fall, another particle is projected downwards with a speed u such that both the particles reach the ground simultaneously. What is the value of u?

Solution Let vertically downward direction be positive. We can use the following equation for the first particle.

$$S = ut + \tfrac{1}{2} at^2$$

$$\Rightarrow \quad 125 = 0 + \tfrac{1}{2}(10)\, t^2$$

$$\Rightarrow \quad t^2 = \frac{125}{5} = 25$$

$$\Rightarrow \quad t = 5 \text{ s.}$$

Second particle is released after 2 s; hence second particle must cover the same distance in 5 – 2 = 3 s.

$$S = ut + \tfrac{1}{2} at^2$$

$$\Rightarrow \quad 125 = u \times 3 + \tfrac{1}{2}(10)\,(3)^2$$

$$\Rightarrow \quad 3u = 125 - 45 = 80$$

$$\Rightarrow \quad u = 80/3 \text{ m/s} = 26.67 \text{ m/s}$$

Problem 8. A particle is moving in a straight line. It covers half of the total distance with velocity v_0. Remaining half distance is covered with a velocity v_1 for half the time and with velocity v_2 for other half of time. Find the average velocity of the particle.

Solution Let us assume that total distance covered by the particle is x. Let t_1 be the time taken to cover first half of the distance and t_2 be the time taken to cover remaining half.

$$t_1 = \frac{x/2}{v_0} = \frac{x}{2v_0}$$

For the next half we can write the following:

$$\frac{x}{2} = v_1 \times \frac{t_2}{2} + v_2 \times \frac{t_2}{2}$$

$$t_2 = \frac{x}{v_1 + v_2}$$

Total time can be written as follows:

$$T = t_1 + t_2$$

$$\Rightarrow \quad T = \frac{x}{2v_0} + \frac{x}{v_1 + v_2}$$

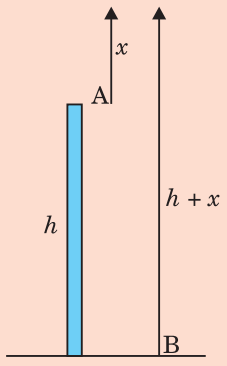
$$\Rightarrow \quad T = \frac{x(v_1 + v_2 + 2v_0)}{2v_0(v_1 + v_2)}$$

Average velocity can be written as follows:

$$v_{av} = \frac{x}{T} = \frac{2v_0(v_1 + v_2)}{(v_1 + v_2 + 2v_0)}$$

Problem 9. There is one tower of height h. One particle A is projected upward from top of the tower with a speed u_1 at time $t = 0$. At time $t = t_0$ another particle B is projected upward from the bottom of the tower with a speed u_2. When will the particles cross each other?

Solution Let us assume upward direction as the positive direction. Let x be the height above the tower where the two particles meet. We can say that displacement of particle A in time t is x and displacement of particle B in time $t - t_0$ is $h + x$ (see figure).



We can use, $S = ut + \tfrac{1}{2} at^2$

For Particle A:

$$x = u_1 t - \frac{1}{2} gt^2$$

$$\Rightarrow \quad \frac{1}{2} gt^2 = u_1 t - x \qquad \ldots(1)$$

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For Particle B:

$$h + x = u_2(t - t_0) - \frac{1}{2}g(t - t_0)^2$$

$$\Rightarrow \quad h + x = u_2(t - t_0) - \frac{1}{2}gt^2 - \frac{1}{2}gt_0^2 + gtt_0$$

Substituting from equation (1), we get

$$\Rightarrow \quad h + x = u_2(t - t_0) - u_1 t + x - \frac{1}{2}gt_0^2 + gtt_0$$

$$\Rightarrow \quad 2h = 2u_2 t - 2u_2 t_0 - 2u_1 t - gt_0^2 + 2gtt_0$$

$$\Rightarrow \quad 2h + 2u_2 t_0 + gt_0^2 = 2t(u_2 - u_1 + gt_0)$$

$$\Rightarrow \quad t = \frac{2h + 2u_2 t_0 + gt_0^2}{2(u_2 - u_1 + gt_0)}$$

Problem 10. Velocity of a particle moving in a straight line is given as: $v = t^2 - t$. Here, v is measured in m/s and time t in seconds. Find the time interval for which the particle undergoes retardation.

Solution Velocity: $v = t^2 - t = t\,(t-1)$ ...(1)

$$\text{Acceleration: } a = \frac{dv}{dt} = 2t - 1 \quad \ldots(2)$$

Motion of particle is considered as retarding motion when v and a have opposite signs that means they are along opposite directions.

From equation (1) we can see that,

Velocity is positive when $t > 1$.

Velocity is negative when $t < 1$.

From equation (2) we can see that,

Acceleration is positive when $t > 1/2$

Acceleration is negative when $t < 1/2$

From above results we can see that when velocity is positive then acceleration cannot be negative.

But there is an interval $\frac{1}{2} < t < 1$, when velocity is negative and acceleration is positive; hence this is the interval of time when particle retards.

NCERT FILE Solved
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NCERT Textbook Exercises

3.1. In which of the following examples of motion can the body be considered approximately a point object:

(a) a railway carriage moving without jerks between two stations.

(b) a monkey sitting on top of a man cycling smoothly on a circular track.

(c) a spinning cricket ball that turns sharply on hitting the ground.

(d) a tumbling beaker that has slipped off the edge of a table.

Ans. In parts (a), (b) the body can be considered approximately as a point object because the distance covered by the body is very much greater than the size of the body.

3.2. The position-time ($x - t$) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in following figure. Choose the correct entries in the brackets below:

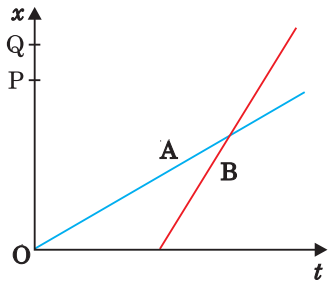
(a) (A/B) lives closer to the school than (B/A).

(b) (A/B) starts from the school earlier than (B/A).

(c) (A/B) walks faster than (B/A).

(d) A and B reach home at the (same/different) time.

(e) (A/B) overtakes (B/A) on the road (once/twice).



Ans. (a) A lives closer to the school than B. (because OP < OQ)

(b) A starts from the school earlier than B. When they start, then $t = 0$ at $x = 0$ for A. But for B, t has a finite value when it starts at $x = 0$.

(c) B walks faster than A. Because, the velocity is given by slope of the $x - t$ graph. The slope is more for B than A.

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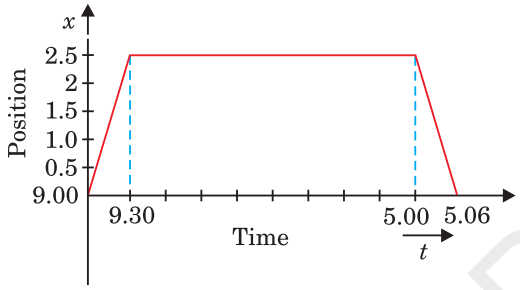
(*d*) A and B reach home at the same time. Because, the value of *t* corresponding to the points P and Q is same in $x - t$ graph.

(*e*) B overtakes A on the road once because B starts from the school after A and overtakes A.

3.3. A woman starts from her home at 9:00 am, walks with a speed of 5 km h^{-1} on a straight road up to her office 2.5 km away, stays at the office up to 5:00 pm, and returns home by an auto with a speed of 25 km h^{-1}. Choose suitable scales and plot the x – *t* graph of her motion.

Ans.



Time taken by woman to reach office = $\frac{\text{distance}}{\text{speed}}$

As distance = 2.5 km, speed = 5 km/h

Therefore, time, $t = 2.5/5 = 0.5$ hr

∴ Woman reach office at 9.30 A.M.

Time taken by woman to return home = $\frac{2.5}{25} = 0.1$ hr

= 6 min

⇒ Woman reaches back to home at 5.06 P.M.

3.4. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the *x* – *t* graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.

Ans. Effective distance covered by the drunkard in 8 steps in 8 s

$= 5 - 3 = 2$ m

The distance travelled by the drunkard in 8 s = 2 metre.

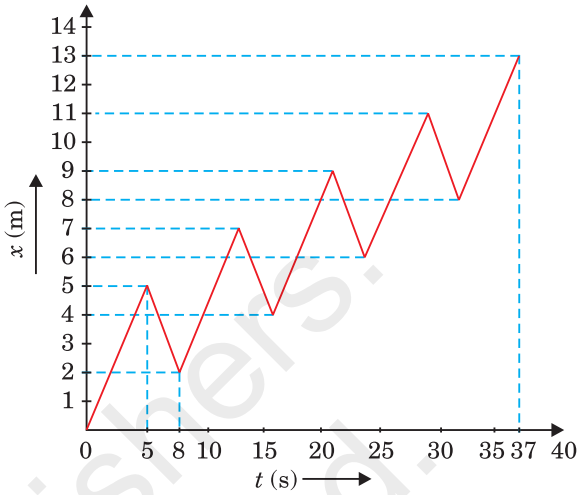
The distance travelled by the drunkard in 32 s is 8 m.

Now after 32 s, if the drunkard will move next 5 steps forward, then he will fall into the pit which is at a distance of 13 m from the start.

Therefore, total time taken by drunkard to fall in the pit

$= 32 + 5 =$ **37 s**

The $(x - t)$ plot is shown in figure.



3.5. A jet airplane travelling at the speed of 500 km h^{-1} ejects its products of combustion at the speed of 1500 km h^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on the ground?

Ans. Let us consider that v_p = velocity of products

v_A = velocity of airplane

$v_p - v_A = -1500$ km h^{-1}

$v_p - 500 = -1500$

$v_p = -1500 + 500$

$= -1000$ km h^{-1} (negative sign indicates that velocity of products is opposite to that of jet)

3.6. A car moving along a straight highway with speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?

Ans. Initial velocity, $u = 126$ km $h^{-1} = \frac{126 \times 5}{18}$ m s^{-1}

$= 35$ m s^{-1}

Final velocity, $v = 0$, $s = 200$ m

Using $v^2 - u^2 = 2as$,

$(0)^2 - (35)^2 = 2 \times a \times 200$

Therefore, $a = \frac{-35 \times 35}{2 \times 200}$

or $a = -\frac{49}{16} =$ **−3.06 m s^{-2}**

As we know, $v = u + at$

$0 = 35 - \frac{49}{16} \times t$

or $t = \frac{35 \times 16}{49} =$ **11.43 s**.

3.7. Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake

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A and accelerates by 1 m s^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them?

Ans.

Velocity of train A = 72 km h^{-1} = $\frac{72 \times 1000}{3600}$

= 20 ms^{-1}

Velocity of train B = 20 m/s, t = 50 s, a = 0

S_A = Distance covered by train A in 50, $S_A = ut + \frac{1}{2}at^2$

$= 20 \times 50 + 0$

= 1000 m

For train B, a = 1 m/s^2

$\therefore$ S_B = Distance covered by train B in 50 s

$$S_B = ut + \frac{1}{2}at^2$$

= 2250 m

$\therefore$ $S_B - S_A = 2250 - 1000 =$ **1250 m**

3.8. On a two-lane road, car A is travelling with a speed of 36 km h^{-1}. Two cars B and C approach car A in opposite direction with a speed of 54 km h^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

Ans. Velocity of car A = 36 km h^{-1} = 10 m s^{-1}

Velocity of car B = 54 km h^{-1} = 15 m s^{-1}

Velocity of car C = 54 km h^{-1} = 15 m s^{-1}

The relative velocity of B *w.r.t.* A = (15 – 10) m s^{-1}

= 5 m s^{-1}

The relative velocity of C *w.r.t.* A = (15 + 10) m s^{-1}

= 25 m s^{-1}

At an instant when both the cars B and C are at same distance of 1 km from car A

then time taken by C to cover a distance of 1000 m = 1000/25 = 40 s

Hence the car B should travel the same distance in a time less than 40 s. Therefore, the minimum acceleration produced by car B can be found by using

$$s = ut + \frac{1}{2}at^2$$

$$1000 = 5 \times 40 + \frac{1}{2} \times a \times 40 \times 40$$

$$800 = \frac{1}{2}a \times 40 \times 40$$

$$a = \frac{800 \times 2}{40 \times 40} = \textbf{1 m s}^{-2}.$$

3.9. Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of 20 km h^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?

Ans. Let V_b = speed of bus

V_c = speed of cyclist.

The relative velocity of bus in the direction of motion of cyclist = $V_b - V_c = V_b - 20$

The relative velocity of bus in opposite direction of motion of cyclist = $V_b + V_c$

$= V_b + 20$

As we know $\frac{\text{distance}}{\text{velocity}}$ = time

$\therefore$ The distance travelled by bus in time $T = V_bT$

$$\frac{V_bT}{V_b - 20} = 18$$

$$\frac{V_bT}{V_b + 20} = 6$$

Solving we get V_b = 40 km h^{-1}

$T =$ **9 min**

3.10. A player throws a ball upwards with an initial speed of 29.4 ms^{-1}.

(*a*) What is the direction of acceleration during the upward motion of the ball?

(*b*) What are the velocity and acceleration of the ball at the highest point of its motion?

(*c*) Choose the x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of X-axis, and give the signs of position, velocity and acceleration of the ball during its upward and downward motion.

(*d*) To what height does the ball rise and after how long does the ball return to the player's hands? (Take g = 9.8 m s^{-2} and neglect air resistance).

Ans. (*a*) As the ball is under the effect of gravity, the direction of acceleration due to gravity during the upward motion of ball is vertically downward.

(*b*) At the highest point the velocity of ball = 0 and acceleration will be equal to acceleration due to gravity and is 9.8 m s^{-2} vertically downward.

(*c*) As the downward direction is taken to be positive direction of X-axis, then during upward motion, sign of position and velocity are negative while the sign of acceleration is positive.

For downward motion of the ball, sign of position, velocity and acceleration is positive.

(*d*) Let t be the time taken by the ball to reach the highest point of motion = time of ascent

For upward motion, $u = -29.4$ m/s, $v' = 0$

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As $v'^2 - u^2 = 2gh$, where h is the maximum height attained by the ball.

$(0)^2 - (-29.4)^2 = 2\,(9.8)\,(-h)$

or $$h = \frac{29.4 \times 29.4}{2 \times 9.8} = \mathbf{44.1\ m}$$

$v' = v + at$

$0 = -29.4 + (9.8)\,t$

or $$t = \frac{29.4}{9.8} = 3\ \text{s}$$

Time of ascent = Time of descent = 3 s

Total time = (3 + 3) = **6 s**.

3.11. Read each statement below carefully and state with reasons and examples if it is true or false:

A particle in one-dimensional motion

(*a*) with zero speed at an instant may have non-zero acceleration at that instant,

(*b*) with zero speed may have non-zero velocity,

(*c*) with constant speed must have zero acceleration,

(*d*) with positive value of acceleration must be speeding up.

Ans. (*a*) True.

When a body is projected upwards then at the highest point, velocity = 0, acceleration is equal to acceleration due to gravity and is 9.8 m s^{-2} vertically downwards.

(*b*) False. As velocity of the body is the speed in a particular direction. If speed is zero then velocity is also zero.

(*c*) True. When a body is moving in a straight line with constant speed then velocity of the body is constant and for constant velocity, acceleration is zero.

(*d*) False. When velocity is negative and acceleration is positive, then body slows down.

3.12. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Plot the speed-time graph of its motion between $t = 0$ to 12 s.

Ans. As ball is dropped downwards.

Therefore, $u = 0$, $a = 10\ \text{ms}^{-2}$, $s = 90$ m

$v = ?$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2}at^2 \qquad [\text{Here } a = g = 10\ \text{m/s}^2]$$

$$\Rightarrow \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 90}{10}} = 4.24\ \text{s}$$

$v^2 - u^2 = 2\,as$

$v^2 - 0 = 2\,as$

$$v = \sqrt{2\,as}$$

$$= \sqrt{2 \times 10 \times 90} = 30\sqrt{2}$$

Rebound velocity = $v' = \frac{9}{10}v$

$$= \frac{9}{10} \times 30\sqrt{2} = 27\sqrt{2}\ \text{m/s}$$

t' = time taken to reach highest point = $\frac{v'}{10} = \frac{27\sqrt{2}}{10}$

$= 2.7\sqrt{2} = 3.81$ s

Total time taken by ball = $t + t' = 4.24 + 3.81 = 8.05$ s

As time of ascent = time of descent.

$\therefore$ The ball will take 3.81 s to come back to floor

Velocity of the ball before striking the floor

$= 27\sqrt{2}$ m/s

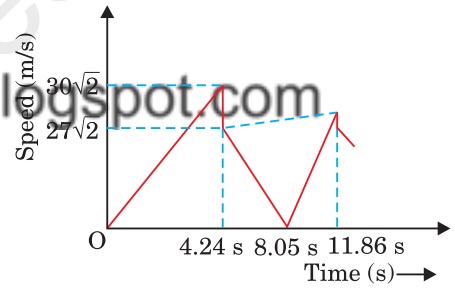
Velocity of the ball after striking the floor

$$= 27\sqrt{2} \times \frac{9}{10}$$

$$= 24.3\sqrt{2}\ \text{m/s}$$

$\therefore$ Total time elapsed for 2nd rebound = 8.05 + 3.81

= 11.86 s



3.13. Explain clearly, with examples, the distinction between:

(*a*) magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval;

(*b*) magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval]. Show in both (*a*) and (*b*) that the second quantity is either greater than or equal to the first. When is the equality sign true? [For simplicity consider one-dimensional motion]

Ans. (*a*) Displacement is a vector quantity. When a body changes its position with respect to time, then a straight line joining from initial to final position of a body is called the *displacement*, independent of the nature of the actual path taken by the body.

Distance is the total path length covered by a body in a certain interval of time.

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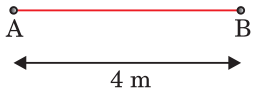
Examples

(*i*) Consider a particle is moving from A to B in 2 s and then from B to A in 2 s.



Here distance covered = AB + BA = 4 + 4 = 8 m

But displacement will be zero as the initial and final position of the particle is same.

In this case, distance is greater than the displacement.

Distance will be equal to displacement when the body is moving in a straight line in same direction.

(*ii*) Average speed = $\frac{\text{distance covered}}{\text{time taken}}$

$= \frac{AB + BA}{t}$

$= \frac{4+4}{2+2} = 2$ m/s

Average velocity = $\frac{\text{displacement}}{\text{time}}$

$= 0$

Hence in this case, average speed is greater than the average velocity.

Average speed is equal to average velocity when the body is moving in a straight line in same direction only.

3.14. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1}. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1}. What is the

(*a*) magnitude of average velocity, and

(*b*) average speed of the man over the interval of time

(*i*) 0 to 30 min, (*ii*) 0 to 50 min, (*iii*) 0 to 40 min? [Note: You will appreciate from this exercise why it is better to define average speed as total path length divided by time, and not as magnitude of average velocity. You would not like to tell the tired man on his return home that his average speed was zero !]

Ans. Time taken by a man to go from home to market = 2.5/5 = 0.5 hr = 30 min.

Time taken by a man to go from market to home = 2.5/7.5 = 1/3 hr = 20 min

Total time = 30 + 20 = 50 min.

Case (*i*) From 0 to 30 min

distance covered = 2.5 km

Net displacement = 2.5 km

(*a*) Magnitude of average velocity

= net displacement/total time taken

$= \frac{2.5 \text{ km}}{1/2 \text{ h}} = 5 \text{ km h}^{-1}$

(*b*) Average speed

$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{2.5 \text{ km}}{1/2 \text{ h}} = 5 \text{ km h}^{-1}$

(*ii*) From 0 to 50 minutes

Distance travelled in 30 min $\left(\frac{1}{2} \text{ h}\right)$ while going to the market = 2.5 km

Distance travelled in 20 min (1/3 h) while coming back to home

$= 7.5 \times \frac{1}{3} = 2.5$ km

Total distance travelled in 50 min

= (2.5 + 2.5) = 5.0 km

(*b*) Average speed

$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{5 \times 60}{50} = 6 \text{ km h}^{-1}$

(*a*) Magnitude of average velocity

= displacement/total time taken

$= \frac{(2.5 - 2.5)}{50/60} = 0$

(*iii*) From 0 to 40 min

Distance travelled in 30 min. while going to market = 2.5 km

Distance travelled in 10 min. while coming back from market

$= 7.5 \times \frac{1}{6} = \frac{5}{4}$ km = 1.25 km

Total distance = 2.5 + 1.25 = 3.75 km

Displacement = 2.5 – 1.25 = 1.25

(*a*) Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{3.75}{40/60} = 5.625 \text{ km h}^{-1}$

Magnitude of average velocity

$= \frac{\text{Net displacement}}{\text{Total time taken}}$

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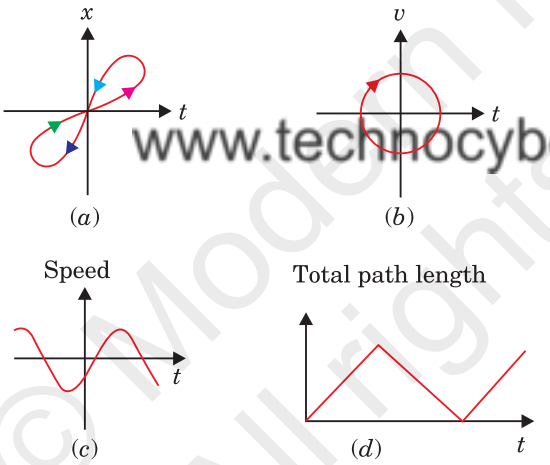
$$= \frac{1.25}{40/60} \text{ km h}^{-1}$$

$$= \mathbf{1.875 \text{ km h}^{-1}}$$

3.15. In Exercises 3.13 and 3.14, we have carefully distinguished between *average* speed and magnitude of *average* velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?

Ans. For instantaneous speed we consider only a small interval of time. During this short interval of time, direction of motion is not supposed to change. Therefore, there is no difference between total distance covered and magnitude of displacement. Hence, instantaneous speed and instantaneous velocity are same.

3.16. Look at the graphs (*a*) to (*d*) carefully and state, with reasons, which of these *cannot* possibly represent one-dimensional motion of a particle.



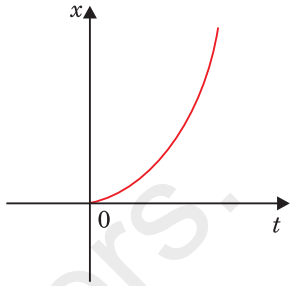
Ans. (*a*) This graph does not represent one-dimensional motion because, a particle cannot have two different positions at the same time.

(*b*) This graph does not represent one-dimensional motion because a particle cannot have velocity in positive and negative direction at the same instant.

(*c*) This graph also does not represent one-dimensional motion, because the speed is always positive in one-dimensional motion.

(*d*) This graph also does not represent one-dimensional motion, because the total path length covered by a particle can never decrease with time.

3.17. Figure shows $x - t$ plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.



Ans. No, it is not correct to say that a particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$. Because the $x - t$ graph cannot show the trajectory of the particle. In this graph when $t = 0$, $x = 0$, the situation can be correlated to the motion of a body dropped freely from a height.

3.18. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car? (Note: Obtain that speed which is relevant for damaging the thief's car).

Ans. Speed of the police van, $v_p = 30$ km h^{-1}

Speed of the thief's car, $v_t = 192$ km h^{-1}

As the bullet is fired from the police van, it will share the velocity of the van.

Muzzle speed of the bullet = 150 m/s = 540 km/h

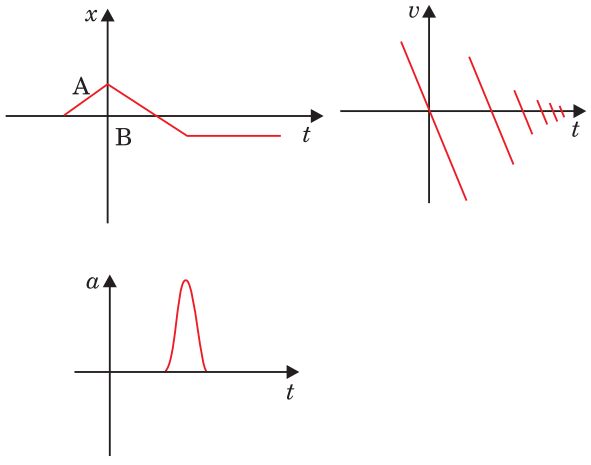
Total speed of the bullet, $v_b = 540 + 30 = 570$ km h^{-1}

Relative velocity of bullet w.r.t. thief car = $v_b - v_t$

$= 570 - 192$

$= 378$ km hr^{-1} = **105 ms^{-1}**

3.19. Suggest a suitable physical situation for each of the following graphs:



Ans. (*a*) The graph shows that initially a body was at rest. Then suddenly acquires constant velocity

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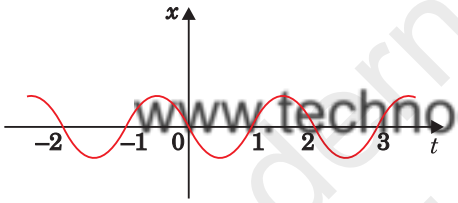
in positive direction. After some time it rebounds and move in negative direction and finally comes to rest.

A similar situation occurs when a ball (at rest) is kicked, which rebounds on striking a wall with a reduced speed, then moves to the opposite wall and gets stopped ultimately.

(*b*) The similar situation occurs when a ball is thrown up with some initial velocity. On striking the floor, it rebounds with reduced speed after each collision with floor.

(*c*) In this graph, velocity of a body is initially constant. Then acceleration increases for a short duration, then falls to zero and after that the body again moves with a constant velocity. The similar situation occurs when a cricket ball is hit by a bat for a very short duration of time.

3.20. Figure gives the *(x – t)* plot of a particle executing one-dimensional simple harmonic motion. (You will learn about this motion in more detail in Chapter 14). Give the signs of position, velocity and acceleration variables of the particle at *t* = 0.3 s, 1.2 s, – 1.2 s.



Ans. (*i*) At $t = 0.3$ s, x is negative,

Velocity is negative.

Acceleration is positive.

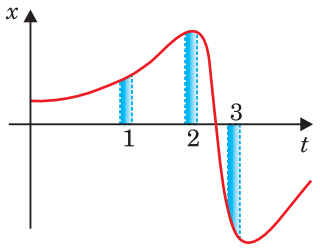
(*ii*) At $t = 1.2$ s, x is positive,

Velocity is positive. But acceleration will be negative.

(*iii*) At time $t = -1.2$ s, x is negative,

Velocity is positive and acceleration is also positive.

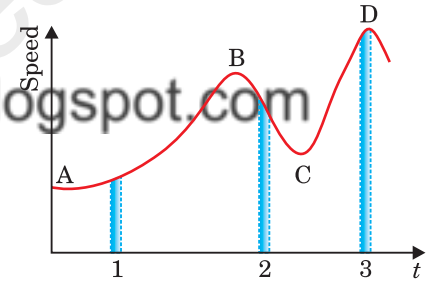
3.21. Figure gives the *x – t* plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.



Ans. As Average speed in a given interval of time is equal to the slope of $x - t$ graph. The slope is greatest in interval 3, therefore, average speed is greatest in interval 3 and least in interval 2, because slope of $x - t$ graph is least there.

Average velocity is positive in interval 1 and 2 because slope of $x - t$ graph is positive there and negative in interval 3 because of negative slope there.

3.22. Figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude? In which interval is the average speed greatest? Choosing the positive direction as the constant direction of motion, give the signs of *v* and *a* in the three intervals. What are the accelerations at the points A, B, C and D?



Ans. As average acceleration is equal to slope of $v - t$ graph. As slope is greatest in interval 2. Therefore average acceleration is greatest in interval 2.

Average speed is greatest in interval 3 due to highest peak at D.

In interval (1), v is positive, slope of $v - t$ graph is positive, hence acceleration is positive.

In interval (2), v is positive, but slope of $v - t$ graph is negative, hence acceleration is negative.

In interval (3), u is positive, but $v - t$ graph is parallel to time axis, therefore, acceleration is zero.

Slope of graph is zero at A, B, C and D, hence acceleration is zero at these points.

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NCERT Additional Exercises

3.23. A three-wheeler starts from rest, accelerates uniformly with 1 m s^{-2} on a straight road for 10 s and then moves with uniform velocity. Plot the distance covered by the vehicle during the n^{th} second (n = 1, 2, 3, ...) versus n. What do you expect this plot to be during accelerated motion: a straight line or a parabola?

Ans. As $u = 0$, $a = 1$ ms^{-2},

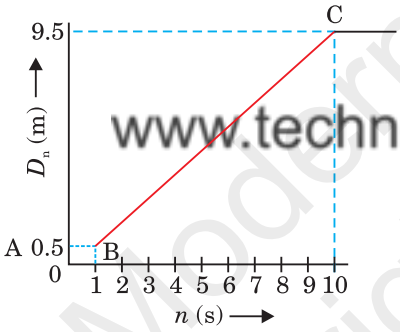
D_n = distance covered in n^{th} sec = $u + \frac{a}{2}(2n-1)$

$= 0 + \frac{1}{2}(2n-1)$

by putting n = 1, 2, 3, ...we can get different values of D_n like

n	D_n
1	0.5
2	1.5
3	2.5
4	3.5
5	4.5
6	5.5
7	6.5
8	7.5
9	8.5
10	9.5

As $D_n \propto n$, therefore graph comes to be a straight line BC upto t = 10 sec. After 10 sec., the graph is a straight line CD parallel to time axis as shown.



3.24. A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 m s^{-1}. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m s^{-1} and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands?

Ans. When the lift is stationary:

Consider the motion of ball going upwards and then coming back to the hands of boy, i.e. displacement is zero.

$\therefore \quad S = 0, u = 49\ ms^{-1}, g = -9.8\ ms^{-2}$

$S = ut + \frac{1}{2}gt^2$

$0 = 49 + \frac{1}{2}(-9.8)t^2$

$\Rightarrow \quad t = 10$ s

When lift starts moving, there is no change in relative velocity of ball w.r.t. boy and hence, time taken by the ball to return to boy's hand will be 10 sec.

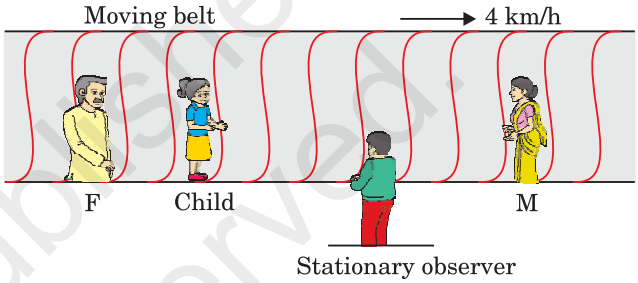
3.25. On a long horizontally moving belt, a child runs to and fro with a speed 9 km h^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km h^{-1}. For an observer on a stationary platform outside, what is the

(*a*) speed of the child running in the direction of motion of the belt?

(*b*) speed of the child running opposite to the direction of motion of the belt?

(*c*) time taken by the child in (*a*) and (*b*)?

Which of the answers alter if motion is viewed by one of the parents?



Ans. Speed of child w.r.t. belt = 9 km h^{-1}.

Consider the direction from left to right is taken to be positive direction.

(*a*) $v_C = +9$ km h^{-1}, $v_B = +4$ km h^{-1}

$\therefore$ Speed of child w.r.t. stationary observer

$= 9 + 4 = 13$ km h^{-1}

(*b*) $v_C = -9$ km h^{-1}, $v_B = +4$ km h^{-1}

$\therefore$ Speed of child w.r.t. stationary observer.

$= -9 + 4 = -5$ km h^{-1}

(*c*) Distance between parents = 50 m

As parents and child are located on the same belt, therefore, speed of child with respect to either parent will be same in either direction and will be 9 km h^{-1}.

$\therefore$ Time taken by the child in case (*a*) and case (*b*) will be

$t = \frac{50 \times 3600}{9 \times 1000}$

$= 20$ s.

If the motion is viewed by any one of the parents, answer to (*c*) is unaltered because the parents and the child are located at the same belt while the answers to (*a*) and (*b*) will change because the relative velocity of the child is 9 km h^{-1} as observed by one of parents.

3.26. Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 m s^{-1} and 30 m s^{-1}. Verify that the graph shown in the figure correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take g = 10 m s^{-2}. Give the equations for the linear

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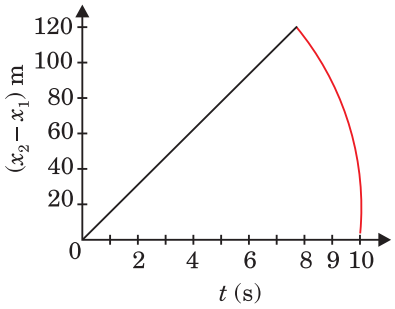
and curved parts of the plot.



Ans. Consider the upward motion of 1st stone:

$x_0 = 200$ m, $u = 15$ ms^{-1}, $g = -10$ ms^{-2}, $x = x$,

$$x_1 = x_0 + ut + \frac{1}{2}gt^2$$

$$x_1 = 200 + 15\,t - 5\,t^2 \qquad ...(i)$$

When first stone hits the ground then

$x_1 = 0$, we get $200 + 15\,t - 5t^2 = 0$

$t^2 - 3t - 40 = 0$

solving we get $t = -5$ s or $t = 8$ s

As $t = -5$ s is meaningless.

Therefore $t = 8$ s

Similarly when 2nd stone hits the ground then

$x_2 = 0$

$x_0 = 200$ m, $u = 30$ ms^{-1}, $g = -10$ ms^{-2}, $x = x_2$

We get $x_2 = x_0 + ut + \frac{1}{2}gt^2$

$$0 = 200 + 30\,t - 5t^2 \qquad ...(ii)$$

$200 + 30\,t - 5t^2 = 0$

or $t^2 - 6t - 40 = 0$

Solving we get $t = -4$ s, or $t = 10$ s.

As $t = -4$ s is meaningless

therefore, $t = 10$ s

From (i) and (ii) we get relative position of second stone w.r.t. first stone.

$x_2 - x_1 = 15\,t$.

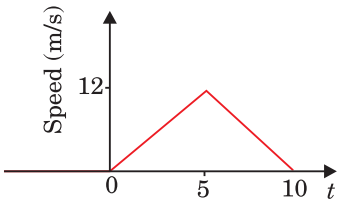
The first stone will be in motion for 8 seconds.

As $x_2 - x_1 \propto t$

Therefore, graph will be a straight line for $t = 8$ sec. and the maximum separation between two stones $= 15\,t = 15 \times 8 = 120$ m.

After 8 sec., only 2nd stone will be in motion and the graph is in accordance with the equation $x_2 = 200 + 30\,t - 15t^2$ for an interval between 8 s. to 10 s.

3.27. The speed-time graph of a particle moving along a fixed direction is shown in figure. Obtain the distance traversed by the particle between (*a*) $t = 0$ s to 10 s (*b*) $t = 2$ s to 6 s.



What is the average speed of the particle over the intervals in (*a*) and (*b*)?

Ans. (a) Distance covered by the particle in 10 s

= Area enclosed by the graph

$$= \frac{1}{2} \text{ Base} \times \text{Height}$$

$$= \frac{1}{2} \times 10 \times 12 = 60 \text{ m}$$

Average speed between $t = 0$ and $t = 10$ s

$$= \frac{\text{Total path length}}{\text{Time}} = \frac{60}{10} = 6 \text{ m/s}$$

(b) So, distance travelled from $t = 2$ s to $t = 6$ s = distance travelled from $t = 2$ s to $t = 5$ s (S_1)+ distance travelled from $t = 5$ s to $t = 6$ s(S_2)

Let a_1 = acceleration of particle from $t = 0$ to $t = 5$ sec.

From $t = 0$ to $t = 5$ s, final velocity, $v = 12$ ms^{-1}

$u = 0$

$\therefore \quad v = u + a'_1 t$

$$a_1 = \frac{v - u}{t} = \frac{12 - 0}{5} = 2.4 \text{ ms}^{-2}$$

Thus velocity after $t = 2$ s

$v = u + at$

$= 0 + 2.4 \times 2 = 4.8$ ms^{-1}

To find distance S_1 travelled between $t = 2$ s and $t = 5$ s we have

$\Delta t = 3$ s, $a_1 = 2.4$ ms^{-2}, $v = 4.8$ ms^{-1}

$$S_1 = v\Delta t + \frac{1}{2}a\Delta t^2$$

$$= 4.8 \times 3 + \frac{1}{2}(2.4) \times (3)^2 = 25.2 \text{ m}$$

To find S_2: (between $t = 5$ s to $t = 6$ s),

$$a_2 = \frac{0 - 12}{10 - 5} = -2.4 \text{ ms}^{-2}$$

For $t = 5$ s to $t = 6$ s we have

$\Delta t = 1$ s, $a_2 = -2.4$ ms^{-2}, $v = 12$ ms^{-1}

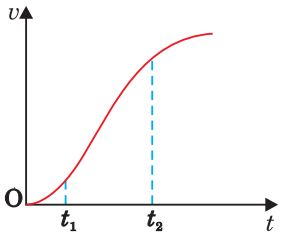
$$\therefore \quad S_2 = vt + \frac{1}{2}a\Delta t^2 \Rightarrow 12 \times 1 + \frac{1}{2}(-2.4) \times 1^2$$

$S_2 = 10.8$ m

Therefore, total distance travelled between $t = 2$ s and $t = 6$ s is $S_1 + S_2 = 25.2 + 10.8 = 36$ m

Thus Average speed $= \frac{36}{6 - 2} = 9$ ms^{-1}

3.28. The velocity-time graph of a particle in one-dimensional motion is shown in the figure.



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Which of the following formulae are correct for describing the motion of the particle over the time-interval t_1 to t_2:

(a) $x(t_2) = x(t_1) + v\,(t_1)\,(t_2 - t_1) + (½)\,a\,(t_2 - t_1)^2$

(b) $v(t_2) = v(t_1) + a\,(t_2 - t_1)$

(c) $v_{average} = (x(t_2) - x(t_1))/t_2 - t_1)$

(d) $a_{average} = (v(t_2) - v(t_1) + t_2 - t_1)$

(e) $x(t_2) = x(t_1) + v_{average}\,(t_2 - t_1) + (½)\,a_{average}\,(t_2 - t_1)^2$

(f) $x(t_2) - x(t_1)$ = area under the $v - t$ curve bounded by the t-axis and the dotted line shown.

Ans. As the motion of the particle over the interval t_1 and t_2 is non-uniform. Therefore, the relations (*c*), (*d*), (*f*) are correct. But (*a*), (*b*), (*e*) represent uniform acceleration.

NCERT Exemplar Problems

Subjective Questions

Note: *Objective Questions (3.1 to 3.11) from Exemplar Problems are given in Competition File page 74.*

Very Short Answer Type Questions

3.12. Refer to the graphs in figure. Match the following:

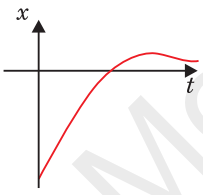
Graph characteristic

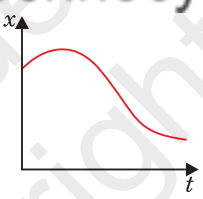
(a) (i) has $v > 0$ and $a < 0$ throughout.

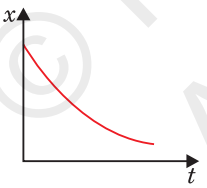
(b) (ii) has $x > 0$ throughout and has a point with $v = 0$ and a point with $a = 0$.

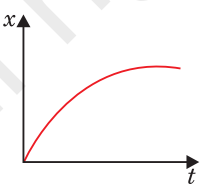
(c) (iii) has a point with zero displacement for $t > 0$.

(d) (iv) has $v < 0$ and $a > 0$.

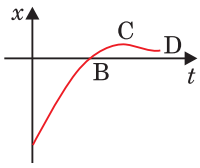
(*a*) 

(*b*) 

(*c*) 

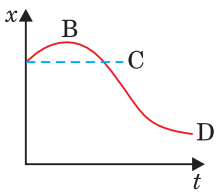
(*d*) 

Ans. For graph (*a*),



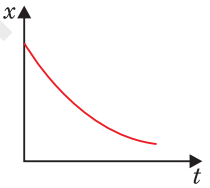
At B, $x = 0$ for $t > 0$, so $a \to iii$.

For graph (*b*),



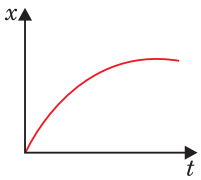
x is positive ($x > 0$) throughout and at B, $v = \frac{dx}{dt} = 0$ and at C, the curvature of the curve changes, i.e. $a = 0$, so $b \to ii$.

For graph (*c*),



Slope $v = \frac{dx}{dt}$ is negative ($v < 0$), the curvature in $x - t$ graph indicates that negative velocity is decreasing with time, i.e. $a > 0$, so $c \to iv$.

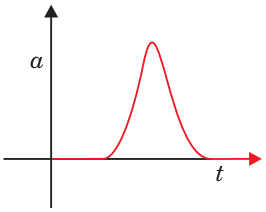
For graph (*d*),



slope $\frac{dx}{dt} = v$ is positive ($v > 0$), and $x - t$ graph says that positive velocity is decreasing with time, i.e. $a < 0$, so $d \to (i)$.

3.13. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time interval. Show the variation of its acceleration with time. (Take acceleration in the backward direction as positive).

Ans. Acceleration of the ball is zero just before it strikes the bat and when it strikes the bat, it gets accelerated due to applied impulsive force of the bat, and $a - t$ graph is as shown in figure.



3.14. Give examples of a one-dimensional motion where

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(a) the particle moving along positive x-direction comes to rest periodically and moves forward.

(b) the particle moving along positive x-direction comes to rest periodically and moves backward.

Ans. The motion must be linked with periodic motion represented by sine or cosine function.

(a) Let $x(t) = t - \sin t$

$\therefore \quad v = \frac{dx}{dt} = 1 - \cos t$

and $\quad a = \frac{dv}{dt} = \sin t$

when $\quad t = 0, x = 0$

when $\quad t = \pi, x = \pi - \sin \pi = \pi > 0$

(b) Let $x(t) = \sin t$

$$v = \frac{dx}{dt} = \cos t$$

As x and v are involving $\sin t$ and $\cos t$, hence, these equations represent periodic motion.

3.15. Give example of a motion where $x > 0, v < 0, a > 0$ at a particular instant.

Ans. Let the motion be represented by equation

$x(t) = a + be^{-kt}$, $a > b > 0$ and $k > 0$

So $\quad x(t) > 0$

$$v = \frac{dx}{dt} = -bke^{-kt}, \text{ i.e. } v < 0$$

and $\quad a = \frac{dv}{dt} = bk^2e^{-kt}$, i.e. $a > 0$

3.16. An object falling through a fluid is observed to have acceleration given by $a = g - bv$ where g = gravitational acceleration and b is constant. After a long time of release, it is observed to fall with constant speed. What must be the value of constant speed?

Ans. When an object is falling with constant speed, v is constant, then

$$a = \frac{dv}{dt} = 0$$

or $\quad g - bv = 0$

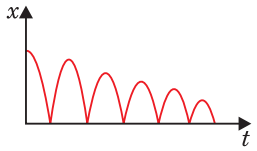
or $\quad v = \frac{g}{b}$

Short Answer Type Questions

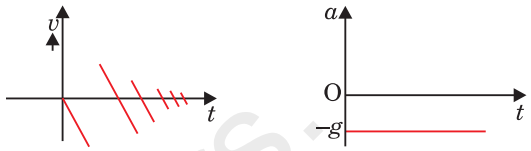
3.17. A ball is dropped and its displacement *vs* time graph is as shown in figure (displacement x is from ground and all quantities are positive upwards).

(a) Plot qualitatively velocity vs time graph.

(b) Plot qualitatively acceleration vs time graph.



Ans. When a ball is dropped, displacement x is always positive. Velocity v is positive in downward direction and v is negative in upward direction and acceleration is always $-g$. So, the required graphs are shown in figure.



(a) (b)

3.18. A particle executes the motion described by $x(t) = x_0(1 - e^{-\gamma t})$; $t \geq 0, x_0 > 0$.

(a) Where does the particle start and with what velocity?

(b) Find maximum and minimum values of $x(t)$, $v(t)$, $a(t)$. Show that $x(t)$ and $a(t)$ increase with time and $v(t)$ decreases with time.

Ans. Give $\quad x(t) = x_0(1 - e^{-\gamma t})$

$$v = \frac{dx}{dt} = x_0\gamma e^{-\gamma t}$$

$$a = \frac{dv}{dt} = -x_0\gamma^2 e^{-\gamma t}$$

(a) At $t = 0, x = x_0(1 - e^{-0}) = x_0(1 - 1) = 0$

and $\quad v = x_0\gamma e^{-0} = x_0\gamma$

(b) $x(t)$ is maximum, when $t = \infty$, so $x(t) = x_0$

$x(t)$ is minimum, when $t = 0$, so $x(t) = 0$

$v(t)$ is maximum, when $t = 0$, so $v(0) = \gamma x_0$

$v(t)$ is minimum, when $t = \infty$, so $v(t) = 0$

$a(t)$ is minimum, when $t = \infty$, so $a(t) = 0$

$a(t)$ is maximum, when $t = 0$, so $a(0) = -x_0\gamma^2$

3.19. A bird is tossing (flying to and fro) between two cars moving towards each other on a straight road. One car has a speed of 18 km/h while the other has the speed of 27 km/h. The bird starts moving from first car towards the other and is moving with the speed of 36 km/h and when the two cars were separated by 36 km. What is the total distance covered by the bird? What is the total displacement of the bird?

Ans. Relative speed of cars = 18 + 27 = 45 km h^{-1}

Time required to meet, $t = \frac{36}{45} = 0.8$ h

So, distance covered by the bird, $S = vt$

$= 36 \times 0.8 = 28.8$ km

3.20. A man runs across the roof-top of a tall building and jumps horizontally with the hope of landing on the roof of the next building which is of a lower height than the first. If his speed is 9 m/s, the (horizontal) distance between the two buildings is 10 m and the height difference is 9 m, will he be able to land on the next building? (take $g = 10$ m/s^2)

Ans. When the man jumps horizontally with $u_x = 9$ m s^{-1}, its motion along vertical is given by

$$y = u_y t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2} \times 10 \times t^2$$

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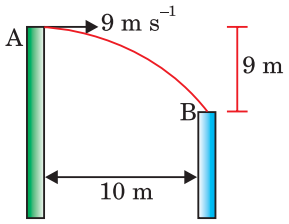
or $9 = 5t^2$

or $t = \frac{3}{\sqrt{5}}$



Horizontal distance travelled,

$$x = u_x t = 9 \times \frac{3}{\sqrt{5}} = \frac{27}{\sqrt{5}} \text{ m} \simeq 12 \text{ m}$$

So, the horizontal distance covered by the man while jumping is greater than the horizontal distance between the two buildings. So, the man will land on the next building.

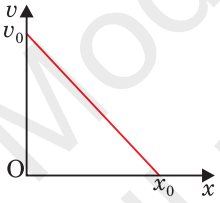
3.21. A ball is dropped from a building of height 45 m. Simultaneously another ball is thrown up with a speed 40 m/s. Calculate the relative speed of the balls as a function of time.

Ans. Initial relative velocity is 40 m/s. Relative acceleration is zero as both are under gravity. Hence relative velocity remains constant (40 m/s).

3.22. The velocity-displacement graph of a particle is shown in figure.

(*a*) Write the relation between v and x.

(*b*) Obtain the relation between acceleration and displacement and plot it.

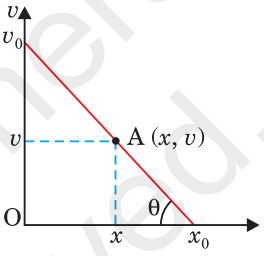


Ans. Here v_0 is initial velocity, if x and v be distance and velocity at any time t at A then from figure, slope is given by

$$\tan\theta = -\frac{v_0}{x_0} = \frac{v_0 - v}{x - 0}$$

or $$v_0 - v = \frac{v_0}{x_0} x$$

or $$v = v_0 - \frac{v_0}{x_0} x \quad \ldots(i)$$

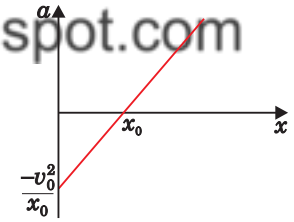


And acceleration, $a = \frac{dv}{dt} = -\frac{v_0}{x_0}\frac{dx}{dt} = -\frac{v_0}{x_0}(v)$

$$= -\frac{v_0}{x_0}\left(v_0 - \frac{v_0}{x_0} x\right)$$

(using equation (i))

Graph between a and x is shown in figure below.



or $$a = -\frac{v_0^2}{x_0} + \frac{v_0^2}{x_0^2} x$$

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HOTS Higher Order Thinking Skills & Advanced Level

QUESTIONS WITH ANSWERS

Q.1. A horizontal conveyor belt is moving at a speed of 5 m/s. A box of mass 20 kg is gently placed on this belt. Box first slips on the belt and finally comes to rest with respect to belt. If box takes time 0.1 s to stop slipping on the belt, then what will be the distance travelled by the box during this interval?

Ans. Box stops slipping on the belt when it acquires same speed of 5 m/s as that of the belt. Initial speed of the box is zero and it acquires a speed of 5 m/s in 0.1 s. We can find average acceleration of box using following equation:

$v = u + at \Rightarrow 5 = 0 + a \times 0.1$

$\Rightarrow a = 50 \text{ m/s}^2$.

We can now use the following equation to calculate distance travelled by the box.

$v^2 = u^2 + 2as \Rightarrow (5)^2 = (0)^2 + 2 \times 50 \times s$

$\Rightarrow s = 0.25 \text{ m}$

Q.2. Police in a jeep is chasing a pickpocket. Police jeep is moving with uniform velocity v. Pickpocket starts a motorcycle when police jeep is at a distance d from him. Motorcycle starts moving with uniform acceleration a. Prove that pickpocket will be caught only if $v \geq \sqrt{2ad}$.

Ans. Let us assume that it takes time t to catch the pickpocket. Distance travelled by the motorcycle in this time interval can be written as follows:

$$s = ut + \frac{1}{2}at^2 \Rightarrow s = \frac{1}{2}at^2$$

Distance travelled by jeep in same time interval should be $d + s$, hence we can write the following:

$$d + s = vt$$

$$\Rightarrow \quad d + \frac{1}{2}at^2 = vt$$

$$\Rightarrow \quad at^2 - 2vt + 2d = 0$$

$$t = \frac{2v \pm \sqrt{4v^2 - 8ad}}{2a}$$

$$\Rightarrow \quad t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$$

For real solution of time we can write the following:

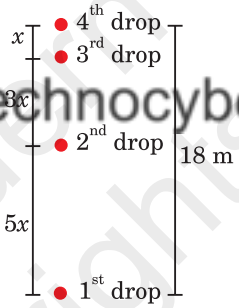
$$v^2 - 2ad \geq 0$$

$$\Rightarrow \quad v^2 \geq 2ad$$

or $$v \geq \sqrt{2ad}$$

Q.3. Water drops are falling at a regular interval from a leaked pipe 18 m above the floor. When first drop touches the ground, fourth drop is just leaving the pipe. What is the height of third drop at the given instant?

Ans. When a body is dropped from state of rest then in equal intervals of time body covers distance in the ratio 1 : 3 : 5 : 7 etc. So we can divide the 18 m height in the ratio of 1 : 3 : 5.



Let x be the distance between 3rd and 4th drop then it will be distance $3x$ between 2nd and 3rd drop and $5x$ will be the distance between 1st and 2nd drop. Total distance in terms of x can be written as follows:

$$9x = 18 \Rightarrow x = 2.$$

Distance of 3rd drop from the bottom is $8x$; hence 16 m above the floor.

Q.4. A balloon is going upwards with uniform velocity of 10 m/s. A stone attached to the balloon gets separated from it at a certain instant of time. Find the separation between stone and the balloon when stone has fallen through a height of 50 m from the point where it got separated from balloon.

Ans. Let $t = 0$ be the instant when stone gets separated from balloon. Velocity of the stone at time t = 0 is 10 m/s upwards. Stone moves under gravity like a freely falling body after it gets separated from the balloon.

If we assume upward direction as positive, then initial velocity can be written as u = +10 m/s. Acceleration due to gravity is downwards, hence acceleration a = – 10 m/s^2. Displacement of the stone is 50 m downwards, hence displacement s = – 50 m. We can use the following equation of motion to find the time.

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad -50 = 10t + \frac{1}{2}(-10)t^2$$

$$\Rightarrow \quad t^2 - 2t - 10 = 0 \qquad \Rightarrow t = \frac{2 \pm \sqrt{4 + 40}}{2}$$

$$\Rightarrow \quad t = 1 \pm \sqrt{11}$$

Negative sign in the above solution will yield negative result hence that is not correct. Using only positive sign, we can write the time taken as follows:

$$\Rightarrow \quad t = 1 + \sqrt{11} = 4.32 \text{ s}$$

Distance travelled by balloon in this time interval

$$= 10 \text{ m/s} \times 4.32 \text{ s} = 43.2 \text{ m}$$

Hence, separation between the stone and the balloon

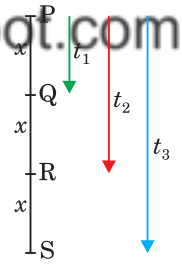
$$= 50 + 43.2 = 93.2 \text{ m}$$

Q.5. P, Q, R and S are the points in a vertical line. It is given that PQ = QR = RS. A particle is released from rest from the point P. Particle takes time t_{PQ}, t_{QR} and t_{RS} to cover three equal distances, respectively. Find ratio $t_{PQ} : t_{QR} : t_{RS}$.

Ans. Let us assume that PQ = QR = RS = x.

Let it take time t_1 to cover the distance PQ, then we can write the following:

$$x = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2x}{g}}$$



Let it take time t_2 to cover the distance PR, then we can write the following:

$$2x = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \sqrt{\frac{4x}{g}}$$

Similarly, assume that it takes time t_3 to cover the distance PS, then we can write the following:

$$3x = \frac{1}{2}gt_3^2 \Rightarrow t_3 = \sqrt{\frac{6x}{g}}$$

Now we can write the following:

$$t_{PQ} = t_1 = \sqrt{\frac{2x}{g}}$$

$$t_{QR} = t_2 - t_1 = \sqrt{\frac{4x}{g}} - \sqrt{\frac{2x}{g}} = \sqrt{\frac{2x}{g}}(\sqrt{2} - 1)$$

$$t_{RS} = t_3 - t_2 = \sqrt{\frac{6x}{g}} - \sqrt{\frac{4x}{g}} = \sqrt{\frac{2x}{g}}(\sqrt{3} - \sqrt{2})$$

Hence ratio can be written as follows:

$$t_{PQ} : t_{QR} : t_{RS} = \mathbf{1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})}$$

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Revision Exercises

Very Short Answer Questions *Carrying 1 mark*

1. When can we say that an object is in motion?
2. How are rest and motion relative to each other? Explain with an example.
3. How are rectilinear motion and translatory motion similar or different to each other?
4. Define circular motion.
5. When can we say that circular motion is periodic?
6. When is the motion of an object categorised as oscillatory motion?
7. How are vibratory motion and simple harmonic motion similar?
8. How are vibratory motion and simple harmonic motion different?
9. Define inertial frame of reference.
10. Earth revolving around the sun is an example of motion in how many dimensions?
11. Give an example of three-dimensional motion.
12. Give some example of scalar quantities.
13. Define vector quantities with example.
14. What do we measure using speedometer?
15. What will be the distance and displacement of the object when it makes one complete round of a circular park of radius 5 m?
16. What is instantaneous speed?
17. Define velocity of an object.
18. What can be deduced from the slope of a position-time graph of uniform motion?
19. What parameters can be determined from velocity–time graph of uniform motion?
20. How is non-uniform motion different from uniform motion?
21. What do we measure by odometer installed in vehicles?
22. How can we measure the instantaneously velocity graphically?
23. What does negative slope of a position-time graph signify?
24. What can be said about the velocity of the object if its displacement-time graph is parallel to displacement axis?
25. Is it possible for an object to possess constant velocity and varying speed?
26. Define acceleration of an object.
27. What do we mean by instantaneous acceleration?
28. Mention one use of velocity-time curve of an accelerating object.
29. Write three equations of motion of object with uniform acceleration.
30. Can we use the above listed equation in SHM or circular motion?
31. Is it possible for a uniformly accelerating object to change its direction of velocity?
32. What will be the instantaneous acceleration of an object thrown upward, when it reaches maximum height?
33. When is the acceleration of a bike larger: when it is suddenly stopped or when it is raced?
34. How does the velocity of a ball thrown upwards change on its way?
35. Give one example when the velocity of an object changes at a constant rate both in magnitude and direction.
36. A girl dropped an orange in a moving bus with acceleration a, what will be the acceleration of the apple with respect to the bus?
37. Find the acceleration of an orange fallen in a bus moving with acceleration a, with respect to the road.
38. Give an example of an object moving with constant value of acceleration but in variable direction.
39. Represent the motion of an object thrown upward with the help of velocity-time graph.
40. Represent the variation of distance with time for an object under free fall.

Fill in the Blanks *Carrying 1 mark*

1. ..... speed is arithmetic mean of individual speeds.
2. Path length is a ..... quantity.
3. When a particle moves in a circle of radius r, the displacement covered in one complete revolution is ..... .
4. Negative acceleration is also called ..... or ..... .
5. When an object is moving with zero acceleration, the velocity-time graph is a straight line ..... to the time axis.
6. ..... is the time a person takes to think and take some action.
7. Time occurs ..... in the unit of acceleration.
8. Average speed cannot be ..... than magnitude of average velocity.
9. The acceleration of the light travelling in vacuum is ..... .
10. If the tangent at any point to the position-time graph is parallel to time axis then instantaneous velocity is ..... .

Short Answer Questions *Carrying 2 or 3 marks*

1. Explain briefly about one-, two- and three-dimensional motion.
2. Differentiate between inertial frame of reference and non-inertial frame of reference.
3. Differentiate between scalar and vector quantities.
4. Write three differences between distance and displacement.

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5. Is it possible that magnitude of displacement of an object and distance covered by an object in same time is same?
6. Adil is running at speed of 2.5 ms^{-1} for five minutes. He then completes the remaining distance by walking for another five minutes at speed of 1 ms^{-1}. Find the average speed of Adil.
7. Write any four differences between speed and velocity.
8. Is it possible for an object to have zero average velocity and zero average speed?
9. Write some uses of velocity-time graph of an object in uniform motion.
10. Prove that average velocity of an object can never be greater than the average speed of an object over a given interval of time.
11. A toy car is moving on a circular track of radius R and completes one rotation in 55 seconds. Calculate the displacement of the car in 1 min 50 sec.
    1 min and 50 sec = 110 sec
12. Two cars are moving in same direction with speed of 40 kmh^{-1}. Find the relative velocity of first car with another. Do they have same velocity?
13. Represent the position-time graph of two objects moving along a straight line such that their relative velocity is (*a*) zero and (*b*) non-zero.
14. Two balls were thrown simultaneously from a top of a building in opposite directions. If one ball is thrown up and another in downward direction, then what will be the ratio of their speeds when they hit the ground.
15. A cricket ball and a ping pong ball are simultaneously thrown upward with velocity *v*. Which of them will pass through the point of projection with greater speed?
16. What is the significance of (*a*) positive and (*b*) negative slope in distance-time graph of an object in uniformly accelerated motion?
17. What is the significance of (*a*) positive and (*b*) negative slope in velocity-time graph of an object in uniformly accelerated motion?
18. Represent the velocity-time graph for a uniformly accelerated motion when acceleration is positive.
19. Write the steps to calculate the distance and displacement covered by a uniformly accelerated object from its velocity-time graph.
20. Calculate the acceleration of an object if its velocity is given by $v = \frac{1}{2}\left(\sqrt{12x+24}\right)$.

Long Answer Questions *Carrying 5 marks*

1. Write and derive all the three equations of motion analytically.
2. Write a short note on velocity and speed. Differentiate between them with the help of examples and illustrations.
3. Write a short note on acceleration, average acceleration, variable acceleration and instantaneous acceleration. Also show how instantaneous acceleration is related to average acceleration.
4. Derive the equations of motion for an object in uniformly accelerated motion in one dimension using calculus.
5. For an object under free fall, discuss the variation of acceleration with time, velocity with time and position of object with time. Also represent them graphically.

Numerical Problems

1. An ant is crawling on the rim of a circular plank of radius 7 cm. Calculate the distance and displacement of ant in (*a*) completing one round, (*b*) in completing 2.5 round and (*c*) half round.
2. A bus on a straight highway moves from stop A to C, 300 m away in 20 sec. It further moves from stop C to D, 180 m away in 7 sec. Calculate the average speed of bus from A to D.
3. A bike is moving on a straight road. After covering a distance of 420 m in 22 sec, it turns back and stops after 12 sec midway. Calculate the average velocity of the bike in first 22 sec and in first 34 sec.
4. The instantaneous position of a moving drone is given by $y = m + nt^2$, where $m = 6\ m$ and $n = 3.2\ ms^{-2}$ and t is time. Find the velocity, the average velocity of drone between $t = 0$ sec and 4 sec. Also calculate the velocity at $t = 3$ sec.
5. A stone is thrown up with a velocity of 18 ms^{-1} from a 25 m high deck. Calculate the maximum height achieved by stone from ground. Also calculate the total time taken by stone to reach the ground.
6. A ball is dropped from a top of a 100 m high building. At the same time another ball is thrown up from the ground with a speed of 20 ms^{-1}. At what time will the balls meet?
7. Two buses are running antiparallel in East-West direction. Calculate the velocity of Bus II with respect to Bus I. The speeds of Bus I and II are 60 kmh^{-1} and 90 $kmhr^{-1}$ respectively. Also calculate the velocity of road with respect to Bus I.

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Hints & Answers for Revision Exercises

Very Short Answer Questions

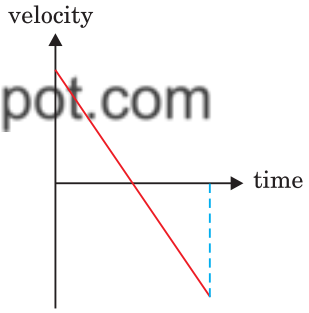
1. An object is said to be in motion when it changes its position with time with respect to its surroundings.
2. Rest and motion are relative terms as an object can be at rest with respect to some surrounding and can be in motion with respect to some other surrounding simultaneously. For example, while sitting in a moving bus we are in rest with respect to the bus but we are in motion with respect to the surroundings outside the bus.
3. When a body moves along any path without changing its orientation or without rotation, its motion is called

translatory motion. Whereas when a body moves along a straight line without rotation then its motion is called rectilinear motion.

5. Uniform circular motion is periodic in nature.
6. The motion of an object is oscillatory when the object moves to and fro repeatedly about the mean position in a fixed interval of time.
7. Both vibratory motion and simple harmonic motion are oscillatory motions.
8. Vibratory motion is when the object is undergoing oscillatory motion with very small amplitude. However, simple harmonic motion is when force acting on the object undergoing oscillatory motion is linear.
9. The frame of reference or system of coordinates describing time, position, displacement, acceleration, etc. which follows Newton's first law of motion is said to be inertial frame of reference.
10. Two dimensions
11. Aeroplane flying in the sky is example of three-dimensional motion.
12. Distance, speed, length, time, etc. are scalar quantities.
13. The physical quantities which have both magnitude and direction are vector quantities. For example, velocity, acceleration, displacement, etc.
14. We measure the instantaneous speed of the vehicle with speedometer.
15. Distance = $2\pi r = 10\ \pi$ m
    Displacement will be zero.
16. The limit of average speed as the time interval Δt of an object at the given instant is very small.
17. Velocity of an object can be defined as the ratio of displacement of the object in the given time t and the time taken, t.
18. The uniform velocity of an object can be calculated using position-time graph of uniform motion.
19. The area of velocity-time graph between any two time instants is equal to the displacement of the object in that time interval.
20. Non-uniform motion is when an object covers unequal displacements in unequal intervals of time. However, in case of uniform motion, the object covers equal displacements in equal intervals of time.
21. Odometer is used to measure instantaneous velocity of the vehicle.
22. We can measure instantaneous velocity from slope of tangent at any instant from position-time graph.
23. It signifies that the velocity of the object is negative.
24. The velocity of the object will be infinite.
25. No, it is not possible.
26. Acceleration of an object can be defined as the rate of change of its velocity with time. Its S.I. unit is ms^{-2}.
27. Instantaneous acceleration of an object is acceleration of that object at a given instant of time, which can be found as the first derivative of velocity and second derivative of its position at given time.
28. The velocity-time curve of a moving object is useful in determining the acceleration of the object.
29. $v = u + at$

$$s = ut + \frac{at^2}{2}$$

$$v^2 = u^2 + 2aS$$

30. No.
31. Yes, it is observed when a ball is thrown up, after reaching the maximum height it comes back and changes the direction of its velocity.
32. The acceleration of the object when it reaches the maximum height is equal to the acceleration due to gravity, i.e. 9.8 ms^{-2}.
33. The acceleration of the bike is larger when it is suddenly stopped because then the change in velocity in given time is large.
34. When the ball is thrown upwards its velocity is directed upwards. However when the ball reaches maximum height, its direction is changed to downward direction.
35. In projectile motion, the velocity of an object changes in both magnitude and direction at a constant rate.
36. The apple will fall freely with acceleration due to gravity with respect to the bus.
37. The acceleration of the orange with respect to the road will be $\sqrt{a^2 + g^2}$ with respect to the road.
38. An object moving in circular path has constant value of acceleration but in variable direction.
39. 

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Fill in the Blanks

1. Average
2. Scalar
3. zero
4. retardation, deceleration
5. parallel
6. Reaction time
7. twice
8. less
9. zero
10. zero

Short Answer Questions

6. $v_1 = 2.5\ ms^{-1}$
   $v_2 = 1\ ms^{-1}$
   $t_1 = 5$ min $= 300$ sec
   $t_2 = 5$ min $= 300$ sec

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$$

$$\Rightarrow \text{average speed} \frac{2.5 \times 300 + 1 \times 300}{300 + 300} = \frac{1050}{600}$$

$$= 1.75\ ms^{-1}$$

11. In 110 sec, the car will complete two rotations on the tracks. Thus the total displacement will be zero.
12. Relative velocity of car one with respect to other car is 40 – 40 = 0.

20. Acceleration can be calculated as:

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v\frac{dv}{dx}$$

From equation,

$$v = \frac{1}{2}\left(\sqrt{12x+24}\right)$$

$$\Rightarrow \quad v^2 = \frac{1}{4}(12x+24)$$

$$\Rightarrow \quad 2v\frac{dv}{dx} = \frac{1}{4} \times 12$$

$$\Rightarrow \quad a = v\frac{dv}{dx} = \frac{12}{8} = 1.5 \text{ ms}^{-2}$$

Numerical Problems

1. *(a)* Given, radius of circular plank, $r = 7$ cm

Distance travelled by ant in one circle $= 2\pi r$

$= 2 \times 3.14 \times 7 = 43.96$ cm

Displacement in one round = **zero**

(b) In 2.5 rounds

Distance $= 2.5 \times 2\pi r$

$= 2.5 \times 43.96 = 109.9$ cm

Displacement $= 7 + 7 =$ **14 cm**

(c) In half round

$$\text{Distance} = \frac{2\pi r}{2} = \frac{43.96}{2} = 21.98 \text{ cm}$$

Displacement $= 7 + 7 =$ **14 cm**

2. $$\text{Average speed} = \frac{\text{Path length}}{\text{Time}}$$

$$= \frac{300+180}{20+7} = \textbf{17.78 ms}^{-1}$$

3. In first 22 sec,

displacement = 420 m

time = 22 sec.

$$\text{average velocity} = \frac{420}{22} = 19.09 \text{ ms}^{-1}$$

In first 34 seconds,

$$\text{displacement} = 420 - \frac{420}{2} = 210 \text{ m}$$

time = 34 sec.

$$\text{average velocity} = \frac{\text{displacement}}{\text{time}}$$

$$= \frac{210}{34} = \textbf{6.18 ms}^{-1}$$

4. average velocity between $t = 0$ and $t = 4$ can be calculated as:

$$\bar{v} = \frac{y(4.0) - y(0)}{4-0} = \frac{m + n(16)}{4}$$

$$\Rightarrow \quad v = \frac{6 + 3.2 \times 16}{4} = 14.3 \text{ ms}^{-1}$$

Velocity of drone at $t = 3$ can be calculated as:

$$y = m + nt^2$$

$$v = \frac{dy}{dt} = 2nt = 2 \times 3.2 \times 3 = \textbf{19.2 ms}^{-1}$$

5. Given $v_0 = +18 \text{ ms}^{-1}$

$a = -g = -9.8 \text{ ms}^{-1}$

$v = 0 \text{ ms}^{-1}$

Let ball reach y cm from deck, then

$$v^2 = v_0^{\,2} + 2a\,(y - y_0),$$

here y_0 is height of deck

$$\Rightarrow \quad 0 = (18)^2 + 2(-9.8)\,(y - y_0)$$

$$(y - y_0)\,19.6 = (18)^2$$

$$\Rightarrow \quad (y - y_0) = \frac{324}{19.6} = 16.53 \text{ m}$$

maximum height from the ground will be 25 + 16.53

$$= \textbf{41.53 m}$$

(b) To calculate the total time taken by stone to reach the ground, we can consider the initial and final position of stone with respect to the origin.

Thus using, $y = y_0 + v_0 t + \frac{1}{2}at^2$

here $y_0 = 25$ m, $y = 0$

$v_0 = 18 \text{ ms}^{-1}$ $a = -9.8 \text{ ms}^{-2}$

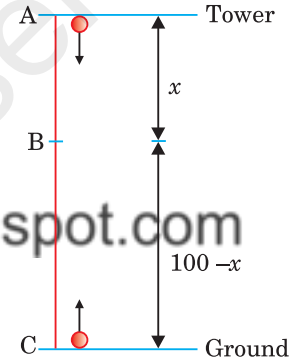
$$\therefore \quad 0 = 25 + 18\,t + \left(\frac{1}{2}\right)(-9.8)t^2$$

$$\Rightarrow \quad 4.9\,t^2 - 18\,t - 25 = 0$$

Solving for quadratic equation

$$t = \textbf{3.80 sec.}$$

6. Let us consider that balls meet at time t at some distance x from the top of building.



For ball falling down, $u = 0$, $a = 9.8 \text{ ms}^{-2}$, $S = x$ and time $= t$

Using $\quad S = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad x = \frac{1}{2} \times 9.8 \times t^2$$

$$\Rightarrow \quad x = 4.9\,t^2 \qquad \ldots(i)$$

For ball thrown upwards, $u = 20 \text{ ms}^{-1}$, $a = -9.8 \text{ ms}^{-2}$, $S = (100 - x)$

(from figure)

$$\therefore \quad S = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad 100 - x = 20\,t - 4.9\,t^2 \qquad \ldots(ii)$$

From (i) and (ii), we get

$$100 = 20\,t$$

$$\Rightarrow \quad t = \textbf{5 sec.}$$

7. Given, $v_I = 60 \text{ kmh}^{-1} = 16.67 \text{ ms}^{-1}$

then, $v_{II} = -90 \text{ kmh}^{-1} = -25 \text{ ms}^{-1}$

relative velocity of Bus II with respect to Bus I can be calculated as:

$$v_{II} - v_I = -25 - 16.67 = -41.67 \text{ ms}^{-1}$$

velocity of road with respect to Bus I can be calculated as:

$$v' = 0 - v_{II} = 0 - (-25) = \textbf{25 ms}^{-1}$$

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Additional Useful Information and Objective Questions

ADDITIONAL USEFUL INFORMATION

When we talk about motion then we always need a frame of reference to record our observations. Generally we use x-y-z coordinate system as our frame of reference and assume that observer is located at the origin. We define positive directions for X, Y and Z axis. Position, displacement, velocity and acceleration depend on the choice of reference. Note that position of the particle depends on the choice of origin but remaining quantities namely displacement, velocity and acceleration do not depend on the choice of the origin. When we change the location of the origin without changing the orientation of coordinate axes then displacement, velocity and acceleration are not affected but the position will be accordingly modified. In many cases reference frame remains fixed but it may be in a state of motion and will accordingly affect observations regarding displacement, velocity and acceleration. Observer always assumes itself in a state of rest and effect of motion of observer is observed on other objects. Concept of rest and motion is always relative and we shall discuss about it in details in next chapter. To learn the concepts behind the frame of references we should start building the notion. Let us take an example of ourselves as one living creature on the surface of earth. We know that the earth is in a continuous state of motion. In fact, earth is moving with much higher speed than you can really guess. But we don't observe the effect of motion of the earth directly. All the objects are attached to the earth and are moving similarly so there is no relative motion and we observe everything at rest with respect to ourselves. We observe motion only when something moves with respect to the earth. Atmosphere of the earth is also held by its gravitational force and moves along with it. So we don't observe the motion of air particles. We observe winds only when the air particles move with respect to the earth due to pressure differences between various locations.

When two particles are moving in the same direction with same speed then one particle will observe the other in a state of rest. Relative motion between the particles is observed only when they move with different velocities. When particles move along the same direction with slightly different speeds then one will observe the other to be moving very slowly but when both the particles move along opposite directions, then they observe faster relative motion between them. When you encounter a train on parallel tracks moving in the same direction, then you observe that the other train is moving at a slower rate with respect to you, and the relative speed is the difference between their speeds. But when you find a train on parallel track moving along opposite direction then you observe the motion of other train to be much faster and the relative speed in this case is sum of the speeds of both the trains.

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OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions *M. C. Q.*

A MULTIPLE CHOICE QUESTIONS *with only one correct answer*

A1. In a one-dimensional motion, particle has zero speed at an instant. At the same instant, particle must have

(a) Zero acceleration (b) Non-zero acceleration

(c) Zero velocity (d) Non-zero velocity

A2. Displacement of a particle moving in a straight line is represented as follows:

$x = at^3 + bt^2 + ct + d$

Ratio of initial velocity to initial acceleration depends

(a) Only on a and b (b) Only on b and c

(c) Only on c and d (d) Only on d and a

A3. A particle is projected vertically upwards and reaches the highest point in time t_1 and takes time t_2 to return to the ground from top. Choose the correct option when air resistance applies constant force on the particle opposite to direction of its motion.

(a) $t_1 = t_2$

(b) $t_1 > t_2$

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(c) $t_1 < t_2$

(d) Information is not sufficient to decide the relation between t_1 and t_2.

A4. Magnitude of average velocity and speed are found to be the same in an interval of time.

(a) Particle must have zero acceleration.

(b) Particle must have non-zero acceleration.

(c) Particle must be moving in a straight line without reversing the direction of motion.

(d) Particle must be in a state of rest.

A5. A body is moving with constant speed. Its acceleration

(a) must be non-zero

(b) must be zero

(c) may be non-zero and constant

(d) may be non-zero and variable

A6. A parrot flies in a straight line for 6 s. Velocity of the parrot is given by $v = |t-3|$. Time (t) is measured in seconds and velocity in m/s. Displacement of the parrot in 6 s is

(a) 0 (b) 5 m

(c) 6 m (d) 9 m

A7. Velocity of an object is variable, then

(a) its acceleration may remain constant.

(b) its speed may remain constant.

(c) its average acceleration may remain constant.

(d) all of these

A8. If speed of an object is variable, then

(a) its acceleration may be zero.

(b) its velocity may be constant.

(c) its velocity must be variable.

(d) its acceleration must be zero.

A9. An object is given an initial velocity of 11 m/s towards the north and a constant acceleration of 2 m/s^2 towards the south. What will be the distance covered by the object in the sixth second of its motion?

(a) 0.25 m (b) 0.5 m

(c) 0 (d) 2 m

A10. An object is projected vertically upward and is found to be at the same height at time t_1 and t_2. Neglect the air resistance. What is the speed of projection?

(a) $\frac{g(t_1+t_2)}{2}$ (b) $\frac{g(t_1-t_2)}{2}$

(c) $g(t_1+t_2)$ (d) $g(t_1-t_2)$

A11. The initial velocity given to a particle is u and acceleration is given by $a = \alpha t^{3/2}$. What will be the velocity of particle after time t.

(a) $u+\alpha t^{5/2}$ (b) $u+\frac{2}{5}\alpha t^{5/2}$

(c) $u+\frac{5}{2}\alpha t^{5/2}$ (d) $u+\alpha t^{3/2}$

A12. A body is given an initial velocity towards the north and constant acceleration is applied on it towards the south. If s_1 and s_2 are the magnitudes of displacements in first 5 s and the next 5 s respectively then

(a) $s_1 = s_2$

(b) $s_1 > s_2$

(c) $s_1 < s_2$

(d) Information is not sufficient to decide the relation between s_1 and s_2.

A13. A particle is thrown up with an initial velocity such that it takes more than one second to reach the top point. What is the distance travelled by the particle during the first second of its decent?

(a) $g/2$

(b) $g/4$

(c) g

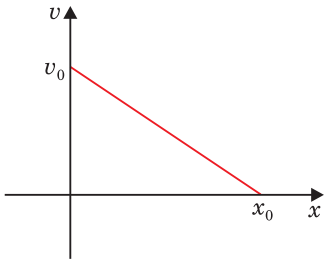
(d) Information is insufficient to calculate the desired distance.

A14. A particle is projected vertically upward from a point near the surface of earth. Upward direction is assumed as positive direction. Acceleration of the particle during its upward and downward journey are respectively

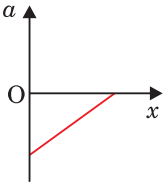
(a) Positive, Negative (b) Negative, Positive

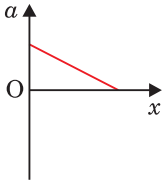
(c) Positive, Positive (d) Negative, Negative

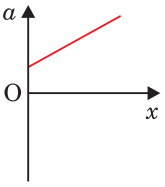
A15. Graph between velocity and displacement is shown in the following figure:

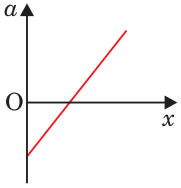


Which of the following graphs represents the correct variation of acceleration with displacement?

(a) 

(b) 

(c) 

(d) 

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A16. Displacement x of a particle varies with time as $\sqrt{x} = t + 5$, where x is in metres and time t is in seconds. Select the correct option.

(a) Acceleration of the particle is constant.

(b) Velocity of the particle at $t = 0$ is 10 m/s.

(c) Particle never reverses its direction of motion for $t > 0$.

(d) All of the above

A17. A particle starts from rest with a constant acceleration. Let S_n represent displacement in the interval from $t = n - 1$ to $t = n$. Here S_n represents displacement in n^{th} second of its motion. Then S_n/S_{n+1} is

(a) $\frac{n-1}{2n+1}$ (b) $\frac{2n-1}{2n+1}$

(c) $\frac{n+1}{2n+1}$ (d) $\frac{2n-1}{2n}$

A18. A particle is dropped from rest from the top of a building of height 100 m. At the same instant another particle is projected upward from the bottom of the building. What should be the speed of projection of the particle projected from the bottom of building so that the particles cross each other after 1 s? Acceleration due to gravity is 10 m/s^2.

(a) 100 m/s (b) 95 m/s

(c) 90 m/s (d) 105 m/s

A19. Two buses are moving in the same direction with a speed of 30 km/h. Separation between the buses is 5 km. A car is moving in the opposite direction and meets the two buses after an interval of 4 min. What is the speed of the car?

(a) 45 km/h (b) 30 km/h

(c) 25 km/h (d) 40 km/h

A20. A particle moves in a straight line and its position x and time t are related as follows:

$$x^2 = 2 + t$$

Acceleration of the particle is given by

(a) $\frac{1}{4x^2}$ (b) $-\frac{1}{4x^{3/2}}$

(c) $-\frac{1}{4x^3}$ (d) $-\frac{1}{4x}$

Answers

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A1.	(c)	**A2.**	(b)	**A3.**	(c)	**A4.**	(c)	**A5.**	(d)	**A6.**	(d)	**A7.**	(d)	**A8.**	(c)	**A9.**	(c)	**A10.**	(a)
A11.	(b)	**A12.**	(d)	**A13.**	(a)	**A14.**	(d)	**A15.**	(a)	**A16.**	(d)	**A17.**	(b)	**A18.**	(a)	**A19.**	(a)	**A20.**	(c)

B MULTIPLE CHOICE QUESTIONS *from Competitive Examinations*

AIPMT/NEET & Other State Boards for Medical Entrance

BA1. A bus starts from rest with an acceleration of 1 m s^{-2}. A man who is 48 m behind the bus starts with uniform velocity of 10 ms^{-1}. Then minimum time after which the man can catch the bus is:

(a) 4 s (b) 10 s

(c) 12 s (d) 8 s (*A.F.M.C. 2001*)

BA2. A particle is thrown vertically upwards. Its velocity at half of the height is 10 m s^{-1}. The maximum height attained by it is (given, g = 10 m/s^2):

(a) 16 m (b) 10 m

(c) 8 m (d) 18 m (*C.B.S.E. 2005*)

BA3. A stone is dropped from rest from the top of a tower 19.6 m high. The distance travelled during the last second of its fall is (given g = 9.8 m/s^2):

(a) 9.8 m (b) 14.7 m

(c) 4.9 m (d) 19.6 m

(e) 10 m (*Kerala P.M.T. 2001*)

BA4. A 120 m long train is moving towards west with speed of 10 m/s. A bird flying towards east with a speed of 5 m/s crosses the train. The time taken by the bird to cross the train will be:

(a) 16 sec (b) 12 sec

(c) 10 sec (d) 8 sec (*Manipal 2002*)

BA5. A stone dropped from a tower of height h reaches the ground after t seconds. From the same tower if two stones are thrown, one upwards and the other downwards, with the same velocity u and they reach the ground after t_1 and t_2 seconds respectively, then:

(a) $t = t_1 - t_2$ (b) $t = \frac{t_1 + t_2}{2}$

(c) $t = \sqrt{t_1 t_2}$ (d) $t = \sqrt{t_1^2 - t_2^2}$ (*A.F.M.C. 2002*)

BA6. If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is:

(a) $\frac{1}{2}gt^2$ (b) $ut - \frac{1}{2}gt^2$

(c) $(u + gt)t$ (d) ut (*C.B.S.E. 2003*)

BA7. A body moving with uniform acceleration describes 40 m in the first 5 sec and 65 m in next 5 sec. Its initial velocity will be:

(a) 4 m/s (b) 2.5 m/s

(c) 5.5 m/s (d) 11 m/s (*A.F.M.C. 2003*)

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BA8. A man throws balls with the same speed vertically upwards one after the other at an interval of 2 second. What should be the speed of the throw so that more than two balls are in the sky at any time? (Given $g = 9.8$ m/s^2)

(a) At least 9.8 m/s

(b) Any speed less than 19.6 m/s

(c) Only with speed 19.6 m/s

(d) More than 19.6 m/s *(C.B.S.E. 2003)*

BA9. A bullet loses 1/20 of its velocity after penetrating a plank. How many planks are required to stop the bullet?

(a) 6 (b) 9

(c) 11 (d) 13 *(A.F.M.C. 2004)*

BA10. The displacement x of a particle varies with time as $x = a\,e^{\alpha t} + be^{\beta t}$ where a, b, α, β are constants and are positive. The velocity of the particle will:

(a) Drop to zero when $\alpha = \beta$

(b) be independent of α and β

(c) go on increasing with time

(d) go on decreasing with time *(C.B.S.E. 2005)*

BA11. A particle moves in a straight line with a constant acceleration. It changes its velocity from 10 ms^{-1} to 20 ms^{-1} while passing through a distance 135 m in t second. The value of t is:

(a) 1.8 (b) 12

(c) 9 (d) 10 *(C.B.S.E. 2008)*

BA12. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}$ m s^{-2} in the third second is:

(a) 4 m (b) $\frac{10}{3}$ m

(c) $\frac{19}{3}$ m (d) 6 m *(C.B.S.E. 2008)*

BA13. A particle moves along straight line OX. At time t (seconds) the distance x (metres) from O is given by: $x = 40 + 12t - t^3$. How long the particle travels before coming to rest?

(a) 16 m (b) 21 m

(c) 40 m (d) 56 m *(C.B.S.E. 2006)*

BA14. Two bodies A and B of masses 1 kg and 3 kg respectively are dropped from heights 16 m and 25 m. The ratio of the time taken by them to reach the ground is:

(a) $\frac{4}{5}$ (b) $\frac{5}{4}$

(c) $\frac{12}{5}$ (d) $\frac{5}{12}$ *(C.B.S.E. 2006)*

BA15. The position x of a particle with respect to time t along X-axis is given by $x = 9t^2 - t^3$ where x is in metres and t in second. What will be the position of this particle when it achieves maximum speed along the $+x$ direction?

(a) 54 m (b) 81 m

(c) 24 m (d) 32 m *(C.B.S.E. 2007)*

BA16. A car accelerates from rest at constant rate for first 10 s and covers a distance 'x'. It covers a distance 'y' in next 10 s at the same acceleration. Which of the following is true?

(a) $x = 3y$ (b) $y = 3x$

(c) $x = y$ (d) $y = 2x$ *(A.F.M.C. 2005)*

BA17. A car moves from X to Y with a uniform speed v_u and returns to Y with a uniform speed v_d. The average speed for this round trip is:

(a) $\sqrt{v_u\, v_d}$ (b) $\frac{2v_d\, v_u}{v_d + v_u}$

(c) $\frac{v_u + v_d}{2}$ (d) $\frac{v_d\, v_u}{v_d + v_u}$ *(C.B.S.E. 2007)*

BA18. A particle moving along X-axis has acceleration f, at time t given by $f = f_0\left(1 - \frac{t}{T}\right)$, where f_0 and T are constants. The particle at $t = 0$ has zero velocity. In the time interval between $t = 0$ and the instant when $f = 0$, the particle's velocity (v_x) changes by:

(a) $\frac{1}{2} f_0\, T^2$ (b) $\frac{1}{2} f_0\, T$

(c) $f_0 T^2$ (d) $f_0 T$ *(C.B.S.E. 2007)*

BA19. A particle starting from the origin (0, 0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the X-axis an angle of:

(a) 45° (b) 60°

(c) 0° (d) 30° *(C.B.S.E. 2007)*

BA20. A particle starts its motion from rest under the action of a constant force. If the distance covered in the first 10 seconds is S_1 and that covered in the first 20 seconds is S_2, then:

(a) $S_2 = 3S_1$ (b) $S_2 = 4S_1$

(c) $S_2 = S_1$ (d) $S_2 = 2S_1$ *(C.B.S.E. 2009)*

BA21. A bus is moving with a speed of 10 m s^{-1} on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus?

(a) 40 m s^{-1} (b) 25 m s^{-1}

(c) 10 m s^{-1} (d) 20 m s^{-1} *(C.B.S.E. 2009)*

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BA22. A ball is dropped from a high rising platform at $t = 0$ starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v. The two balls meet at t = 18 s. What is the value of v? (Take g = 10 m/s^2)

(a) 40 m/s (b) 60 m/s

(c) 75 m/s (d) 55 m/s (*C.B.S.E. 2010*)

BA23. A particle moves a distance x in time t according to equation $x = (t + 5)^{-1}$. The acceleration of particle is proportional to:

(a) (distance)$^{-2}$ (b) (velocity)$^{2/3}$

(c) (velocity)$^{3/2}$ (d) (distance)^2

(*C.B.S.E. 2010*)

BA24. A body A starts from rest with an acceleration 'a_1'. After 2 seconds another body B starts from rest with an acceleration 'a_2'. If they travel equal distances in 5th second after the start of A, their ratio $\frac{a_1}{a_2}$ is equal to:

(a) $\frac{5}{9}$ (b) $\frac{5}{7}$

(c) $\frac{9}{5}$ (d) $\frac{9}{7}$ (*A.I.I.M.S. 2001*)

BA25. A ball dropped from the 9th storey of a multistoried building reaches the ground in 3 second. In the first second of its free fall, it passes through n storeys, where n is equal to (Take g = 10 m s^{-2}).

(a) 1 (b) 2

(c) 3 (d) 4 (*J.I.P.M.E.R. 2001*)

BA26. One car moving on a straight road covers one-third of the distance with 20 km h^{-1} and the rest with 60 km h^{-1}. The average speed is:

(a) 40 km h^{-1} (b) 80 km h^{-1}

(c) 46 $\frac{2}{3}$ km h^{-1} (d) 36 km h^{-1}

(*J.I.P.M.E.R. 2001*)

BA27. A particle starts from rest and has an acceleration of 2 m/s^2 for 10 seconds. After that, it travels for 30 seconds with constant speed and then undergoes a retardation of 4 m/s^2 and comes back to rest. The total distance covered by the particle is:

(a) 650 m (b) 700 m

(c) 750 m (d) 800 m (*A.I.I.M.S. 2002*)

BA28. A very large number of balls are thrown vertically upwards in quick successions in such a way that the next ball is thrown when the previous one is at the maximum height. If the maximum height is 5 m, then number of balls thrown per minute is (take g = 10 m/s^2):

(a) 80 (b) 120

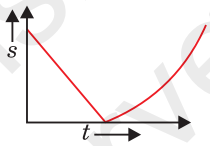
(c) 40 (d) 60 (*J.I.P.M.E.R. 2002*)

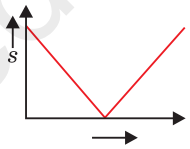
BA29. The position x of a particle varies with time t as $x = a\,t^2 - b$. For what value of t acceleration is zero?

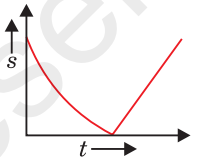
(a) $\frac{2a}{3b}$ (b) a/b

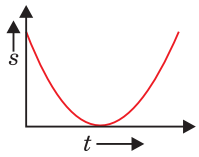
(c) $\frac{a}{3b}$ (d) never (*B.H.U. 2002*)

BA30. A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its flight if the air resistance is not ignored? (Assume air resistance constant)

(a) 

(b) 

(c) 

(d) 

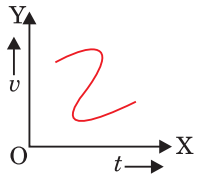
(*A.I.I.M.S. 2003*)

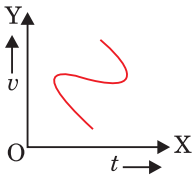
BA31. A body falls from a height h = 200 m. The ratio of distance travelled in each 2 sec, during $t = 0$ to $t = 6$ second of the journey is:

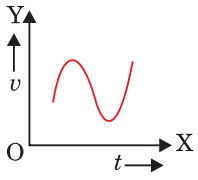
(a) 1 : 4 : 9 (b) 1 : 2 : 4

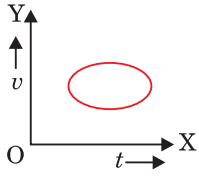
(c) 1 : 3 : 5 (d) 1 : 2 : 3 (*B.H.U. 2003*)

BA32. Which of the following velocity-time graphs shows a realistic situation for a body in motion?

(a) 

(b) 

(c) 

(d) 

(*A.I.I.M.S. 2004*)

BA33. A ball is thrown up vertically with velocity v_0, it reaches a maximum height 'h'. If one wishes to tripple the maximum height the ball should be thrown with a velocity of:

(a) 3 v_0 (b) 9 v_0

(c) $\sqrt{3}\,v_0$ (d) $\frac{3}{2}v_0$ (*A.I.I.M.S. 2005*)

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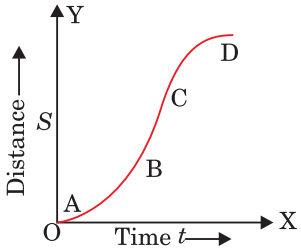
BA34.



A particle shows distance-time curve as given in this figure. The maximum instantaneous velocity of the particle is around the point:

(*a*) C (*b*) D

(*c*) A (*d*) B (*C.B.S.E. 2008*)

BA35. A car starts from rest, moves with an acceleration '*a*' and then decelerate at a constant rate '*b*' for some time to come to rest. If the total time taken is t, the maximum velocity of the car is given by:

(*a*) $\frac{abt}{a+b}$ (*b*) $\frac{a^2t}{a+b}$

(*c*) $\frac{at}{a+b}$ (*d*) $\frac{b^2t}{a+b}$ (*B.H.U. 2006*)

BA36. A boy standing at the top of a tower of 20 m height drops a stone. Assuming $g = 10\ m\ s^{-2}$, the velocity with which it hits the ground is:

(*a*) 10.0 m/s (*b*) 20.0 m/s

(*c*) 40.0 m/s (*d*) 5.0 m/s (*C.B.S.E. 2011*)

BA37. The motion of a particle along a straight line is described by equation:

$$x = 8 + 12t - t^3$$

where x is in metre and t in second. The retardation of the particle when its velocity becomes zero is:

(*a*) 12 ms^{-2} (*b*) 24 ms^{-2}

(*c*) zero (*d*) 6 ms^{-2} (*C.B.S.E. 2012*)

BA38. A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respectively. The relation between h_1, h_2 and h_3 is:

(*a*) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$ (*b*) $h_2 = 3h_1$ and $h_3 = 3h_2$

(*c*) $h_1 = h_2 = h_3$ (*d*) $h_1 = 2h_2 = 3h_3$ (*N.E.E.T. 2013*)

BA39. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = \beta x^{-2n}$

where β and n are constants and x is the position of the particle. The acceleration of the particle as a function of x is given by:

(*a*) $-2n\beta^2 x^{-2n-1}$ (*b*) $-2n\beta^2 x^{-4n-1}$

(*c*) $-2\beta^2 x^{-2n+1}$ (*d*) $-2n\beta^2 e^{-4n+1}$ (*A.I.P.M.T. 2015*)

BA40. If the velocity of a particle is $v = At + Bt^2$ where A and B are constant, then the distance travelled by it between 1 s and 2 s is:

(*a*) $\frac{3}{2}A + \frac{7}{3}B$ (*b*) $\frac{A}{2} + \frac{B}{3}$

(*c*) $\frac{3}{2}A + 4B$ (*d*) $3A + 7B$ (*N.E.E.T. 2016*)

BA41. A car moves a distance of 200 m. It covers the first half of the distance at a speed of 40 km h^{-1} and the second half of the distance at a speed v. The average speed is 48 km h^{-1}. Find the value of v.

(*a*) 56 km h^{-1} (*b*) 60 km h^{-1}

(*c*) 50 km h^{-1} (*d*) 48 km h^{-1} (*C.B.S.E. 1991*)

BA42. A body travels such that square of time is proportional to the displacement. Its acceleration is:

(*a*) zero (*b*) infinite

(*c*) constant (*d*) variable (*A.F.M.C. 1993*)

BA43. A body is vertically projected at 100 m s^{-1}. It returns after ($g = 10\ m\ s^{-2}$)

(*a*) 10 s (*b*) 20 s

(*c*) 8 s (*d*) 16 s (*A.F.M.C. 1993*)

BA44. A boy walks to his school at a distance 6 km with a speed of 2.5 km h^{-1} and walks back with a constant speed of 4 km h^{-1}. His average speed for the round trip is:

(*a*) $\frac{24}{13}$ km h^{-1} (*b*) $\frac{40}{13}$ km h^{-1}

(*c*) 3 km h^{-1} (*d*) 4.8 km h^{-1} (*A.I.I.M.S. 1995*)

BA45. A ball of mass m_1 and another of mass m_2 are dropped from equal heights. If the time taken by the balls are t_1 and t_2 respectively then,

(*a*) $t_1 = \frac{t_2}{2}$ (*b*) $t_1 = t_2$

(*c*) $t_1 = 4t_2$ (*d*) $t_1 = \frac{t_2}{4}$ (*B.H.U. 1997*)

BA46. A ball is dropped on to the floor from a height 10 m and it rebounds to a height of 2.5 m. If the ball is in contact with the floor for 0.01 s, the average acceleration during the contact is ($g = 9.8\ m/s^2$):

(*a*) 1400 m s^{-2} (*b*) 2100 m s^{-2}

(*c*) 700 m s^{-2} (*d*) 2800 m s^{-2} (*C.P.M.T. 1997, B.H.U. 1997*)

BA47. A cyclist accelerates from rest to a velocity of 72 km hr^{-1} in 10 sec. If the cyclist is in straight track, the acceleration of the cyclist is:

(*a*) 7.2 m s^{-2} (*b*) 120 m s^{-2}

(*c*) 2 m s^{-2} (*d*) 0.2 m s^{-2} (*Kerala P.M.T. 2003*)

BA48. A car has speed of 40 km/h. On applying brakes it stops after 15 m. If its speed was 80 km h^{-1} it would have stopped after:

(*a*) 15 m (*b*) 30 m

(*c*) 45 m (*d*) 60 m (*I.P.U. Delhi 2003*)

BA49. A particle is moving such that its position coordinates (x, y) are (2 m, 3 m) at time $t = 0$, (6 m,

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7 m) at time $t = 2$ s and (13 m, 14 m) at time $t = 5$ s Average $\left(\vec{v}_{av}\right)$ from $t = 0$ to $t = 5$ s is

(a) $\frac{1}{5}\left(13\hat{i} + 14\hat{j}\right)$ (b) $\frac{7}{3}\left(\hat{i} + \hat{j}\right)$

(c) $2\left(\hat{i} + \hat{j}\right)$ (d) $\frac{11}{5}\left(\hat{i} + \hat{j}\right)$

(A.I.P.M.T. 2014)

BA50. Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be

(a) $\frac{t_1 + t_2}{2}$ (b) $\frac{t_1 t_2}{t_2 - t_1}$

(c) $\frac{t_1 t_2}{t_2 + t_1}$ (d) $t_1 - t_2$ *(N.E.E.T. 2017)*

BA51. The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in metres and t in seconds. The acceleration of the particle at $t = 2$ s is

(a) 0 (b) 5 m/s^2

(c) – 4 m/s^2 (d) – 8 m/s^2 *(N.E.E.T. 2017)*

BA52. A person travelling in a straight line moves with a constant velocity v_1 for certain distance 'x' and with a constant velocity v_2 for next equal distance. The average velocity v is given by the relation

(a) $v = \sqrt{v_1 v_2}$ (b) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

(c) $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$ (d) $\frac{v}{2} = \frac{v_1 + v_2}{2}$

(N.E.E.T. 2019)

JEE (Main) & Other State Boards for Engineering Entrance

BB1. A parachutist after bailing out falls 50 m without friction. When parachute opens it decelerates at 2 m s^{-2}. He lands with a speed of 3 m s^{-1}. At what height did he bail out?

(a) 191 m (b) 182 m

(c) 293 m (d) 111 m *(A.I.E.E.E. 2005)*

BB2. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal

(a) $\frac{2a_1a_2}{a_1 + a_2}t$ (b) $t\sqrt{2a_1a_2}$

(c) $\frac{a_1 + a_2}{2}t$ (d) $t\sqrt{a_1a_2}$

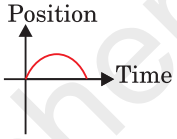
(J.E.E. Main 2019)

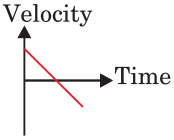
BB3. An automobile, travelling at 40 km/h, can be stopped at a distance of 40 m by applying brakes. If the same automobile is travelling at 80 km/h, the minimum stopping distance in metres is (assume no skidding):

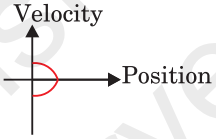
(a) 150 m (b) 100 m

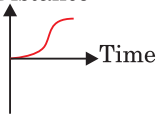
(c) 75 m (d) 160 m *(J.E.E. Main 2018)*

BB4. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

(a) 

(b) 

(c) 

(d) 

(J.E.E. Main 2018)

BB5. The velocity of particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time is:

(a) $v + g/2 + f/3$ (b) $v + g + f$

(c) $v + g/2 + f$ (d) $v + 2g + 3f$. *(A.I.E.E.E. 2007)*

BB6. An object moving with a speed of 6.25 m/s is decelerated at a rate given by:

$\frac{dv}{dt} = -2.5\sqrt{v}$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be:

(a) 2 s (b) 4 s

(c) 8 s (d) 1 s *(A.I.E.E.E. 2011)*

BB7. A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball at $\frac{T}{3}$ second?

(a) $\frac{h}{9}$ metres from the ground

(b) $\frac{7h}{9}$ metres from the ground

(c) $\frac{8h}{9}$ metres from the ground

(d) $\frac{17h}{18}$ metres from the ground *(A.I.E.E.E. 2004)*

BB8. A particle of mass m is acted upon by a force F given by the empirical law $F = \frac{R}{t^2}v(t)$. If this law is to be tested experimentally by observing the motion starting from rest, the best way is to plot:

(a) $v(t)$ against t^2 (b) $\log v(t)$ against $\frac{1}{t^2}$

(c) $\log v(t)$ against t (d) $\log v(t)$ against $\frac{1}{t}$

(J.E.E. Main 2016)

BB9. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken

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by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path.

The relation between H, u and n is:

(*a*) $2gH = n^2u^2$ (*b*) $gH = (n-2)^2u^2$

(*c*) $2gH = nu^2(n-2)$ (*d*) $gH = (n-2)u^2$

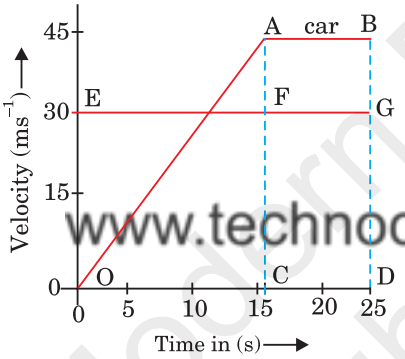
(*J.E.E. Main 2014*)

BB10. A person climbs up a stalled escalator in 60 s. If standing on the same but escalator running with constant velocity he takes 40 s. How much time is taken by the person to walk up the moving escalator?

(*a*) 37 s (*b*) 27 s

(*c*) 24 s (*d*) 45 s (*J.E.E. Main 2014*)

BB11. The velocity-time graphs of a car and a scooter are shown in the figure. (*i*) The difference between the distance travelled by the car and the scooter in 15 s and (*ii*) the time at which the car will catch up with the scooter are, respectively.



(*a*) 112.5 m and 22.5 s

(*b*) 337.5 m and 25 s

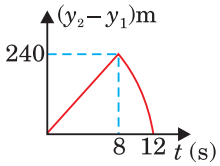
(*c*) 225.5 m and 10 s

(*d*) 112.5 m and 15 s (*J.E.E. Main 2018*)

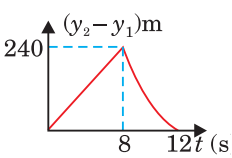
BB12. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10$ m/s^2)

(The figures are schematic and not drawn to scale)

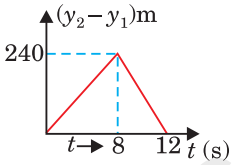
(*a*)



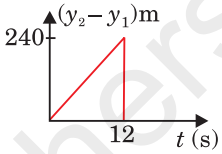
(*b*)



(*c*)



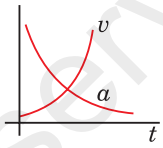
(*d*)



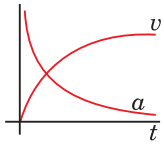
(*J.E.E. Main 2015*)

BB13. Which of the following option correctly describes the variation of the speed v and acceleration 'a' of a point mass falling vertically in a viscous medium that applies a force $F = -kv$, where 'k' is a constant, on the body? (Graphs are schematic and not drawn to scale.)

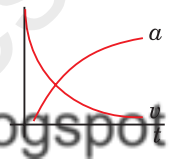
(*a*)



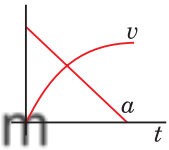
(*b*)



(*c*)



(*d*)



(*J.E.E. Main 2016*)

BB14. A car moving with a velocity of 20 ms^{-1} is stopped in a distance of 40 m. If the same car is travelling at double the velocity, the distance travelled by it for same retardation is

(*a*) 640 m (*b*) 320 m

(*c*) 160 m (*d*) 1280 m

(*Karnataka C.E.T. 2017*)

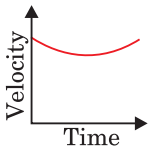
BB15. A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration 2 m/s^2 and the car has acceleration 4 m/s^2. The car will catch up with the bus after a time of:

(*a*) $\sqrt{110}$ s (*b*) $\sqrt{120}$ s

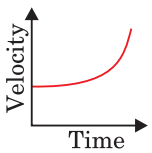
(*c*) $10\sqrt{2}$ s (*d*) 15 s (*J.E.E. Main 2017*)

BB16. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity?

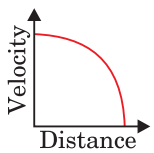
(*a*)



(*b*)



(*c*)



(*d*)

(*J.E.E. Main 2017*)

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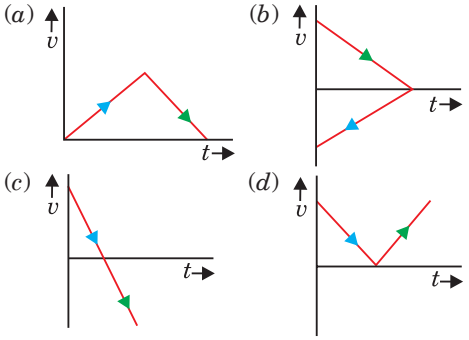
BB17. A body is thrown vertically upwards. Which one of the following graphs correctly represents the velocity vs time?



(*J.E.E. Main 2017*)

BB18. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$, where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be:

(a) $a + \frac{b^2}{c}$ (b) $a + \frac{b^2}{4c}$

(c) $a + \frac{b^2}{3c}$ (d) $a + \frac{b^2}{2c}$

(*J.E.E. Main 2019*)

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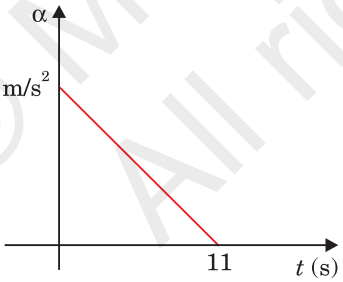
JEE (Advanced) for IIT Entrance

BC1. A particle starts from rest. Its acceleration (α) versus time (t) is as shown in the figure. The maximum speed of the particle will be:



(a) 110 m/s (b) 55 m/s

(c) 550 m/s (d) 660 m/s

(*I.I.T. Screening 2004*)

BC2. A small block slides without friction down an inclined plane starting from rest. Let S_n be the distance travelled from time $t = n - 1$ to $t = n$.

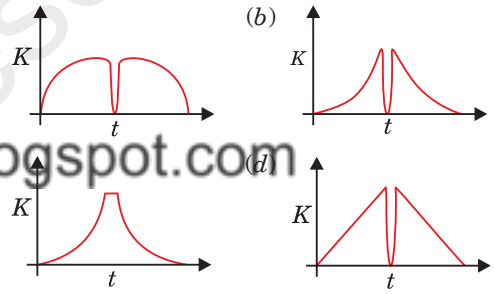
Then $\frac{S_n}{S_{n+1}}$ is:

(a) $\frac{2n-1}{2n}$ (b) $\frac{2n+1}{2n-1}$

(c) $\frac{2n-1}{2n+1}$ (d) $\frac{2n}{2n+1}$

(*I.I.T. Screening 2004*)

BC3. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to scale.



(*J.E.E. Advanced 2014*)

BC4. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the *instantaneous* density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to

(a) R (b) R^3

(c) $\frac{1}{R}$ (d) $R^{2/3}$

(*J.E.E. Advanced 2017*)

Answers

BA1. (d)	**BA2.** (b)	**BA3.** (b)	**BA4.** (d)	**BA5.** (c)	**BA6.** (a)	**BA7.** (c)	**BA8.** (d)	**BA9.** (b)	**BA10.** (c)
BA11. (c)	**BA12.** (b)	**BA13.** (d)	**BA14.** (a)	**BA15.** (a)	**BA16.** (b)	**BA17.** (b)	**BA18.** (b)	**BA19.** (b)	**BA20.** (b)
BA21. (d)	**BA22.** (c)	**BA23.** (c)	**BA24.** (a)	**BA25.** (a)	**BA26.** (d)	**BA27.** (c)	**BA28.** (d)	**BA29.** (d)	**BA30.** (b)
BA31. (c)	**BA32.** (c)	**BA33.** (c)	**BA34.** (a)	**BA35.** (a)	**BA36.** (b)	**BA37.** (a)	**BA38.** (a)	**BA39.** (b)	**BA40.** (a)
BA41. (b)	**BA42.** (c)	**BA43.** (b)	**BA44.** (b)	**BA45.** (b)	**BA46.** (b)	**BA47.** (c)	**BA48.** (d)	**BA49.** (d)	**BA50.** (c)
BA51. (c)	**BA52.** (c)	**BB1.** (c)	**BB2.** (d)	**BB3.** (d)	**BB4.** (d)	**BB5.** (a)	**BB6.** (a)	**BB7.** (c)	**BB8.** (d)
BB9. (c)	**BB10.** (c)	**BB11.** (a)	**BB12.** (a)	**BB13.** (b)	**BB14.** (c)	**BB15.** (c)	**BB16.** (c)	**BB17.** (c)	**BB18.** (c)
BC1. (b)	**BC2.** (c)	**BC3.** (b)	**BC4.** (a)						

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C MULTIPLE CHOICE QUESTIONS *with more than one correct answer*

C1. A particle is moving along X-axis according to the following equation:

$x = u(t-4) + a(t-3)^2$

All terms in above equation are measured in MKS system.

(*a*) Acceleration of the particle is a.

(*b*) Acceleration of the particle is $2a$.

(*c*) Velocity of particle at $t = 3$ s is u.

(*d*) At $t = 0$ particle is at the origin.

C2. Let v and a represent instantaneous velocity and acceleration of a particle respectively.

(*a*) Acceleration a can be zero when velocity $v \neq 0$

(*b*) Acceleration a can be nonzero when velocity $v = 0$.

(*c*) Acceleration a can be zero when velocity $v = 0$.

(*d*) Acceleration a must be zero when velocity $v = 0$.

C3. A particle may have

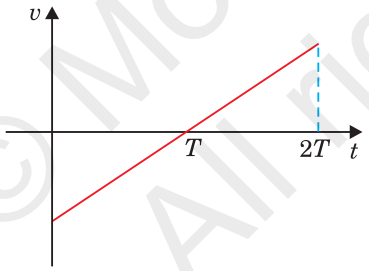
(*a*) variable velocity without variable speed.

(*b*) variable speed without variable velocity.

(*c*) zero acceleration with variable velocity.

(*d*) nonzero acceleration with constant speed.

C4. Velocity-time graph for a particle is shown in following figure for time interval 0 to 2 T.



(*a*) During the motion, particle reverses its direction of motion.

(*b*) Particle is moving with constant acceleration.

(*c*) Net displacement in time interval 0 to $2T$ is zero.

(*d*) Speeds of particle at $t = 0$ and $t = 2T$ are same.

C5. An observer moves along the line joining two stationary objects with constant speed. He observes that the two objects

(*a*) move with same speed.

(*b*) move with the same velocity.

(*c*) move in the same direction.

(*d*) move along the opposite directions.

C6. If a particle moving in a vertical straight line is

(*a*) moving up and speeding up then its acceleration must be upward.

(*b*) moving up and slowing down then its acceleration must be downward.

(*c*) moving down and speeding up then its acceleration must be downward.

(*d*) moving down and slowing down then its acceleration must be upward.

C7. For a particle moving in a straight line

(*a*) magnitude of velocity is equal to the speed of the particle.

(*b*) if position and velocity have same sign then particle is moving away from the origin.

(*c*) magnitude of average velocity is equal to the average speed for a given interval.

(*d*) if speed of the particle remains zero in a time interval then acceleration is equal to zero at any instant of the time during that interval.

C8. For a particle moving in a straight line

(*a*) if velocity is negative and acceleration is positive then speed increases.

(*b*) if velocity is positive and acceleration is negative then speed decreases.

(*c*) if velocity is zero at an instant then acceleration must also be zero at that instant.

(*d*) it is possible that speed of a particle is never zero in an interval of time, but average speed is zero.

C9. Displacement of a particle moving along X-axis is given by $x = at^2 - bt^3$.

(*a*) Particle starts from rest and again comes to rest after time $2a/3b$.

(*b*) Particle starts from origin and again returns to origin after time $t = a/b$.

(*c*) Particle starts with zero acceleration and acceleration again becomes zero after time $t = a/3b$.

(*d*) Acceleration of the particle becomes zero at $t = a/3b$.

C10. Select the correct statements for a particle in a state of motion.

(*a*) If speed of the particle changes then velocity of the particle must change and it must have non-zero acceleration.

(*b*) If velocity of the particle changes then its speed must also change and particle must have non-zero acceleration.

(*c*) When particle moves in a straight line with changing speed then its velocity may remain constant.

(*d*) When velocity of the particle changes then its speed may or may not change but particle must have some acceleration.

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C11. A particle is moving on a straight line and its average velocity is found to be zero in an interval of time.

(a) Average speed of the particle may also be zero for a given interval of time.

(b) Velocity of the particle can never be zero in given interval of time.

(c) Velocity of the particle must be zero at a particular instant.

(d) Acceleration of particle may be zero.

C12. A particle is projected upwards with initial velocity u. Assume acceleration due to gravity is 10 m/s^2. It is found that particle covers 5 m in last second before reaching the maximum height. What can be the possible value of u?

(a) 20 m/s (b) 35 m/s

(c) 5 m/s (d) 40 m/s

C13. Displacement of a particle moving in a straight line is written as follows:

$$x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + 7$$

What is the possible acceleration of particle when particle is in a state of rest?

(a) 1 m/s^2 (b) -1 m/s^2

(c) -5 m/s^2 (d) $+5$ m/s^2

Answers

C1. (b, c)	**C2.** (a, b, c)	**C3.** (a, d)	**C4.** (a, b, c, d)	**C5.** (a, b, c)
C6. (a, b, c, d)	**C7.** (a, b, d)	**C8.** (b)	**C9.** (a, b, d)	**C10.** (a, d)
C11. (c)	**C12.** (a, b, d)	**C13.** (a, b)		

D MULTIPLE CHOICE QUESTIONS
based on a given passage/comprehension

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Passage I

A particle starts moving in a straight line with initial velocity v_0. Applied forces cause a retardation of αv, where v is magnitude of instantaneous velocity and α is a constant.

Answer the following questions:

D1. How long the particle will take to come to rest.

(a) $\frac{\ln 2}{\alpha}$ (b) $\alpha \ln 2$

(c) $\frac{1}{\alpha}$ (d) infinite

D2. How long the particle will take to reduce its speed to half of its initial value?

(a) $\frac{\ln 2}{\alpha}$ (b) $\alpha \ln 2$

(c) $\frac{1}{\alpha}$ (d) infinite

D3. Total distance covered by the particle is

(a) infinity (b) $v_0\alpha$

(c) v_0/α (d) $v_0/e\alpha$

Passage II

There is a tower of height 20 m and a particle is projected up from top of the tower with an initial speed 20 m/s. Top of the tower is marked as point A, from where particle is projected. Point of maximum height is denoted as B. When particle reaches the point A during downward journey then we call the same point as C. Point at the bottom of tower is marked as D where particle finally strikes. Acceleration due to gravity $g = 10$ m/s^2.

Answer the following questions:

D4. How much time the particle takes to cross the point C after being projected from point A.

(a) 1 s (b) 2 s

(c) 3 s (d) 4 s

D5. Maximum height above the ground attained by particle is

(a) 20 m (b) 30 m

(c) 40 m (d) 60 m

D6. Time taken by the particle to reach the ground is

(a) 2 s (b) 4 s

(c) $2\sqrt{2}$ s (d) $(2+2\sqrt{2})$ s

Passage III

A particle is travelling along X-axis and its x-coordinate is related to time as follows:

$$x = 5t^2 - 20$$

Here x is measured in metres and time t in seconds.

Answer the following questions:

D7. When does the particle cross the origin?

(a) 2 s (b) 3 s

(c) 1 s (d) never

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D8. When does the particle reverse its direction of motion?

(*a*) 2 s (*b*) 3 s

(*c*) 1 s (*d*) never

D9. When does the magnitude of velocity become equal to that of acceleration?

(*a*) 2 s (*b*) 3 s

(*c*) 1 s (*d*) never

Answers

Passage I	**D1.** (*d*)	**D2.** (*a*)	**D3.** (*c*)
Passage II	**D4.** (*d*)	**D5.** (*c*)	**D6.** (*d*)
Passage III	**D7.** (*a*)	**D8.** (*d*)	**D9.** (*c*)

ASSERTION REASON TYPE QUESTIONS

The questions given below consist of an assertion and the reason. Use the following key to choose appropriate answer:

(*a*) If both assertion and reason are *correct* and reason is a *correct explanation* of the assertion.

(*b*) If both assertion and reason are *correct* but reason is *not the correct explanation* of assertion.

(*c*) If assertion is *correct* but reason is *incorrect*.

(*d*) If assertion is *incorrect* but reason is *correct*.

(*e*) If both assertion and reason are *incorrect*.

1. **Assertion:** When an object is accelerating, it is either speeding up or slowing down.
   Reason: When an object moves on a circular path with uniform speed, the object accelerates.
2. **Assertion:** A ball is thrown vertically upwards and air resistance is considered. Time of ascent is less than the time of decent.
   Reason: Air resistance always acts opposite to velocity.
3. **Assertion:** If velocity of a particle at a certain instant is zero then its acceleration must also be zero at the same instant.
   Reason: When a particle is projected upward under gravity then at the top point its instantaneous velocity becomes zero.
4. **Assertion:** If speed of a particle is never zero in an interval of time then it may have zero average speed for the same interval of time.
   Reason: If a particle is moving in a circular path then its average speed in one complete circle is zero.
5. **Assertion:** For a particle in motion, it is not possible to have constant velocity and variable acceleration.
   Reason: A particle under acceleration cannot have constant velocity.
6. **Assertion:** Average speed of the particle in a given interval of time may be equal to the magnitude of the average velocity.
   Reason: Instantaneous speed of the particle is equal to the magnitude of instantaneous velocity.
7. **Assertion:** Average speed in an interval cannot be less than the magnitude of average velocity in the same interval.
   Reason: For a particle in motion distance travelled is always greater than or equal to the magnitude of the displacement.
8. **Assertion:** For a particle in state of motion, average velocity can be zero, but average speed cannot be zero in any finite interval of time.
   Reason: For a particle in state of motion, displacement in an interval of time can be zero but distance can never be zero.
9. **Assertion:** For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.
   Reason: If the observer and the object are moving at velocities $\vec{V}_1$ and $\vec{V}_2$ respectively with the reference to a laboratory frame, the velocity of the object with respect to the observer will be $\vec{V}_2 - \vec{V}_1$.

Answers

1. (*d*) **2.** (*a*) **3.** (*d*) **4.** (*e*) **5.** (*a*) **6.** (*b*) **7.** (*a*) **8.** (*a*) **9.** (*b*)

MATCHING TYPE QUESTIONS

Each question has a matching list. The codes for the lists have choices (*a*), (*b*), (*c*) and (*d*), out of which only one is correct.

1. For a particle moving in a straight line assume s, v, a and t represents displacement, velocity, acceleration and time respectively.

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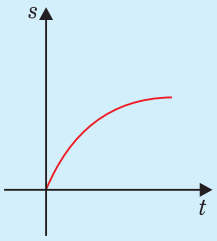
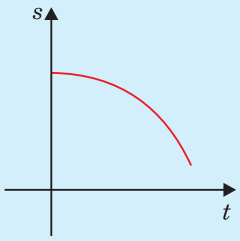
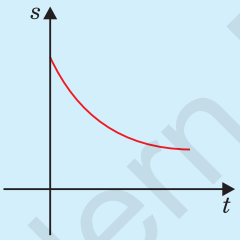
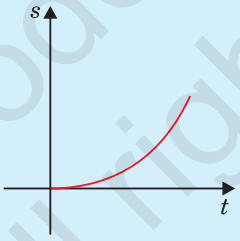
Competition File

List-I		List-II	
P	v: positive a: positive	1	
Q	v: positive a: negative	2	
R	v: negative a: positive	3	
S	v: negative a: negative	4	

	P	Q	R	S
(a)	3	2	4	1
(b)	1	3	3	4
(c)	2	1	3	4
(d)	4	1	3	2

2. For a particle moving along the vertical direction, assume s, v, a and t represent displacement, velocity, acceleration and time respectively. Assume the vertical upward direction as the positive direction.

List-I		List-II	
P	v: positive a: positive	1	Particle is moving down and slowing down
Q	v: positive a: negative	2	Particle is moving up and speeding up
R	v: negative a: positive	3	Particle is moving down and speeding up
S	v: negative a: negative	4	Particle is moving up and slowing down

	P	Q	R	S
(a)	3	2	4	1
(b)	1	3	3	4
(c)	2	4	1	3
(d)	4	1	2	3

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Answers

1. (d) **2.** (c)

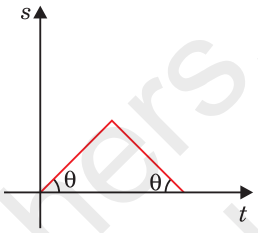
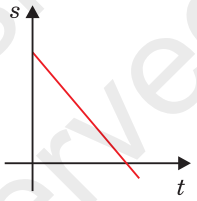
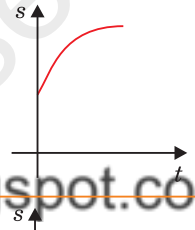
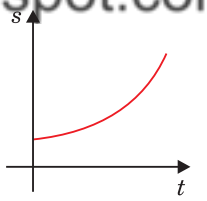
MATRIX MATCH TYPE QUESTIONS

Each question contains statements given in two columns, which are to be matched. Statements in Column-I are labelled as A, B, C and D whereas statements in Column-II are labelled as p, q, r and s. Match the entries of Column-I with appropriate entries of Column-II. Each entry in Column-I may have one or more than one correct option from Column-II. The answers to these questions have to be appropriately bubbled as illustrated in the given example. If the correct matches are A → (q, r), B → (p, s), C → (r, s) and D → (q), then correctly bubbled matrix will look like the following:

	p	q	r	s
A	p	**q**	**r**	s
B	**p**	q	r	**s**
C	p	q	**r**	**s**
D	p	**q**	r	s

Competition File

1. Match the Column-I with Column-II:

Column-I		Column-II	
(A)	Constant speed	(p)	
(B)	Constant velocity	(q)	
(C)	Increasing speed	(r)	
(D)	Negative acceleration	(s)	

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Answers

1. A → (p, q) B → (q) C → (s) D → (r)

INTEGER TYPE QUESTIONS

The answer to each of the following questions is a single digit integer ranging from 0 to 9. Darken the correct digit.

1. A balloon starts rising above the surface of earth with an acceleration 1.25 m/s^2. After 8 s, a stone is released from the balloon. Calculate the time taken by the stone in seconds to reach the ground.

0	1	2	3	4	5	6	7	8	9

2. A particle is projected vertically upwards from an elevated point. Magnitude of the velocity at height h above the starting point is found to be half of the magnitude of velocity at h height below the starting point. If maximum height reached by the particle above its initial point is mh/n then find $(m - n)$.

0	1	2	3	4	5	6	7	8	9

3. Particle is moving in a straight line. Distance x is related to the time t by the equation, $t = \sqrt{x} + 3$. Distance x is measured in metres and time t in seconds. After how many seconds will the particle come to the rest?

0	1	2	3	4	5	6	7	8	9

4. A particle is moving in a straight line. All the physical quantities are to be measured in MKS system. Square of the magnitude of its instantaneous velocity is found to be ten times its instantaneous displacement. What is the acceleration of the particle?

0	1	2	3	4	5	6	7	8	9

Competition File

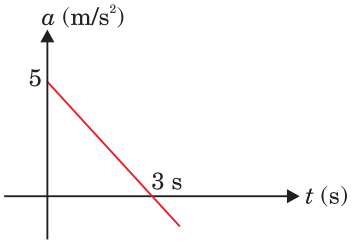
5. What is the velocity of particle in previous question when its displacement is zero?

0	1	2	3	4	5	6	7	8	9

6. A particle is moving in a straight line and its velocity varies with its displacement as $v = \sqrt{4+4s}$ m/s. Assume $s = 0$ at $t = 0$. Find the displacement of the particle in metres at $t = 1$ s.

0	1	2	3	4	5	6	7	8	9

7. A particle is given an initial velocity of 20 m/s. Acceleration of the particle changes with time and its variation is shown in the following figure:



How many seconds the particle will take to acquire its initial velocity again?

0	1	2	3	4	5	6	7	8	9

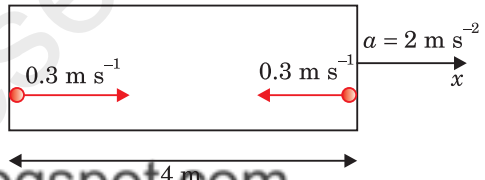
8. A particle is moving in a straight line and relation between time and displacement is $t = \alpha x^2 + \beta x$. If retardation is found to be proportional to the v^n, where v is instantaneous velocity, find the value of n.

0	1	2	3	4	5	6	7	8	9

9. A balloon starts from the state of rest from the ground with constant acceleration g/n. After time T, a stone is dropped from the balloon. If stone takes time T to reach the ground then calculate value of n.

0	1	2	3	4	5	6	7	8	9

10. A rocket is moving in a gravity-free space with a constant acceleration of 2 m s^{-2} along $+x$ direction (see the figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in $+x$ direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in $-x$ direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is



(*J.E.E. Advanced 2014*)

0	1	2	3	4	5	6	7	8	9

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Answers

1. [4] **2.** [2] **3.** [3] **4.** [5] **5.** [0] **6.** [3] **7.** [6] **8.** [3] **9.** [3] **10.** [2]

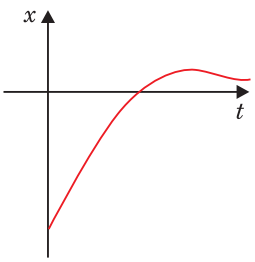
NCERT Exemplar Problems

Objective Questions

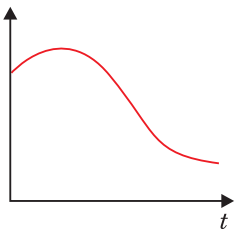
Multiple Choice Questions (Type-I)

3.1. Among the four graphs (as shown below), there is only one graph for which average velocity over the time interval $(0, T)$ can vanish for a suitably chosen T. Which one is it?

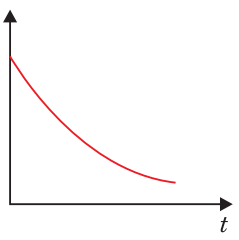
(*a*)



(*b*)



(*c*)



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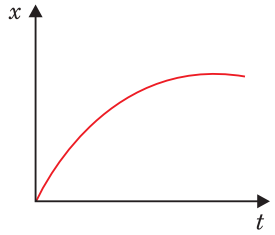
(*d*) 

3.2. A lift is coming from 8th floor and is just about to reach 4th floor. Taking ground floor as origin and positive direction upwards for all quantities, which one of the following is correct?

(*a*) $x < 0, v < 0, a > 0$ (*b*) $x > 0, v < 0, a < 0$

(*c*) $x > 0, v < 0, a > 0$ (*d*) $x > 0, v > 0, a < 0$.

3.3. In one-dimensional motion, instantaneous speed v satisfies $0 \le v < v_0$.

(*a*) The displacement in time T must always take non-negative values.

(*b*) The displacement x in time T satisfies $-v_0T < x < v_0T$.

(*c*) The acceleration is always a non-negative number.

(*d*) The motion has no turning points.

3.4. A vehicle travels half the distance L with speed V_1 and the other half with speed V_2, then its average speed is

(*a*) $\dfrac{V_1 + V_2}{2}$ (*b*) $\dfrac{2V_1 + V_2}{V_1 + V_2}$

(*c*) $\dfrac{2V_1V_2}{V_1 + V_2}$ (*d*) $\dfrac{L(V_1 + V_2)}{V_1V_2}$

3.5. The displacement of a particle is given by $x = (t - 2)^2$ where x is in metres and t in seconds. The distance covered by the particle in first 4 seconds is

(*a*) 4 m (*b*) 8 m

(*c*) 12 m (*d*) 16 m.

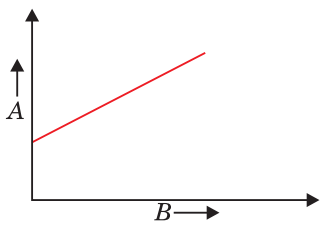
3.6. At a metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on the escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be

(*a*) $(t + t_2)/2$ (*b*) $t_1t_2/(t_2 - t_1)$

(*c*) $t_1t_2/(t_2 + t_1)$ (*d*) $t_1 - t_2$.

Multiple Choice Questions (Type-II)

3.7. The variation of quantity A with quantity B, plotted in the figure, describes the motion of a particle in a straight line.



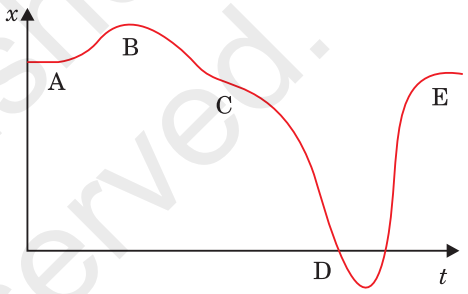
(*a*) Quantity B may represent time.

(*b*) Quantity A is velocity if motion is uniform

(*c*) Quantity A is displacement if motion is uniform.

(*d*) Quantity A is velocity if motion is uniformly accelerated.

3.8. A graph of x versus t is shown in the figure given below. Choose correct alternatives from below.



(*a*) The particle was released from rest at $t = 0$

(*b*) At B, the acceleration $a > 0$.

(*c*) At C, the velocity and the acceleration vanish.

(*d*) Average velocity for the motion between A and D is positive

(*e*) The speed at D exceeds that at E.

3.9. For the one-dimensional motion, described by $x = t - \sin t$

(*a*) $x(t) > 0$ for all $t > 0$

(*b*) $v(t) > 0$ for all $t > 0$

(*c*) $a(t) > 0$ for all $t > 0$

(*d*) $v(t)$ lies between 0 and 2.

3.10. A spring with one end attached to a mass and the other to a rigid support is stretched and released.

(*a*) Magnitude of acceleration, when just released is maximum.

(*b*) Magnitude of acceleration, when at equilibrium position, is maximum.

(*c*) Speed is maximum when mass is at equilibrium position.

(*d*) Magnitude of displacement is always maximum whenever speed is minimum.

3.11. A ball is bouncing elastically with a speed 1 m/s between walls of a railway compartment of size 10 m in a direction perpendicular to walls. The train is moving at a constant velocity of 10 m/s parallel to the direction of motion of the ball. As seen from the ground

(*a*) the direction of motion of the ball changes every 10 seconds

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(*b*) speed of ball changes every 10 seconds

(*c*) average speed of ball over any 20 seconds interval is fixed

(*d*) the acceleration of ball is the same as from the train.

Answers

3.1. (*b*)	**3.2.** (*c*)	**3.3.** (*b*)	**3.4.** (*c*)	**3.5.** (*b*)	**3.6.** (*c*)
3.7. (*a*, *c*, *d*)	**3.8.** (*a*, *c*, *e*)	**3.9.** (*a*, *d*)	**3.10** (*a*, *c*)	**3.11** (*b*, *c*, *d*)	

Hints & Solutions for *Difficult Objective Type Questions*

A. *MCQs with only one correct answer*

A1. (*c*): Acceleration of the particle can be zero as well as non-zero, but when speed of the particle is zero then its velocity is zero for sure at the same instant. Hence option (*c*) is correct.

A2. (*b*): Velocity of the particle can be written as follows:

$$v = \frac{dx}{dt} = 3at^2 + 2bt + c$$

Acceleration of the particle can be written as follows:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 6at + 2b$$

Velocity and acceleration at $t = 0$ can be written as follows:

$$v_0 = c$$

$$a_0 = 2b$$

Hence ratio of initial velocity and initial acceleration can be written as follows:

$v_0/a_0 = \boldsymbol{c/2b}$.

Hence option (*b*) is correct.

A3. (*c*): When the particle moves upwards then its weight as the well as the air resistance both acts downwards; hence the retardation for the upward journey can be calculated as follows:

$$Mg + F = Ma_1$$

$$\Rightarrow \quad a_1 = g + \frac{F}{M} \qquad \ldots(1)$$

When particle moves downward then weight acts downwards but air resistance acts upwards; hence the acceleration during the downward journey can be calculated as follows:

$$Mg - F = Ma_2$$

$$\Rightarrow \quad a_2 = g - \frac{F}{M} \qquad \ldots(2)$$

If h is the maximum height attained by the particle then we can write the following:

$$h = \frac{1}{2}a_1t_1^2 = \frac{1}{2}a_2t_2^2$$

$$\frac{t_2}{t_1} = \sqrt{\frac{a_1}{a_2}}$$

From equations (1) and (2) we can see that $a_1 > a_2$; hence $\boldsymbol{t_2 > t_1}$.

Hence option (*c*) is correct.

A4. (*c*): Displacement and distance are necessarily equal when particle moves in a straight line path without turning back.

Hence option (*c*) is correct.

A5. (*d*): When acceleration of a particle is zero then it moves with constant speed but the speed may remain constant even if the acceleration is non-zero, for example acceleration of the particle moving in a circular path with constant speed is non-zero and variable because the direction of acceleration continuously changes.

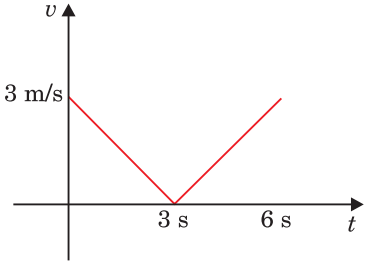
Hence option (*d*) is correct.

A6. (*d*): $v = |t - 3|$

$\Rightarrow \quad v = 3 - t$, when $t < 3$ s

And, $v = t - 3$, when $t > 3$ s.

Graph between velocity and time is shown in figure.



Displacement of the parrot is equal to the area under the graph.

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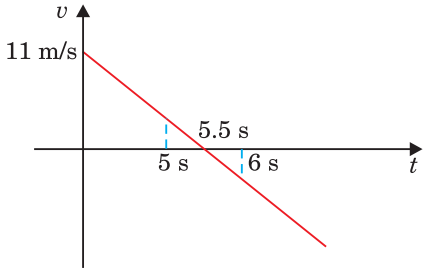
Displacement = $2\left(\frac{1}{2}\times 3\times 3\right)$ m = 9 m

Hence option (*d*) is correct.

A7. (*d*)

A8. (*c*)

A9. (*c*): If we assume north direction as the positive direction then graph between velocity and time is shown in figure.



We can see that velocity of the particle becomes zero at 5.5 s. Sixth second of motion means the time interval between 5 s and 6 s. We can see that the area under the graph for this interval is equally divided above and below the time axis; hence the net displacement in this interval becomes zero. Hence option (*c*) is correct.

A10. (*a*): Let us assume the upward direction as the positive direction and h be the desired height mentioned in the question. We can use the following equation of motion.

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad h = ut - \frac{1}{2}gt^2$$

$$\Rightarrow \quad gt^2 - 2ut + 2h = 0$$

t_1 and t_2 are the two roots of the above equation. We can write sum of the roots as follows:

$$t_1 + t_2 = \frac{2u}{g}$$

$$\Rightarrow \quad u = \frac{\boldsymbol{g(t_1 + t_2)}}{\boldsymbol{2}}$$

Hence option (*a*) is correct.

A11. (*b*): $\frac{dv}{dt} = \alpha t^{3/2}$

$$\Rightarrow \quad dv = \alpha t^{3/2} dt$$

$$\Rightarrow \quad \int_u^v dv = \alpha \int_0^t t^{3/2} dt$$

$$\Rightarrow \quad [v]_u^v = \alpha\left[\frac{2}{5}t^{5/2}\right]_0^t$$

$$\Rightarrow \quad v - u = \frac{2}{5}\alpha[t^{5/2} - 0]$$

$$\Rightarrow \quad v = \boldsymbol{u + \frac{2}{5}\alpha t^{5/2}}$$

Hence option (*b*) is correct.

A12. (*d*): Acceleration is opposite to the velocity; hence body comes to rest at a certain instant and then it reverses it motion. Unless we know the time when particle comes to rest, we cannot decide the relation between s_1 and s_2. Magnitude of initial velocity and acceleration both are required. Hence option (*d*) is correct.

A13. (*a*): Particle comes to rest at the highest point and then it starts falling down due to the gravity. Distance travelled in t seconds can be written as follows:

$$H = \frac{1}{2}gt^2 = \frac{1}{2}g(1)^2 = \boldsymbol{g/2}.$$

Hence option (*a*) is correct.

A14. (*d*): During the motion acceleration remains downward. Hence option (*d*) is correct.

A15. (*a*): Velocity in terms of displacement (x) can be written from the graph as follows:

$$\Rightarrow \quad v = v_0 - \frac{v_0}{x_0}x$$

We can differentiate the above equation to get the acceleration.

$$\Rightarrow \quad a = \frac{dv}{dt} = 0 - \frac{v_0}{x_0}\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad a = -\frac{v_0}{x_0}(v)$$

Substituting v in above equation we get the following:

$$\Rightarrow \quad a = -\frac{v_0}{x_0}\left(v_0 - \frac{v_0}{x_0}x\right)$$

$$\Rightarrow \quad a = \frac{\boldsymbol{v_0^2}}{\boldsymbol{x_0^2}}\boldsymbol{x} - \frac{\boldsymbol{v_0^2}}{\boldsymbol{x_0}}$$

As per above equation, option (*a*) is correct.

A16. (*d*): $\sqrt{x} = t + 5$

$$\Rightarrow \quad x = t^2 + 10t + 25$$

$$\Rightarrow \quad v = \frac{dx}{dt} = 2t + 10$$

$$\Rightarrow \quad a = \frac{dv}{dt} = \boldsymbol{2\ m/s^2}$$

Hence option (*d*) is correct.

A17. (*b*): Displacement in n^{th} second can be written as follows:

$$S_n = \frac{1}{2}a(2n - 1)$$

Hence we can write

$$S_{n+1} = \frac{1}{2}a[2(n+1) - 1] = \frac{1}{2}a(2n + 1)$$

$$\frac{S_n}{S_{n+1}} = \boldsymbol{\frac{2n - 1}{2n + 1}}$$

Hence option (*b*) is correct.

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A18. (a): Distance covered by the particle which is dropped from top of the building in one second.

$$h = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times(1)^2 = 5\text{ m}.$$

Hence particle projected from the bottom must cover $100 - 5 = 95$ m in one second to cross the other particle at this instant. Assuming the upward direction as the positive direction we can use the following equation:

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad 95 = u\,(1) + \frac{1}{2}\,(-10)\,(1)^2$$

$$\Rightarrow \quad u = \mathbf{100\ m/s.}$$

Hence option (a) is correct.

A19. (a): Let speed of the car is u. And it is moving opposite to the buses hence relative speed of approach becomes $30 + u$. Since relative speed is constant hence we can write the time taken as follows:

$$t = \frac{\text{distance}}{\text{relative speed}} = \frac{5}{30+u}$$

Time is given to be 4 min; hence we can write the following:

$$\frac{4}{60} = \frac{5}{30+u}$$

$$\Rightarrow \quad 30 + u = 75$$

$$\Rightarrow \quad u = \mathbf{45\ km/h}$$

Hence option (a) is correct.

A20. (c): $x = (2+t)^{1/2}$

$$\Rightarrow \quad v = \frac{dx}{dt} = \frac{1}{2}(2+t)^{-1/2}$$

$$\Rightarrow \quad a = \frac{d^2x}{dt^2} = \frac{1}{2}\times -\frac{1}{2}(2+t)^{-3/2}$$

$$\Rightarrow \quad a = -\frac{1}{4(2+t)^{3/2}}$$

$$\Rightarrow \quad a = -\frac{\mathbf{1}}{\mathbf{4}x^{\mathbf{3}}}$$

Hence option (c) is correct.

B. *MCQs from Competitive Examinations*

AIPMT/NEET & Other State Boards for Medical Entrance

BA1. (d): $a = 1\text{ m s}^{-2}$,

Bus starts from rest and travelled distance 'S' in time 't'

So, $$S = 0 + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2}t^2 = \frac{1}{2}t^2$$

A man is standing 48 m from bus, he also takes time t to catch the bus with velocity $v = 10$ m/s

$$\Rightarrow \quad (48 + S) = vt$$

$$48 + \frac{1}{2}t^2 = 10\,t$$

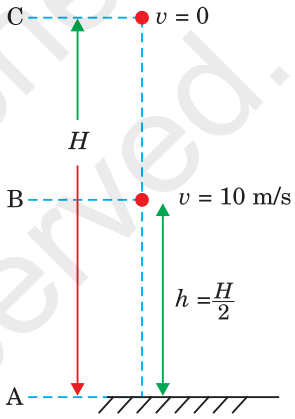
$$t^2 - 20\,t + 96 = 0$$

$$t = 12\text{ seconds}$$

$$t = \mathbf{8\ seconds}$$

The minimum time taken by man is 8 seconds.

BA2. (b): Let a particle gain maximum height H in time t with initial speed is u.



At $h = \frac{H}{2}$, $v = 10$ m/s

Acceleration acting from A to B is $-g$

$$v^2 = u^2 + 2a\ (\text{AB})$$

$$10^2 = u^2 - 2g\left(\frac{H}{2}\right) \qquad \ldots(i)$$

For motion A to C

$$v^2 = u^2 + 2a\ (\text{AC})$$

$$0 = u^2 - 2gH$$

$$u^2 = 2gH \qquad \ldots(ii)$$

From equation (ii) and (i)

$$100 = 2gH - 2g\frac{H}{2}$$

$$gH = 100\text{ m}$$

$$10\,H = 100\text{ m}$$

$$H = \mathbf{10\ m}$$

BA3. (b): Let particle take t sec to reach ground.

$$h = \frac{1}{2}gt^2$$

$$\Rightarrow \quad 19.6 = \frac{1}{2}\times 9.8\times t^2$$

$$\Rightarrow \quad t = 2\text{ s}$$

We have to find distance covered in last sec.

Distance covered in 1 sec can be written as

$$h_1 = \frac{1}{2}gt^2 = \frac{1}{2}\times 9.8\times(1)^2 = 4.9\text{ m}$$

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Hence remaining distance will be covered in last second.

$$x = h - h_1 = 19.6 - 4.9 = 14.7 \text{ m}$$

Option (*b*) is correct.

BA4. (*d*): The relative velocity of bird with respect to train $v = 10 + 5 = 15$ m/s

The distance covered by bird is 120 m with same velocity.

Time taken by bird, $t = \dfrac{120}{15} = \mathbf{8 \text{ seconds}}$

BA5. (*c*): For stone dropped from rest we can write

$$h = \frac{1}{2}gt^2 \qquad ...(i)$$

For stone projected upward we can write

$$h = -ut_1 + \frac{1}{2}gt_1^2 \qquad ...(ii)$$

For stone projected downward we can write

$$h = ut_2 + \frac{1}{2}gt_2^2 \qquad ...(iii)$$

Multiplying equation (*ii*) with t_2 and equation (*iii*) with t_1 and then adding both we get the following:

$$h(t_1 + t_2) = \frac{1}{2}gt_1t_2(t_1 + t_2)$$

$$\Rightarrow \qquad h = \frac{1}{2}gt_1t_2 \qquad ...(iv)$$

Comparing equations (*i*) and (*iv*) we get

$$t = \sqrt{\boldsymbol{t_1\, t_2}}$$

Hence option (*c*) is correct.

BA6. (*a*): Upward motion and downward motion are identical. Hence distance covered during the last '*t*' second is same as distance covered in first '*t*' sec, after reaching maximum height.

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow \qquad h = 0 + \frac{1}{2}gt^2 \Rightarrow h = \boldsymbol{\frac{1}{2}gt^2}$$

Hence option (*a*) is correct.

BA7. (*c*):

$$S = ut + \frac{1}{2}at^2$$

$$40 = 5u + a\frac{(25)}{2} \qquad ...(i)$$

$$S' = 40 + 65 = 105,$$

$$t' = 5 + 5 = 10 \text{ s}$$

$$S' = ut' + \frac{1}{2}a\,t'^2$$

$$105 = 10u + \frac{1}{2}a(100) \qquad ...(ii)$$

From equation $(i) \times 4 - (ii)$

$$160 = 20u + \frac{1}{2}a(100)$$

$$-105 = -10u - \frac{1}{2}a(100)$$

$$55 = 10\,u$$

$$u = \mathbf{5.5 \text{ m/s}}$$

BA8. (*d*): Ball takes time from initial point to final point (where velocity is zero) $t = \dfrac{u}{g}$ and come back at its initial point in same time, total time is $2t$, given condition, at least two balls must be in air.

So, $2t > 4$ seconds

$$\frac{2u}{g} > 4$$

$$u > \mathbf{19.6 \text{ m/s}}$$

BA9. (*b*): Let initial velocity of bullet be u.

After penetrating plank of thickness x

Final velocity of bullet $v = u - \dfrac{u}{20} = \dfrac{19}{20}u$

Let us assume the bullet crossed n number of planks

Total thickness of plank is nx

For a plank, $v^2 = u^2 + 2ax$

$$\left(\frac{19}{20}u\right)^2 = u^2 + 2ax \quad \Rightarrow 2ax = \frac{-39}{400}u^2$$

After penetrating n plank, bullet is stopped

$$\therefore \qquad v'^2 = u^2 + 2a(nx)$$

$$0 = u^2 + (2ax)n$$

$$0 = u^2 + \left(\frac{-39u^2}{400}\right)n$$

$$0 = u^2 - \frac{39u^2}{400}n$$

$$u \neq 0, \qquad n = \frac{400}{39} \approx \mathbf{10.26}$$

So, the bullet will stop at 11^{th} plank.

BA10. (*c*): $x = a\,e^{\alpha t} + be^{\beta t}$

$$v = \frac{dx}{dt} = a\alpha e^{\alpha t} + b\beta e^{\beta t}$$

$$a = \frac{dv}{dt} = a\alpha^2 e^{\alpha t} + b\beta^2 e^{\beta t}$$

We can see that v and a both remain positive all the time and hence velocity of particle goes on increasing with time.

Hence option (*c*) is correct.

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BA11. (c): $u = 10$ m/s, $v = 20$ m/s, $S = 135$ m

$$\Rightarrow \quad \frac{v^2 - u^2}{2S} = a$$

$$\Rightarrow \quad \frac{400 - 100}{2 \times 135} = a$$

$$a = \frac{10}{9} \text{ m/s}^2$$

$$v = u + a\,t$$

$$20 = 10 + \frac{10}{9}t$$

$$t = 9 \text{ seconds}$$

BA12. (b): $a = 4/3$ ms^{-2}, $u = 0$

$t_n = 3$rd second

Distance travelled by particle in 3rd second.

$$S_{3rd} = S_3 - S_2$$

$$= \frac{1}{2}a(3)^2 - \frac{1}{2}a(2)^2$$

$$= \frac{4}{3 \times 2}(5)$$

$$= \frac{10}{3} \text{ m}$$

BA13. (d): $x = 40 + 12t - t^3$

$$\frac{dx}{dt} = 0 + 12 - 3t^2$$

$$v = 12 - 3t^2,$$

when velocity is zero (body at rest)

$$12 - 3t^2 = 0$$

$$t^2 = 4$$

$$t = +2, -2$$

Only positive time is taken, $\therefore t = 2$ s

The distance travelled by particles before coming to rest in time $t = 2$ seconds is

$$x = 40 + 12(2) - 2^3$$

$$= 40 + 24 - 8$$

$$= 56 \text{ m}$$

BA14. (a): Both balls are dropped so, $u_A = 0$, $u_B = 0$

$$S_A = \frac{1}{2}g(t_A)^2, \; S_B = \frac{1}{2}g\,(t_B)^2$$

$$\frac{S_A}{S_B} = \frac{t_A^2}{t_B^2}$$

$$\frac{t_A}{t_B} = \sqrt{\frac{S_A}{S_B}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

BA15. (a): Given, x is the position with respect to time t

$$x = 9t^2 - t^3$$

$$v = \frac{dx}{dt} = 18t - 3t^2$$

$$v = 18t - 3t^2$$

$$\frac{dv}{dt} = 18 - 6t$$

For maxima or minima, $\frac{dv}{dt} = 0$, the time

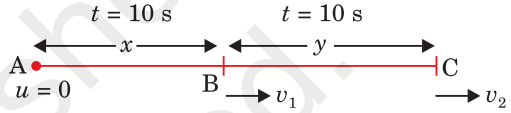
$$18 - 6t = 0$$

$$t = 3 \text{ seconds}$$

The distance covered in time $t = 3$ seconds

$$x = 9(3)^2 - (3)^3 = 54 \text{ m}$$

BA16. (b):



Let acceleration acting in both cases be a

Motion from A to B:

$$x = ut + \frac{1}{2}at^2$$

$$x = 0 + \frac{1}{2}a(10)^2 = 50\,a \qquad \ldots(i)$$

Again $v_1 = u + at$

$$v_1 = 10a$$

Motion from B to C:

$$y = v_1 t + \frac{1}{2}at^2$$

$$= 10\,at + \frac{1}{2}at^2$$

$$= 100\,a + \frac{1}{2}a(100)$$

$$y = 150\,a \qquad \ldots(ii)$$

Divide equation (i) by (ii)

$$\frac{x}{y} = \frac{50\,a}{150\,a}$$

$$y = 3x$$

BA17. (b): Let x be the distance between X and Y.

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\Rightarrow \quad v_{av} = \frac{x + x}{\frac{x}{v_u} + \frac{x}{v_d}}$$

$$\Rightarrow \quad v_{av} = \frac{2v_d\,v_u}{v_d + v_u}$$

BA18. (b): Given acceleration, $f = f_0\left(1 - \frac{t}{T}\right)$

$$\frac{dv}{dt} = f_0\left(1 - \frac{t}{T}\right)$$

$$v = \int f_0\left(1 - \frac{t}{T}\right)dt$$

$$v = f_0\left(t - \frac{t^2}{2T}\right) + C$$

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Given, $t = 0$, $v = 0$

$$0 = f_0(0 - 0) + C \qquad \Rightarrow C = 0$$

$$v = f_0\left(t - \frac{t^2}{2T}\right)$$

Time at which acceleration is zero is given by

$$f_0\left(t - \frac{t^2}{T}\right) = 0$$

$$\Rightarrow \qquad t = T$$

The particle velocity in the time interval $t = 0$ to $t = T$

$$v_x = \int_{t=0}^{t=T} f_0\left(1 - \frac{t}{T}\right) dt$$

$$= \left[f_0\left(t - \frac{t^2}{2T}\right)\right]_0^T$$

$$= f_0\left(T - \frac{T}{2}\right) - f_0(0)$$

$$= f_0\left(\frac{T}{2}\right) - 0$$

$$= \frac{1}{2} f_0(T)$$

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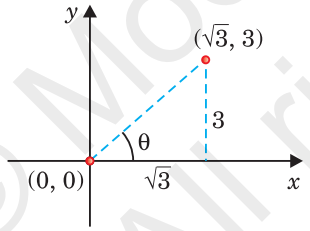
BA19. (*b*): $\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$



$\theta = 60°$

BA20. (*b*): Initial speed $u = 0$

Given constant force, $F = m\,a$

Acceleration acting in the particle, $a = \frac{F}{m}$ = constant

The acceleration acting remains constant on the particle

$$S_1 = 0 + \frac{1}{2}at^2$$

$$S_1 = \frac{1}{2}a(10)^2 = 50\,a$$

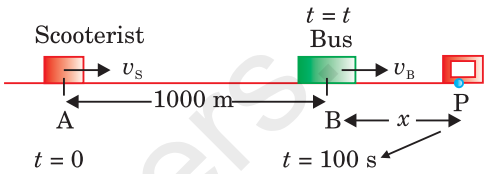
$$S_2 = 0 + \frac{1}{2}a(20)^2 = 200\,a$$

$$\frac{S_1}{S_2} = \frac{50\,a}{200\,a} = \frac{1}{4}$$

$$S_2 = 4\,S_1$$

BA21. (*d*): $v_B = 10$ m/s

Let both reach at point P in time $t = 100$ second. Motion of scooterist from A to P



$$\frac{\text{total distance}}{\text{total time}} = v_s$$

$$\frac{1000 + x}{100} = v_s \qquad \ldots(i)$$

Motion of bus from B to P in same time

$$\frac{\text{total distance}}{\text{total time}} = v_B$$

$$\frac{x}{100} = 10 \text{ m/s}$$

$$x = 1000 \text{ m} \qquad \ldots(ii)$$

From equation (*i*) and (*ii*)

$$\frac{2000}{100} = v_s$$

$$v_s = 20 \text{ m/s}$$

BA22. (*c*): Let a ball takes time t to meet the another ball, the distance covered by ball is S. ($t = 18$ s)

$$S = \frac{1}{2}gt^2 = \frac{1}{2}g(18)^2 \qquad \ldots(i)$$

Again a ball is thrown with velocity v and meet with previous ball, the displacement covered by ball will be same

$$S = vt' + \frac{1}{2}gt'^2$$

$$S = v(t - 6) + \frac{1}{2}g(t - 6)^2$$

Substituting from (*i*)

$$\frac{1}{2}g(18)^2 = v(18 - 6) + \frac{1}{2}g(18 - 6)^2$$

$$\frac{g}{2}(18)^2 - \frac{g}{2}(12)^2 = v \times 12$$

$$v = \frac{g \times 30 \times 6}{2 \times 12}$$

$$= \frac{10 \times 30 \times 6}{2 \times 12}$$

$$v = 75 \text{ m/s}$$

BA23. (*c*): displacement, $x = \frac{1}{(t + 5)}$

Acceleration $\frac{dx}{dt} = \frac{-1}{(t + 5)^2} = -x^2$

$$v = -x^2$$

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$$\frac{dv}{dt} = -2x\frac{dx}{dt}$$

$$a = -2xv$$

$$= -2x(-x^2)$$

$$a = 2x^3$$

$$= 2[(-v)^{1/2}]^3$$

$$a = 2(-v)^{3/2}$$

$$a \propto v^{3/2}$$

BA24. (a): For body A

Distance travelled in 5th second

$$y = \frac{1}{2}a_1(2n-1) = \frac{9}{2}a_1 \quad [\because n = 5 \text{ s}]$$

For body B, the distance travelled in 3rd second $n' = 3$ s

$$y = \frac{1}{2}a_2(2n'-1)$$

$$y = \frac{1}{2}a_2(5)$$

$$\frac{9}{2}a_1 = \frac{5}{2}a_2$$

$$\frac{a_1}{a_2} = \frac{5}{9}$$

BA25. (a): Let height of one storey be h.

Total height = $9h$

A ball is dropped from height $9\,h$ in $t = 3$

$$\therefore \quad 9h = 0 + \frac{1}{2}g(t)^2 \quad [u = 0]$$

$$9h = 5 \times 3 \times 3$$

$$h = 5 \text{ m}$$

Let, in the first second it crosses the n stories of total height nh

$$\therefore \quad nh = 0 + \frac{1}{2}g(1)^2$$

$$nh = 5$$

$$n = 1 \text{ storey}$$

BA26. (d): Let the total distance be x. The car covers $\frac{x}{3}$ in time t_1, with velocity $v_1 = 20$ km/h

$$\therefore \quad t_1 = \frac{x/3}{20} = \frac{x}{60}$$

Again it covers $\frac{2x}{3}$ in time t_2 with velocity $v_2 = 60$ km/h

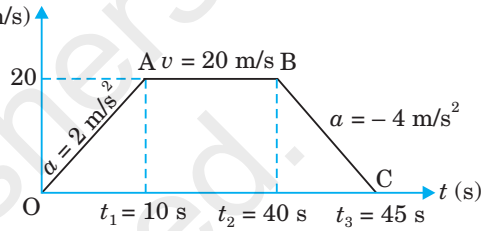
$$\therefore \quad t_2 = \frac{2x}{3 \times 60} = \frac{x}{90}$$

$$\text{Average speed} = \frac{\text{total distance coverd}}{\text{total time}}$$

$$= \frac{\frac{x}{3} + \frac{2x}{3}}{\frac{x}{60} + \frac{x}{90}}$$

$$= \frac{60 \times 90}{(90+60)} = \mathbf{36 \text{ km/h}}$$

BA27. (c):



From O to A, $a = 2$ m/s^2, $t_1 = 10$ s

$$v = u + at_1$$

$$= 0 + 2 \times 10$$

$$= 20 \text{ m/s}$$

From A to B, $v = 20$ m/s till $t_2 = 40$ s. At point C, $v = 0$ and from B to C, $a = -4$ m/s^2

$$\therefore \quad v = u + at_3$$

$$0 = 20 - 4\,t_3$$

$$t_3 = 5 \text{ seconds}$$

Total distance covered by the particle

$$= \text{Area under } v - t \text{ graph}$$

$$= \frac{1}{2} \times 20 \times 10 + 30 \times 20 + \frac{1}{2} \times 20 \times 5$$

$$= 100 + 600 + 50$$

$$= \mathbf{750 \text{ m}}$$

BA28. (d): Maximum height gained by ball is 5 m

$$H = 5 \text{ m}$$

Initial velocity of ball be u

$$\Rightarrow \quad v^2 = u^2 + 2aH$$

$$0 = u^2 - 2gH \quad [v = 0]$$

$$u = \sqrt{2gH} = 10 \text{ m/s}$$

Time taken by ball to gain height H is

$$v = u + at$$

$$10 - 10\,t = 0$$

$$t = 1 \text{ sec.}$$

The time interval for balls between one another is 1 second. So, 60 balls are thrown in one minute.

BA29. (d): $x = at^2 - b$

$$v = \frac{dx}{dt} = 2\,at - 0$$

Acceleration of particle,

$$\frac{dv}{dt} = 2a - 0$$

$$= 2a$$

Since acceleration is constant hence it is never zero.

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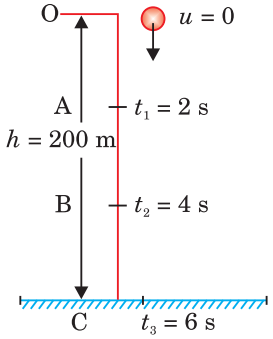
BA30. (*b*): Speed-time graph will be straight line for both upward and downward journey but slope of upward journey will be more.

BA31. (*c*): The distance travelled from O to A is



$$OA = ut + \frac{1}{2}g(2)^2 = \frac{1}{2}g(4) = 2g \qquad [u = 0]$$

From A to B is

$$AB = \frac{1}{2}g(4)^2 - \frac{1}{2}g(2)^2$$

$$= \frac{1}{2}g(12)$$

B to C

$$BC = S_3 - S_2 = \frac{1}{2}g(6)^2 - \frac{1}{2}g(4)^2$$

$$BC = \frac{1}{2}g(36-16)$$

$$BC = \frac{1}{2}g(20)$$

$$OA : AB : BC = \left(\frac{1}{2}g(4)\right) : \frac{1}{2}g(12) : \frac{1}{2}g(20)$$

$$= 4 : 12 : 20$$

$$= \mathbf{1 : 3 : 5}$$

BA32. (*c*): In graphs *a*, *b* and *d* there are two velocities at same instant and it is not possible. Hence only option (*c*) is correct.

BA33. (*c*): A ball is thrown with initial velocity v_0

At maximum height *h*, final velocity must be zero,

$$v^2 = u^2 + 2gh$$

$$0 = v_0^2 - 2gh$$

$$\Rightarrow \quad v_0 = \sqrt{2gh}$$

Now, $h' = 3h$ and initial velocity be v'. Final velocity will be zero, at maximum height h'

$$v^2 = (v')^2 - 2g(3h)$$

$$0 = (v')^2 - 2g(3h)$$

$$v' = \sqrt{2g \times 3h} = \sqrt{3}\ \boldsymbol{v_0}$$

BA34. (*a*): $v_{ins} = \frac{ds}{dt} = \tan\theta$

The point at which slope of (*s*, *t*) graph is maximum, will have maximum instantaneous velocity

Point C is more steeper than A, B and D

BA35. (*a*): Acceleration = + *a*, for time t_1

From initial to maximum velocity.

$$v_{max} = 0 + at_1$$

$$t_1 = \frac{v_{max}}{a}$$

It then decelerates with constant rate *b* and stops in time t_2

$$0 = v_{max} - bt_2$$

$$t_2 = \frac{v_{max}}{b}$$

If the total time is given *t*,

$$t = t_1 + t_2$$

$$v_{max} = \frac{\boldsymbol{abt}}{\boldsymbol{(a+b)}}$$

BA36. (*b*): $h = 20$ m, $u = 0$

ball hit the ground with velocity *v*

$$\therefore \qquad v^2 = u^2 + 2gh$$

$$v^2 = 0 + 2 \times 10 \times 20$$

$$v = \mathbf{20\ m/s}$$

BA37. (*a*): $x = 8 + 12t - t^3$

$$v = \frac{dx}{dt} = 12 - 3t^2$$

$$a = \frac{dv}{dt} = -6t$$

For $v = 0$

$$12 - 3t^2 = 0$$

$$t^2 = 4$$

$$t = +2, -2 \qquad \Rightarrow t = 2\ s$$

The retardation of particle at $t = 2$ s is

$$a = -6 \times 2 = \mathbf{-12\ m/s^2}$$

BA38. (*a*): For height h_1

$$S_5 = h_1 = 0 + \frac{1}{2}g(5)^2 \qquad \ldots(i)$$

$$h_2 = S_{10} - S_5$$

$$h_2 = \frac{1}{2}g(10)^2 - \frac{1}{2}g(5)^2 = \frac{1}{2}g(75) \qquad \ldots(ii)$$

$$h_3 = S_{15} - S_{10} = \frac{1}{2}g(15)^2 - \frac{1}{2}g(10)^2$$

$$= \frac{1}{2}g(125) \qquad \ldots(iii)$$

$$h_1 : h_2 : h_3 = \frac{1}{2}g(25) : \frac{1}{2}g(75) : \frac{1}{2}g(125) = 1 : 3 : 5$$

$$h_1 = \frac{\boldsymbol{h_2}}{\mathbf{3}} = \frac{\boldsymbol{h_3}}{\mathbf{5}}$$

BA39. (*b*): $v(x) = \beta x^{-2n}$

$$\frac{dv(x)}{dt} = \beta(-2n)x^{-2n-1}\frac{dx}{dt}$$

$$= \beta(-2n)x^{-2n-1}(\beta x^{-2n}) \qquad \left[v(x) = \frac{dx}{dt}\right]$$

$$a(x) = \mathbf{-\beta^2(2n)x^{-4n-1}}$$

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BA40. (a): $v = At + Bt^2$

$$\frac{dx}{dt} = At + Bt^2$$

$$\int dx = \int_{t=1}^{t=2} (At + Bt^2)dt$$

$$= A\left(\frac{t^2}{2}\right)_1^2 + B\left(\frac{t^3}{3}\right)_1^2$$

$$= \frac{3A}{2} + \frac{7B}{3}$$

BA41. (b): For the first half let time taken be t_1

$$t_1 = \frac{100\text{ m}}{40\text{ kmh}^{-1}}$$

For the second half let time taken be t_2

$$t_2 = \frac{100}{v}$$

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time}}$$

$$48 = \frac{(200\times10^{-3})\text{ km}}{\frac{10^{-3}\times100}{40\text{ km/h}} + \frac{100\times10^{-3}}{v}}$$

$$48 = \frac{2(40v)}{v+40}$$

$$48v + 40\times48 = 80\,v$$

$$v = \mathbf{60\ km\ h^{-1}}$$

BA42. (c): A body travels

$$x = Kt^2$$

$$v = \frac{dx}{dt} = K(2t)$$

$$a = \frac{dv}{dt} = \mathbf{2K} = \text{Constant}$$

BA43. (b): Time taken for flight

$$T = \frac{2u}{g}$$

$$= \frac{2\times100}{10}$$

$$= 20 \text{ seconds}$$

BA44. (b): Time taken by a boy when walking to his school is t_1

$$\text{where } t_1 = \frac{6}{2.5} \text{ hour.}$$

Again time taken when walking from school to home is t_2

$$\text{where } t_2 = \frac{6}{4} \text{ hour}$$

$$\text{Average speed} = \frac{12\text{ km}}{\frac{6}{2.5}+\frac{6}{4}} = \frac{2\times2.5\times4}{6.5}\text{ m/s}$$

$$= \frac{200}{65} = \frac{40}{13}\text{ km/h}$$

BA45. (b): For mass m_1, time taken to fall from height h is

$$t_1 = \sqrt{\frac{2h}{g}}$$

For mass m_2

$$t_2 = \sqrt{\frac{2h}{g}}$$

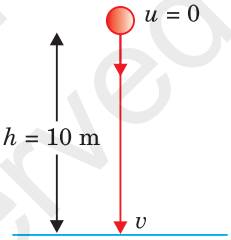
$$t_1 = \boldsymbol{t_2}$$

BA46. (b): $h = 10$ m

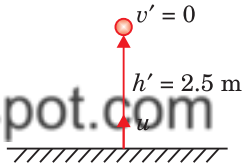
$$v^2 = 0 + 2g(10)$$

$$v^2 = 196 \text{ m/s}$$

$$v = 14 \text{ m/s (downwards)}$$



$h' = 2.5$ m, let the velocity with which the ball rebounds be u. At maximum height $v' = 0$



$$0 = u^2 - 2g(2.5)$$

$$u^2 = 5g$$

$$u = \sqrt{5g} = 7 \text{ m/s (upwards)}$$

During the collision of ball with the ground in time

$$t = 0.01 \text{ s}$$

$$a = \frac{dv}{dt} = \frac{u-(-v)}{t} = \frac{7+14}{0.01} = \mathbf{2100\ m/s^2}$$

BA47. (c): $u = 0$, $v = 72$ km/h, $t = 10$ sec

$$v = \frac{72000}{60\times60} = 20 \text{ m/s}$$

$$v = u + at = 20 \text{ m/s}$$

$$20 = 0 + a(10)$$

$$a = \mathbf{2\ m/s^2}$$

BA48. (d): $u = 40$ km/h, $S = 15$ m $= \frac{100}{9}$ m/s

$$v^2 = u^2 + 2aS$$

$$0 = \left(\frac{100}{9}\right)^2 + 2\times a\times15$$

$$30\,a = -\left(\frac{100}{9}\right)^2 \Rightarrow a = -\frac{100\times100}{30\times81} = \frac{-10^4}{2430}\text{ m/s}^2$$

Now, $v = 80$ km/h $= \frac{200}{9}$ m/s

$$v = 0,\ (v)^2 = u^2 + 2aS'$$

$$(v)^2 = u^2 + 2aS'$$

$$0 = \left(\frac{200}{9}\right)^2 - 2\times\frac{10^4}{2430}\,S'$$

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$$\frac{2\times10^4}{2430}\,S' = \frac{200\times200}{9\times9}$$

$$S' = \mathbf{60\ m}$$

BA49. (d): Average velocity, $\vec{v}_{av} = \dfrac{\text{total displacement}\,(\vec{s})}{\text{total time taken}\,(t)}$

Initial position, $\vec{r}_1 = 2\hat{i} + 3\hat{j}$

Final position, $\vec{r}_2 = 13\hat{i} + 14\hat{j}$

Displacement, $\vec{s} = \vec{r}_2 - \vec{r}_1$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

$$\vec{v}_{av} = \frac{(13\hat{i} + 14\hat{j}) - (2\hat{i} + 3\hat{j})}{5 - 0}$$

$$\vec{v}_{av} = \frac{11\hat{i} + 11\hat{j}}{5}$$

$$\vec{v}_{av} = \mathbf{\frac{11}{5}(\hat{i} + \hat{j})}$$

BA50. (c): Let the total length of the path covered by the escalator be d.

Let the speed of the girl when escalator is stationary be $v_1 = \dfrac{d}{t_1}$

Let the speed of escalator be $v_2 = \dfrac{d}{t_2}$

Let the time taken by the girl to reach the metro station while walking on the moving escalator be t.

Velocity of the girl on the moving escalator, $v = \dfrac{d}{t}$

$$v = v_1 + v_2$$

$$\frac{d}{t} = \frac{d}{t_1} + \frac{d}{t_2}$$

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} \quad \Rightarrow t = \mathbf{\frac{t_1 t_2}{t_1 + t_2}}$$

BA51. (c): The component of velocity along X-axis,

$$v_x = \frac{dx}{dt} = 5 - 4t \text{ m/s}$$

The component of velocity along Y-axis,

$$v_x = \frac{dy}{dt} = 10 \text{ m/s}$$

Acceleration of the particle along the X-axis,

$$a_x = \frac{dv_x}{dt} = -4 \text{ m/s}^2$$

Acceleration of the particle along the Y-axis,

$$a_y = \frac{dv_y}{dt} = 0$$

The net acceleration of the particle is $\mathbf{-4\ m/s^2}$ and it remains same.

BA52. (c): Let total distance be $2x$. Particle moves distance x with speed v_1 and remaining x with v_2. Let t_1 and t_2 be the time taken by the particle to cover corresponding distances.

As $t_1 = \dfrac{x}{v_1}$ and $t_2 = \dfrac{x}{v_2}$

$$\therefore \quad v = \frac{x + x}{t_1 + t_2} = \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}} = \frac{2v_1 v_2}{v_1 + v_2}$$

$$\therefore \quad = \frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$$

Hence option (c) is correct.

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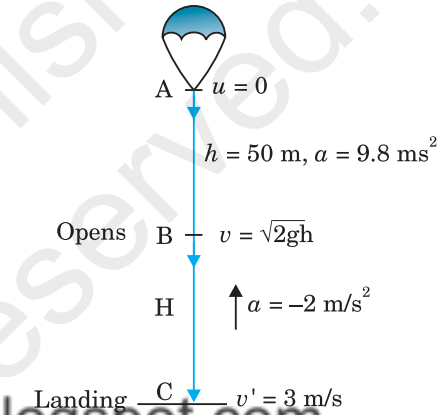
BB1. (c): Taking $a = g = 9.8$ m/s^2



For motion along A to B

$$v^2 = u^2 + 2gh \ (u = 0)$$

$$v = \sqrt{2gh} = \sqrt{2\times9.8\times50}$$

$$v = \sqrt{980} \text{ m/s}$$

Now motion along B to C

$$v'^2 = v^2 + 2aH$$

$$H = \left(\frac{v'^2 - v^2}{2a}\right) = \left(\frac{9 - 980}{2\times(-2)}\right)$$

$$= \frac{971}{4} = 242.75 \text{ m}$$

Total height $= h + H = 50 + 242.75$

$= 292.75 \approx \mathbf{293\ m}$

BB2. (d): Let t_A and t_B are times taken by cars A and B to reach finish point.

$$t_A = (t_B - t)$$

$$s = \frac{1}{2}a_1 t_A^2 \text{ and } s = \frac{1}{2}a_2 t_B^2$$

$$v_A = \sqrt{2sa_1},\ v_B = \sqrt{2sa_2}$$

Difference in velocity,

$$v_A - v_B = (\sqrt{2sa_1} - \sqrt{2sa_2}) = v \qquad ...(i)$$

Difference in time.

$$(t_A - t_B) = \left(\sqrt{\frac{2s}{a_1}} - \sqrt{\frac{2s}{a_2}}\right) = t \qquad ...(ii)$$

Solve equation (i) and (ii) we get

$$v = \mathbf{t\sqrt{a_1 a_2}}$$

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BB3. (d): If a is the retardation then

$$v^2 - u^2 = -2aS$$

To stop after travelling a distance S we can write the following ($v = 0$)

$$2aS = u^2$$

$$\Rightarrow \frac{S_1}{S_2} = \frac{u_1^2}{u_2^2} \Rightarrow S_2 = \frac{u_2^2}{u_1^2}\ (40) = \left(\frac{80}{40}\right)^2 (40\ \text{m})$$

$$= \mathbf{160\ m}$$

Hence option (d) is correct.

BB4. (d): Options (a), (b) and (c) represent the graphs for a situation when velocity of the particle first decreases and then becomes zero for a moment and then it increases in the opposite direction. But in the graph (d) we can see that speed is first increasing and then decreasing; hence this is not consistent with others. So option (d) is incorrect.

BB5. (a): $v = v_0 + gt + ft^2$

$$\frac{dx}{dt} = v_0 + gt + ft^2$$

$$\int_0^x dx = \int_{t=0}^{t=1} \left(v_0 + gt + ft^2\right) dt$$

$$x = \left[v_0 t + g\left(\frac{t^2}{2}\right) + f\left(\frac{t^3}{3}\right)\right]_0^1$$

$$x = \left(\boldsymbol{v_0} + \frac{\boldsymbol{g}}{\boldsymbol{2}} + \frac{\boldsymbol{f}}{\boldsymbol{3}}\right)$$

BB6. (a): $\frac{dv}{dt} = -2.5\sqrt{v}$

$$\int_{v=6.25}^{v=0} \frac{dv}{\sqrt{v}} = \int_{t=0}^{t} -2.5\, dt$$

$$\left[2v^{1/2}\right]_{6.25}^{0} = -2.5\, t$$

$$-2\sqrt{6.25} = -2.5\, t$$

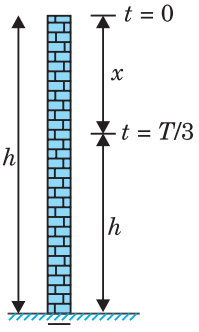
$$2(2.5) = 2.5\, t \qquad \Rightarrow t = \mathbf{2\ s}$$

BB7. (c): Given that the time taken by the ball to reach the ground from a tower of height h, $t = T$

$$h = \frac{1}{2} g\, T^2$$

$$\frac{1}{2} g\, T^2 = h$$

Let x be the distance covered by the ball from $t = 0$ to $t = T/3$ s



$$x = 0 + \frac{1}{2} g \left(\frac{T}{3}\right)^2 = \frac{1}{9}\left(\frac{1}{2} g T^2\right)$$

$$x = \frac{h}{9}$$

height of the point reached by the ball from ground:

$$h' = h - x = h - \frac{h}{9} = \frac{\boldsymbol{8h}}{\boldsymbol{9}}$$

BB8. (d): $F = \frac{R}{t^2} v(t)$

$$m\frac{dv}{dt} = \frac{R}{t^2} v(t)$$

$$\int \frac{dv}{v} = \frac{R}{m} \int \frac{1}{t^2} dt$$

$$ln\,(v) = \frac{-R}{m}\left(\frac{1}{t}\right) + c$$

$$2.303 \log\left[v(t)\right] = \frac{-R}{m}\left(\frac{1}{t}\right) + c$$

The best way is to draw a plot between $\log[v(t)]$ against $\left(\frac{1}{t}\right)$.

BB9. (c): Given that the height of the tower is H and the particle is thrown vertically upwards with a speed u.

Let the time taken by the particle to reach the highest point from the tower be t.

At the highest point the velocity of the particle becomes zero.

$$0 = -u + gt \Rightarrow t = \frac{u}{g} \qquad \ldots(i)$$

Then the time taken by the particle to hit the ground will be, $t_1 = nt$.

From equation (i)

$$t_1 = \frac{nu}{g} \qquad \ldots(ii)$$

$$H = -ut_1 + \frac{1}{2}\, gt_1^2$$

From equation (ii)

$$H = -un \times \frac{u}{g} + \frac{1}{2} gn^2 \left(\frac{u}{g}\right)^2$$

$$H = -\frac{u^2 n}{g} + \frac{n^2 u^2}{2g}$$

$$2gH = -2u^2 n + n^2 u^2$$

$$2gH = \boldsymbol{u^2 n(n-2)}$$

BB10. (c): Let the total length of the path covered by the escalator be d metres.

Speed of the person climbing up stalled escalator,

$$v_1 = \frac{d}{60}\ \text{m/s}$$

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Speed of the escalator, $v_2 = \frac{d}{40}$ m/s

Let the time taken by the person to walk up the moving escalator be t seconds

Thus, the velocity of the person will be, $v = \frac{d}{t}$ m/s

$$v = v_1 + v_2$$

$$\frac{d}{t} = \frac{d}{60} + \frac{d}{40}$$

$$\frac{1}{t} = \frac{2+3}{120}$$

$$\Rightarrow \quad t = \frac{120}{5} = \mathbf{24\ s}$$

BB11. (a): Distance travelled by car in 15 sec $= \frac{1}{2}(45)(15)$

$= \frac{675}{2}$ m $= 337.5$ m,

Distance travelled by scooter in 15 seconds $= 30 \times 15 = 450$

Difference of the distance travelled in 15s is $= 450 - 337.5 = 112.5$ m

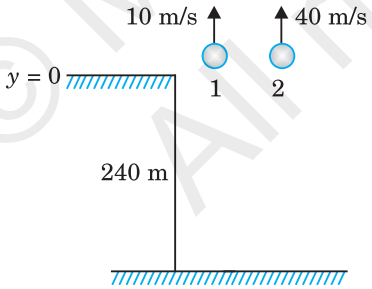
Let car catches scooter in time t: $\frac{675}{2} + 45(t-15) = 30t$

$337.5 + 45t - 675 = 30t \Rightarrow 15t = 337.5 \Rightarrow t = \mathbf{22.5\ sec}$

Hence option (a) is correct.

BB12. (a): Initial speed of stone-1, $u_1 = 10$ m/s

Initial speed of stone-2, $u_2 = 40$ m/s



Let us first calculate the time taken by each stone to reach to the ground.

Time taken by stone-1 to reach the ground

$$-240 = 10t - \frac{1}{2} \times 10t^2$$

$$5t^2 - 10t - 240 = 0 \Rightarrow t^2 - 2t - 48 = 0$$

$$t = 8, -6$$

Thus, $t = 8$ s

Time taken by stone-2 to reach the ground

$$-240 = 40t - \frac{1}{2} \times 10t^2$$

$$5t^2 - 40t - 240 = 0 \Rightarrow t^2 - 8t - 48 = 0$$

$$t = 12, -4$$

Thus, $t = 12$ s

For time, $0 \le t \le 8$ s

Let y_1 be the instantaneous position of the stone-1 at time t:

$$y_1 = 10t - 5t^2$$

Let y_2 be the instantaneous position of the stone-2 at time t, then its displacement can be written as:

$$y_2 = 40t - 5t^2$$

Thus, the difference of the positions is given by:

$$y_2 - y_1 = 30t$$

For time $8\ \text{s} \le t \le 12$ s

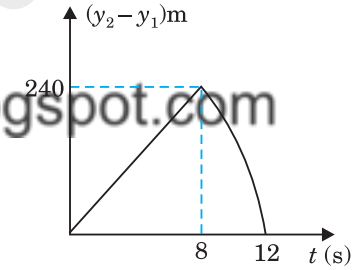
$$y_1 = -240$$

$$y_2 = 40t - 5t^2$$

Thus, the difference of the positions is given by:

$$y_2 - y_1 = -5t^2 + 40t + 240$$

Hence, the correct schematic diagram is as given in option (a).



BB13. (b): Let the mass of the body be m.

Then the acceleration of the body due to viscous force can be expressed as:

$$a_1 = \frac{F}{m} = -\frac{kv}{m}$$

Net acceleration of the body is given by:

$$a = g - \frac{kv}{m}$$

As we know that the acceleration and the velocity are related as:

$$a = \frac{dv}{dt}$$

Or,

$$\frac{dv}{dt} = g - \frac{kv}{m}$$

$$\frac{dv}{g - \frac{kv}{m}} = dt$$

Integrating both sides

$$\ln\left(\frac{g - \frac{kv}{m}}{g}\right) = -\frac{k}{m}t$$

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Let $\frac{k}{m} = \alpha$

$$\ln\left(\frac{g-\alpha v}{g}\right) = -\alpha t$$

$$\left(\frac{g-\alpha v}{g}\right) = e^{-\alpha t}$$

$$\Rightarrow \quad g - \alpha v = ge^{-\alpha t}$$

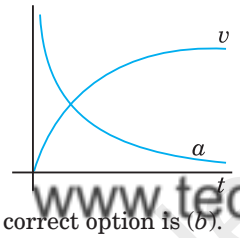
$$\Rightarrow \quad v = \frac{g}{\alpha}(1 - e^{-\alpha t}) \qquad ...(i)$$

$$a = \frac{dv}{dt}$$

$$\Rightarrow \quad a = \frac{g}{\alpha}(\alpha e^{-\alpha t})$$

$$\Rightarrow \quad a = ge^{-\alpha t} \qquad ...(ii)$$

From equations (i) and (ii) we can conclude that the appropriate graph is as follows:



Hence, the correct option is (b).

BB14. (c): $u = 20 \text{ m s}^{-1}$

$S = 40 \text{ m}$

Let the acceleration of the car be a

$$v^2 - u^2 = 2aS$$

$$0 - 20^2 = 2a \times 40$$

$$\Rightarrow \quad a = -5 \text{ m s}^{-2}$$

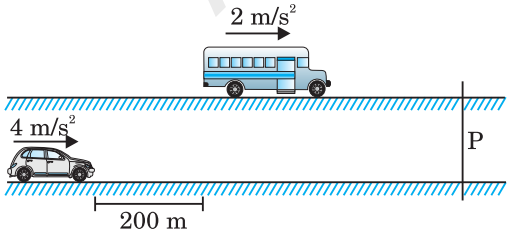
If the initial velocity is doubled, then let the distance travelled by the car before stopping be s.

$$v^2 - u^2 = 2as$$

$$0 - 40^2 = 2(-5)s$$

$$\Rightarrow \quad s = \mathbf{160 \text{ m}}$$

BB15. (c):



Let the bus and the car meet at P.

Let the bus and the car meet after time t.

The distance travelled by the car is given by:

$$S_1 = \frac{1}{2} \times 4 \times t^2 = 2t^2$$

The distance travelled by the bus is given by:

$$S_2 = \frac{1}{2} \times 2 \times t^2 = t^2$$

$$S_1 - S_2 = 2t^2 - t^2 = t^2$$

$$t^2 = 200$$

$$t = \sqrt{200} = \mathbf{10\sqrt{2}\ s}$$

BB16. (c): As it is given that the acceleration is constant and in negative direction velocity is in positive direction.

The acceleration and velocity are related as:

$$a = \frac{dv}{dt} \qquad ...(i)$$

Taking acceleration to be equal to a negative constant $-K$

$$-K = \frac{dv}{dt}$$

$$dv = -Kdt$$

Integrating both sides

$$\int dv = -K\int dt$$

$$v = -Kt + C'$$

None of the graphs given in the options satisfy the above derived relation between the velocity and the time.

Multiplying dx in numerator and denominator in equation (i)

$$a = \frac{dv}{dt} \times \frac{dx}{dx} = \frac{dv}{dx} \times \frac{dx}{dt} = v\frac{dv}{dx}$$

$$v\frac{dv}{dx} = -K$$

$$vdv = -Kdx$$

Integrating both sides

$$\int vdv = \int -Kdx$$

$$\frac{v^2}{2} = -Kx + C$$

$$\Rightarrow \quad x = \frac{C}{K} - \frac{\boldsymbol{v^2}}{\boldsymbol{2K}}$$

The graph given in option (c) is in accordance with the above derived relation.

BB17. (c): As the velocity is thrown vertically upwards, we know that a constant acceleration g is acting in the downward direction.

Let the initial velocity of the body be u, thus the instantaneous value of the velocity is given by:

$$v = u - gt$$

The graph corresponding to the above derived result is as given in option (c).

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BC1. (b): Here we can see that acceleration remains positive for the first 11 s from the start; hence its speed will keep increasing. Change in speed during 11 s is equal to area under the acceleration–time graph and will

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also be equal to maximum speed acquired by the particle. Note that particle starts from rest.

$$v_{max} = \frac{1}{2} \times 10 \times 11 = 55 \text{ m/s}$$

Hence option (b) is correct.

BC2. (c): Acceleration of the block will be constant and let it be a. Block starts from rest.

$$S_n = \frac{1}{2}an^2 - \frac{1}{2}a(n-1)^2 = \frac{a}{2}(2n-1)$$

Substituting $n + 1$ in place of n we can write the following:

$$S_{n+1} = \frac{a}{2}(2n+2-1) = \frac{a}{2}(2n+1)$$

$$\frac{S_n}{S_{n+1}} = \frac{2n-1}{2n+1}$$

Hence option (c) is correct.

BC3. (b): Kinetic energy can be written as follows:

$$K = \frac{1}{2}mu^2 = \frac{1}{2}mg^2t^2$$

At the time of collision kinetic energy of the ball first decreases and then increases. Hence option (b) is correct.

BC4. (a): Total mass of the object = Volume × Density

$$M = \rho V$$

Differentiating both sides

$$\frac{dM}{dt} = \rho\frac{dV}{dt} + V\frac{d\rho}{dt}$$

As the mass remains constant

$$\rho\frac{dV}{dt} + V\frac{d\rho}{dt} = 0$$

$$\rho\frac{dV}{dt} = -V\frac{d\rho}{dt}$$

$$\frac{1}{V}\frac{dV}{dt} = -\frac{1}{\rho}\frac{d\rho}{dt}$$

As it is given in the question that $\frac{1}{\rho}\frac{d\rho}{dt}$ is a constant, let it be equal to K.

$$\frac{1}{V}\frac{dV}{dt} = -K$$

$$\frac{dV}{dt} = -KV \qquad \ldots(i)$$

The instantaneous volume of the sphere is given by

$$V = \frac{4}{3}\pi R^3$$

$$\frac{dV}{dt} = 4\pi R^2\frac{dR}{dt}$$

From equation (i)

$$4\pi R^2\frac{dR}{dt} = -KV$$

$$4\pi R^2\frac{dR}{dt} = -K\left(\frac{4}{3}\pi R^3\right)$$

$$\frac{dR}{dt} = -\frac{K}{3}R \qquad \ldots(ii)$$

The velocity of any point on the sphere will be

$$v = \frac{dR}{dt}$$

Thus, $v = -\frac{K}{3}R$

Hence, the correct option is (a)

C. *MCQs with more than one correct answer*

C1. (b, c): $x = u(t-4) + a(t-3)^2$

$$\Rightarrow \quad v = \frac{dx}{dt} = u + 2a(t-3)$$

$$\Rightarrow \quad a = \frac{dv}{dt} = 2a$$

We can see that options (b) and (c) are correct.

C2. (a, b, c)

C3. (a, d)

C4. (a, b, c, d): Initially velocity is negative and then after time T it becomes positive, so the particle has reversed its direction of motion. Option (a) is correct.

Slope of the v-t graph is acceleration and we can see that the slope of given graph is constant; hence the acceleration of particle remains constant. Option (b) is correct.

Area below and above the time axis are equal in given time interval hence the net displacement becomes zero. Option (c) is correct.

Speed of the particle after time $2T$ is same as speed of particle $t = 0$. Option (d) is correct.

C5. (a, b, c)

C6. (a, b, c, d): When velocity and acceleration are in the same direction then magnitude of velocity increases. When velocity and acceleration are along the opposite directions then magnitude of velocity decreases.

C7. (a, b, d): Options (a) and (b) can be understood.

When speed of the particle remains zero then it means particle is at rest in a time interval; hence velocity also remains zero and hence acceleration remains zero during that interval of time.

C8. (b)

C9. (a, b, d):

$$x = at^2 - bt^3$$

$$\Rightarrow \quad v = \frac{dx}{dt} = 2at - 3bt^2$$

$$\Rightarrow \quad a = \frac{dv}{dt} = 2a - 6bt$$

$x = 0 \Rightarrow$ the $t^2(a - bt) = 0 \Rightarrow t = 0$ and $t = a/b$

Hence the particle returns to origin after time $t = a/b$.

$v = 0 \Rightarrow 2at - 3bt^2 = 0 \Rightarrow t(2a - 3bt) = 0 \Rightarrow t = 0$ and $t = 2a/3b$

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Hence the particle comes to rest after time $2a/3b$.

$a = 0 \Rightarrow 2a - 6bt = 0 \Rightarrow t = a/3b$

Hence acceleration of the particle becomes zero after time $t = a/3b$.

C10. (a, d)

C11. (c): Note that the particle is moving in a straight line and to make average velocity zero in an interval of time, particle must return to the initial point of motion. Return to the initial point along the same straight line is possible only when the particle reverses its motion. To reverse the motion, particle must come to the rest at least for an instant. And it is not possible without acceleration. Hence only option (c) is correct.

C12. (a, b, d): If particle is projected with such an speed that time taken to reach top point remains greater than 1 s, then distance covered during the last second of upward journey is same as distance travelled during first second of its downward journey. Hence it is independent of initial velocity of projection but time taken to reach the top point must be greater than 1 s. Only option (c) is not possible. Hence options (a), (b) and (d) are correct.

C13. (a, b):

Given, $x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + 7$

$\Rightarrow \quad v = \frac{dx}{dt} = t^2 - 5t + 6 \quad \ldots(1)$

$\Rightarrow \quad a = \frac{dv}{dt} = 2t - 5 \quad \ldots(2)$

For particle to be in a state of rest we can write the following:

$v = 0 \Rightarrow t^2 - 5t + 6 = 0 \Rightarrow (t-3)(t-2) = 0$

$\Rightarrow \quad t = 2 \text{ s}, 3 \text{ s}$

Hence there are two instants when particle is at rest and corresponding acceleration at these instants can be calculated by substituting in equation (2).

For $t = 2$ s $\Rightarrow a = 2 \times 2 - 5 = -1$ m/s^2

For $t = 3$ s $\Rightarrow a = 2 \times 3 - 5 = 1$ m/s^2

Hence we can see that both options (a) and (b) both are possible.

D. *MCQs based on passage / comprehension*

Passage I

D1 (d) **D2** (a) **D3** (c)

$$\frac{dv}{dt} = -av$$

$$\Rightarrow \quad \frac{dv}{v} = -\alpha dt$$

$$\Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v} = -\alpha \int_0^t dt$$

$$\Rightarrow \quad [\ln v]_{v_0}^{v} = -\alpha t$$

$$\Rightarrow \quad \ln \frac{v}{v_0} = -\alpha t$$

$$\Rightarrow \quad v = v_0 e^{-\alpha t} \quad \ldots(i)$$

Further we can proceed as follows:

$$\Rightarrow \quad \frac{dx}{dt} = v_0 e^{-\alpha t}$$

$$\Rightarrow \quad dx = v_0 e^{-\alpha t} dt$$

$$\Rightarrow \quad \int_0^x dx = v_0 \int_0^t e^{-\alpha t} dt$$

$$\Rightarrow \quad x = v_0 \left[\frac{e^{-\alpha t}}{-\alpha} \right]_0^t$$

$$\Rightarrow \quad x = \frac{v_0}{\alpha}[1 - e^{-\alpha t}] \quad \ldots(ii)$$

We can calculate time taken by the particle when its speed becomes half using equation (i).

$$v = v_0 e^{-\alpha t}$$

$$\Rightarrow \quad \frac{v_0}{2} = v_0 e^{-\alpha t} \Rightarrow e^{\alpha t} = 2$$

$$\Rightarrow \quad \alpha t = \ln 2$$

$$\Rightarrow \quad t = \frac{\ln 2}{\alpha}$$

Substituting t = infinity in equation (ii), we get $x = \boldsymbol{v_0/\alpha}$

Passage II

D4 (d) **D5** (c) **D6** (d)

Particle is projected from point A with an initial velocity u = 20 m/s in upward direction, and time taken by the particle to return to same point is given by:

$$t = \frac{2u}{g} = \frac{2 \times 20}{10} = 4 \text{ s}.$$

Maximum height reached by particle from top of the tower can be written as follows:

$$h = \frac{u^2}{2g} = \frac{20 \times 20}{2 \times 10} = 20 \text{ m}.$$

Height of the tower is 20 m hence maximum height attained by the particle above the ground is 40 m.

Time to reach the bottom of tower can be calculated using following relation:

$$S = ut + \frac{1}{2} at^2$$

Assuming the upward direction as the positive direction we can substitute the values in above equation.

$$-20 = 20t + \frac{1}{2}(-10)t^2$$

$$\Rightarrow \quad t^2 - 4t - 4 = 0$$

$$\Rightarrow \quad t = \frac{4 \pm \sqrt{16+16}}{2} = \frac{4 \pm 4\sqrt{2}}{2} = 2 \pm 2\sqrt{2}$$

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Negative sign will produce negative result and is not acceptable; hence using the positive sign we can write the time taken by the particle to reach the bottom as follows:

$$t = (2 + 2\sqrt{2})\ \text{s}$$

Passage III

D7 (*a*) **D8** (*d*) **D9** (*c*)

$$x = 5t^2 - 20$$

X-coordinate of the particle becomes zero when it crosses the origin.

$$x = 5t^2 - 20 = 0 \Rightarrow t = 2\ \text{s}$$

Hence option (*a*) is correct for D7.

Velocity of the particle can be written as follows:

$$v = \frac{dx}{dt} = 10t$$

We can see that velocity of the particle is always positive hence it never reverses its direction of motion.

Hence option (*d*) is correct for D8.

Acceleration of particle can be written as follows:

$$a = \frac{dv}{dt} = 10$$

For magnitude of velocity to become equal to that of acceleration, we can write the following:

$$10t = 10 \Rightarrow t = 1\ \text{s}.$$

Hence option (*c*) is correct for D9.

ASSERTION REASON TYPE

1. (*d*): Velocity is a vector quantity; hence it can change due to the change in magnitude as well as the direction. In uniform circular motion the particle accelerates but its speed remains constant.

2. (*a*): While ascending or descending, particle covers the same distance. Acceleration during the downward motion ($g - F/m$) is less than retardation during upward motion ($g + F/m$). Hence using $h = \frac{1}{2} gt^2$ we can understand that time of ascent is less than the time of decent.

4. (*e*): If speed is never zero in an interval of time then the particle must have covered a certain distance; hence its average speed cannot be zero. Similarly when the particle is moving in a circular path then distance covered by the particle in any interval is not zero hence the average speed cannot be zero.

9. (*b*): Both the statements are true but statement 2 is not the correct explanation to the statement 1. Relative velocity of near stationary objects and as well as far stationary objects is same with respect to the moving train but still brain perceives far objects nearly stationary.

When some object moves in front of our eyes then observed displacement of object is proportional to the displacement of image of object formed by eye lens on the retina. When object at a very large distance moves in front of eyes then displacement of corresponding image on retina remains proportionally very small and hence the objects appear stationary.

INTEGER TYPE

1. [4]: Velocity attained by the balloon in 8 s: $v = u + at$

$\Rightarrow v = 0 + 1.25 \times 8 = 10$ m/s.

Above calculated velocity will be the initial velocity of the stone.

Displacement of balloon in 8 s:

$$S = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times 1.25 \times (8)^2 = 40\ \text{m}$$

Now we can say that stone has initial velocity 10 m/s directed upward and final displacement to reach the ground is 40 m downward. Acceleration due to gravity is always downward.

Let us assume downward direction to be positive and using the following equation:

$$S = ut + \frac{1}{2} at^2$$

$$\Rightarrow \quad 40 = -10t + (1/2) \times 10 \times t^2$$

$$\Rightarrow \quad 5t^2 - 10t - 40 = 0$$

$$\Rightarrow \quad t^2 - 2t - 8 = 0$$

$$\Rightarrow \quad t = \frac{2 \pm \sqrt{4 + 32}}{2} = \frac{2 \pm 6}{2}$$

Only positive sign will provide correct answer.

$\Rightarrow t =$ **4 s.**

Hence answer is 4.

2. [2]: Let u be the magnitude of initial velocity which is directed upwards. Then as per given information, we can write the following:

$$\sqrt{u^2 - 2gh} = \frac{1}{2}\sqrt{u^2 + 2gh}$$

$$\Rightarrow \quad 4u^2 - 8gh = u^2 + 2gh$$

$$\Rightarrow \quad u^2 = \frac{10gh}{3}$$

Maximum height attained by the body can be written as follows:

$$H = \frac{u^2}{2g} = \frac{5h}{3}.$$

Comparing it with the given result we get $m = 5$ and $n = 3$; hence value of $m - n = 2$.

Hence answer is **2.**

3. [3]: Given equation can be written as follows:

$$\sqrt{x} = t - 3$$

$$\Rightarrow \quad x = t^2 - 6t + 9$$

Velocity can be written as follows:

$$v = 2t - 6$$

$$v = 0 \Rightarrow t = 3\ \text{s}.$$

Hence answer is **3.**

4. [5]: Let s be the displacement and v the velocity then we can write the following:

$$v^2 = 10s$$

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We can understand that above equation is true for constant acceleration and we can compare it with standard equation $v^2 = u^2 + 2as$ then we get to know the following:

Initial velocity $u = 0$,

And $2a = 10 \Rightarrow a = 5$ m/s^2.

Hence answer is **5.**

5. [0]

6. [3]: $v = \sqrt{4 + 4s}$

Squaring the above equation we get the following:

$$v^2 = 4 + 4s$$

We can see that the above equation is same as that for constant acceleration. We can compare the above equation with $v^2 = u^2 + 2as$ to get the following:

Initial velocity: $u = 2$ m/s,

Acceleration: $a = 2$ m/s^2.

Displacement in time $t = 1$ can be calculated with the following equation:

$$S = ut + \frac{1}{2} at^2$$

$$\Rightarrow S = 2 \times 1 + \frac{1}{2} \times 2 \times (1)^2 = \mathbf{3\ m.}$$

Hence answer is 3.

7. [6]: Area under the acceleration-time graph represents the change in velocity. Area above the time axis is taken as positive and below the time axis, it is taken as negative. To acquire initial velocity again, change in velocity must be zero. If we can extend the graph for 6 s then we can see that total area becomes zero due to equal amounts of positive and negative areas. Hence particle acquires back its initial velocity at $t = \mathbf{6\ s.}$

Answer is 6.

8. [3]: $t = \alpha x^2 + \beta x$

$$\Rightarrow \frac{dt}{dx} = 2\alpha x + \beta$$

Hence instantaneous velocity of the particle can be written as follows:

$$\Rightarrow \quad v = \frac{dx}{dt} = \frac{1}{2\alpha x + \beta} \qquad \ldots(1)$$

We can differentiate the above relation of velocity to get the acceleration.

$$\Rightarrow \quad a = \frac{dv}{dt} = -\frac{1}{(2\alpha x + \beta)^2} \times 2\alpha \times \frac{dx}{dt}$$

In above relation dx/dt is instantaneous velocity v and also we can use equation (1) to rewrite the above relation as follows:

$$\Rightarrow \quad a = -v^2 \times 2\alpha \times v$$

$$\Rightarrow \quad a = -\alpha v^3$$

We can see that $n = \mathbf{3}$.

Hence answer is 3.

9. [3]: Height above the ground attained by balloon in time T can be written as follows:

$$h = \frac{1}{2}\left(\frac{g}{n}\right)T^2 \qquad \ldots(1)$$

Velocity attained by the balloon in time T will be same as initial velocity of stone.

$$v = u + at \Rightarrow v = 0 + \frac{g}{n}T = \frac{gT}{n}$$

Let us assume downward direction as positive direction for the stone. We can write the following for the stone.

Initial velocity: $u = -\dfrac{gT}{n}$

Displacement: $s = h = \dfrac{1}{2}\left(\dfrac{g}{n}\right)T^2$

Acceleration: $a = g$

Time: $t = T$

Equation: $S = ut + \frac{1}{2} at^2$

$$\Rightarrow \quad \frac{gT^2}{2n} = \left(-\frac{gT}{n}\right)T + \frac{g}{2}T^2$$

$$\Rightarrow \quad \frac{1}{2n} = -\frac{1}{n} + \frac{1}{2}$$

$$\Rightarrow \frac{3}{2n} = \frac{1}{2} \Rightarrow n = \mathbf{3}$$

Hence answer is 3.

NCERT Exemplar Problems: MCQs Type-I

3.1. (*b*): In graph (*b*), for one value of x, there are two values of t.

The correct option is (*b*).

3.2. (*c*): So $x > 0$, $v < 0$, before reaching 4th floor lift is retarded, i.e. acceleration is opposite to velocity and hence it is directed upward, so $a > 0$.

The correct option is *(c)*.

3.3. (*b*): In time T, maximum distance travelled = v_0T.

For one-dimensional motion, the displacement x and T satisfy $-v_0T < x < v_0T$.

The correct option is *(b)*.

3.4. (*c*): Time taken to travel a distance of $L/2$ with velocity V_1 is

$$t_2 = \frac{L/2}{V_1} \text{ and}$$

Time taken to travel a remaining distance of $L/2$ with velocity V_2 is

$$t_2 = \frac{L/2}{V_2}$$

$$\text{Average velocity, } v_{av} = \frac{L}{t_1 + t_2} = \frac{L}{\frac{L}{2}\left[\frac{1}{V_1} + \frac{1}{V_2}\right]}$$

$$= \frac{2V_1V_2}{V_1 + V_2}.$$

The correct option is *(c)*.

3.5. (*b*): Given, Displacement, $x = (t - 2)^2$

$$\text{Velocity, } v = \frac{dx}{dt} = 2\,(t - 2)\ \text{m s}^{-1}$$

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Competition File

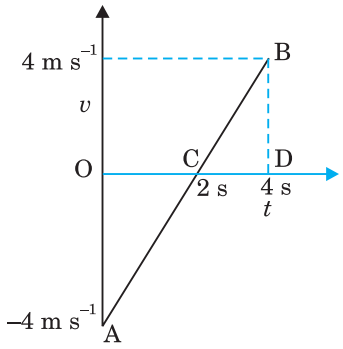
Acceleration, $a = \frac{dv}{dt} = 2 \text{ m s}^{-2}$

At $t = 0, v = -4 \text{ m s}^{-1}$

At $t = 2, v = 0$

At $t = 4, v = 4 \text{ m s}^{-1}$

Velocity-time graph is as shown in the figure.



Distance travelled = Area AOC + Area BCD

$$= \frac{1}{2}(4 \times 2) + \frac{1}{2}(4 \times 2)$$

$$= 8 \text{ m}.$$

The correct option is (b).

3.6. (c): Let L be the total distance travelled on the escalator.

Velocity of girl, $vg = \frac{L}{t_1}$

Velocity of escalator, $va = \frac{L}{t_2}$

Relative velocity of girl w.r.t. escalator

$$= vg + va$$

$$= \frac{L}{t_1} + \frac{L}{t_2}$$

If t is the time taken by her to walk up on the moving escalator, then

$$\frac{L}{t} = \frac{L}{t_1} + \frac{L}{t_2}$$

or $$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$$

or $$t = \frac{t_1 t_2}{t_1 + t_2}$$

The correct option is (c).

NCERT Exemplar Problems: MCQs Type-II

3.7. (a, c, d): Option (a) is correct as quantity B may represent time.

Here option (b) is wrong because for uniform motion, $v - t$ graph is simply a straight line parallel to the time axis (velocity remains constant in uniform motion). If the motion is uniform, displacement-time graph is a straight line with a positive slope. Thus, option (c) can be correct. Also, if the motion is uniformly accelerated (acceleration is constant), this implies that velocity is increasing with time and hence, velocity-time graph is a straight line with a positive slope. Thus, option (d) is correct.

The correct options are (a), (c) and (d).

3.8. (a, c, e): At point A, slope of graph is zero, so $v = 0$.

At B, $x - t$ graph is parallel to time axis, so $v = 0$ and $a = 0$.

At C there is change in slope, so v and a vanish

Between A and D, average velocity is negative

Also, slope at D > slope at E

So, speed at D will be more than at E.

The correct options are (a), (c) and (e).

3.9. (a, d): Since $x = t - \sin t$

$v = 1 - \cos t$

and $a = \sin t$

$x(t) > 0$, for all $t > 0$

At $t = 0$, $v(t) = 0$. So, $v(t)$ can be zero for one value of t

At $t = 0$ and π, $a(t) = 0$

If $t = 0$, $v(t) = 0$, if $t = \pi$, $v(t) = 1 - (-1) = 2$

So $v(t)$ lies between 0 and 2.

Here we can see that $v(t)$ is also equal to zero at certain instants and hence is not always greater than zero. So option (b) is not correct.

The correct options are (a) and (d).

3.10 (a, c): For a stretched spring, restoring force $F = -kx$

Potential energy, P.E. $= \frac{1}{2}kx^2$

When particle is just released, displacement $x = x_{max}$

$$\therefore \quad a = \frac{F_{max}}{m} = \frac{kx_{max}}{m} = \text{maximum}$$

At equilibrium, $x = 0$, so $a = 0$, and the P.E. is converted into K.E. and hence speed of mass is maximum at equilibrium position.

The correct options are (a) and (c).

3.11 (b, c, d): The ball is moving with the moving train. So, the direction of motion of the ball is same as that of the train.

Speed of the ball as seen from the ground before collision with the wall on the other side of the train = 10 + 1 = 11 m/s.

Speed of the ball after elastic collision with the wall = 10 – 1 = 9 m/s

The collision is perfectly elastic. So, momentum and kinetic energy are both conserved. So, average speed of the ball over an interval of 20 sec is fixed. The velocity of train is constant (= 10 m/s). So, the ball is in inertial frame and hence, the acceleration of the ball is the same as from the train.

The correct options are (b), (c) and (d).

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Chapter Practice Test

for Board Examination

Time Allowed: 2 hours **Maximum Marks: 35**

1. When can we say that circular motion is periodic? (1)

2. What are vector quantities? Give some examples. (1)

3. What is the use of speedometer? (1)

4. What does negative slope of a position-time graph signify? (1)

5. Is it possible for a uniformly accelerating object to change its direction of velocity? (1)

6. Define instantaneous acceleration with example. (2)

7. How are rest and motion relative? Explain with an example. (2)

8. What parameters can be determined from velocity-time graph of motion? (2)

9. Calculate the acceleration of an object if its velocity is given by $v = \left(\sqrt{12t - 2}\right)$. (3)

10. A stone is dropped from 100 m high cliff. Another stone is thrown upward from ground with a velocity 10 m/s. At what time both the stones will meet each other? (3)

11. A car is moving on a straight road. After covering a distance of 420 m in 18 s, it turns back and stops after 8 s half the way. Calculate the average velocity of the car in first 20 s. (3)

12. What is significance of (*a*) positive and (*b*) negative slope in distance-time graph of an object in uniformly accelerated motion? (3)

13. Derive the equations of motion for constant acceleration using method of calculus. (3)

14. Differentiate between (*a*) distance and displacement (*b*) speed and velocity. (3)

15. For an object moving along X-axis the position is given by $S = a + bt^2$. Calculate its velocity at time $t = 0$ and $t = 3$ s, if value of $a = 8$ m and $b = 3$ m s^{-2}. Also calculate the average velocity between time interval $t = 3$ s and $t = 6$ s. (3)

16. Briefly explain relative velocity of an object w.r.t. another with example. Represent the motion of two objects with (*a*) zero (*b*) non-zero relative velocity using position-time graph. (3)

► To check your performance, see HINTS AND SOLUTIONS TO SOME QUESTIONS at the end of Part I of the book.

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